The Synchrotron-Čerenkov Effect

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The rate of radiated photons per electron passing through a helium-filled Čerenkov counter was observed to oscillate as a function of the gas pressure when a weak, static magnetic field is applied. The data agree well with an analysis which treats Čerenkov and synchrotron radiation as limiting manifestations of a unified process.

When a charged particle traverses a magnetic field it emits synchrotron radiation. If that particle moves with velocity $v > c/n$ where n is the index of refraction of the surrounding medium then Čerenkov radiation occurs. What happens when a charged particle moves in both a refractive medium and a magnetic field?

The common perception of the two types of radiation is that they are quite distinct, and one might expect the corresponding intensities to add incoherently. But consider the situation very close to the Čerenkov threshold, when the Čerenkov angle, $\theta = \cos^{-1} c/nv$, is near $1/\gamma$, where $E = \gamma mc^2$ is the particle's energy. This angle is also characteristic of synchrotron radiation. If the magnetic field strength is such the the frequency spectra of the synchrotron and Čerenkov radiations also overlap it is impossible to determine which radiation process is responsible for the detected photons. In such a case an interference effect is likely, which has been observed for the first time in the experiment which is the subject of this report.[1]

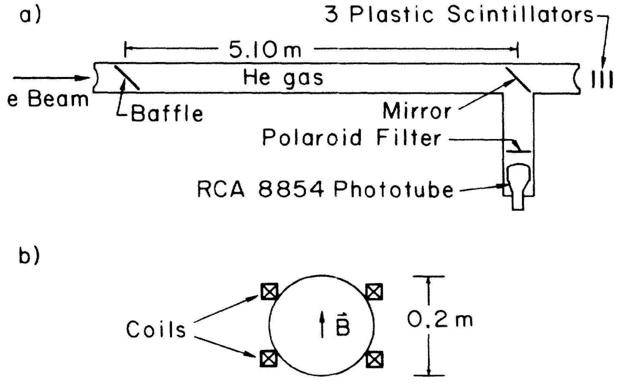

FIG. 1. **a)** Side view of the apparatus; **b)** view along the beam, showing the orientation of the magnetic field.

The detector, sketched in Fig. 1, consisted of a 5.1-m-long Čerenkov counter which was filled with helium gas at various pressures so as to vary the index of refraction and hence the Čerenkov angle. A weak magnetic field was applied transverse to the axis of the counter. As the Čerenkov angle was varied the angular distribution of Čerenkov radiation swept through that of synchrotron radiation and the total radiation rate was observed to oscillate. Čerenkov radiation is unpolarized on average, while synchrotron radiation is largely polarized perpendicular to the applied magnetic field, so the interference effect was more pronounced for the latter polarization.

Radiation emitted at any point along the electron's trajectory was collected in a single photomultiplier tube (RCA 8854). Light was reflected onto the face of the phototube by a plane mirror oriented at 45° to the beam. To insure that light emitted close to the mirror would be detected, the electron beam passed through the mirror. A Polaroid filter (Type HN38S) was placed before the phototube. The detector was sensitive to light of wavelengths from 390 to 520 nm (FWHM). This relatively narrow bandwidth served to minimize the effect of dispersion in the helium gas, enhancing the interference effect at some loss of rate. The gas pressure was monitored with a Datametrics Model 590A-100T-2Q1 transducer.

Electrons passing through the detector were counted in a coincidence of three scintillation counters. The flux through these counters was typically 10^4 per second, corresponding to a peak flux of 10^6 per second. This low intensity was obtained in a parasite mode using those electrons deflected by a 25-μm wire placed in the main beam.

Fig. 2 compares the data collected with 344-MeV electrons for the magnet on (at 51.3 gauss) and off. In both cases the photon polarization was perpendicular to the direction of the magnetic field lines. The solid circles are seen to oscillate about the positions of the squares, with three periods being resolved. The first minimum of the magnet-on data occurred at a pressure of 36 torr, at which the Čerenkov angle was $\sim 0.6/\gamma$.

The features displayed in Fig. 2 are confirmed in the data shown in Fig. 3, which were taken with 378-MeV electrons. For the latter, the magnetic field was always 56.4 gauss, but the Polaroid filter was oriented both parallel and perpendicular to the field lines. Again an oscillatory behavior is observed as a function of pressure for the perpendicular polarization, but the size of the oscillations relative to the case of parallel polarization is not as great as that relative to the magnet-off data shown in Fig. 2.

In the magnet-off data there appear to be about 8×10^{-5} detected photons per electron below Čerenkov threshold, independent of pressure. We attribute this to transition radiation at the surface of the baffle and the mirror which limit the region of light collection. The magnet-on data in Figs. 2 and 3 also tend to a finite intercept of about 8×10^{-5} detected photons per electron at zero pressure. We again attribute this primarily to transition radiation, which is calculated below to be a factor of ten stronger than synchrotron radiation in the rather weak magnetic field. We estimate that scintillation of the helium gas, which varies linearly with pressure, was about 1/4 as large as transition radiation just below Čerenkov threshold.

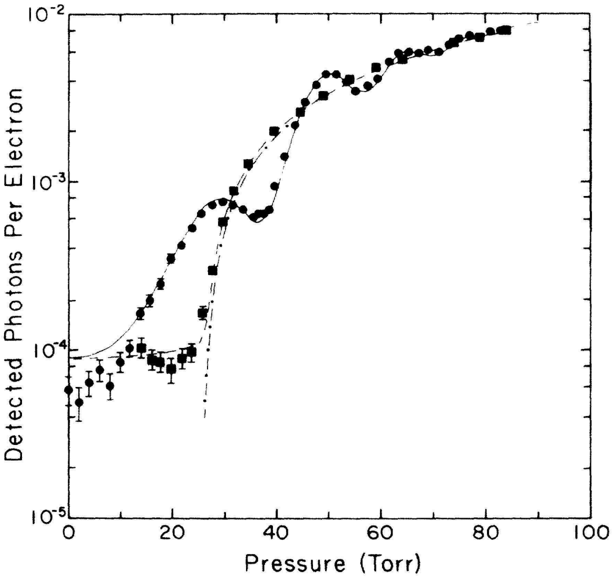

FIG. 2. The number of photons detected per 344-MeV electron. The solid circles (squares) represent data collected with a magnetic field of 51.3 (0) gauss. The Polaroid filter transmitted photons with polarization perpendicular to the direction of magnetic field lines. The three smooth curves are from calculations described in the text.

In all data taken with non-zero magnetic field there is a large enhancement of the radiation rate below Čerenkov threshold. This is reproduced by the detailed calculations described below, but can be understood qualitatively using the Huygens' construction familiar in discussions of Čerenkov radiation. For an electron with a circular trajectory in a magnetic field, the secondary wavelets superimpose to form a sharp wavefront on the inside of the circle even for velocities slightly below c/n.

The dash-dot curve in Fig. 2 shows the usual dependence of the Čerenkov radiation rate as a function of gas pressure:

$$\frac{d^2N}{d\hbar\omega\, dL}\left(\frac{\text{photons}}{\text{eV} - \text{m}}\right) = 3.70 \times 10^4 \left(2\Delta n - \frac{1}{\gamma^2}\right),$$

where $\hbar\omega$ is the photon energy, L is the radiator length, and Δn is the deviation of the index of refraction from 1. The shape of this curve agrees with the data only for pressures well above the Čerenkov threshold.

The interference effect observed here has been predicted some years ago by Erber and his collaborators.[2,3] Their calculation of the combined effects of Čerenkov and synchrotron radiation can be cast into practical units as the number of photons of polarization j emitted by an electron per meter of flight path and per eV of optical bandwidth:[4]

$$\frac{d^2N_j}{d\hbar\omega\, dL}\left(\frac{\text{photons}}{\text{eV} - \text{m}}\right) = 0.121\, y^{2/3}\, P_j(x).$$

Here E is is electron energy, H is the magnetic field,

$$x = 3.06 \times 10^5\, y^{-2/3}\left(2\Delta n - \frac{1}{\gamma^2}\right),$$

and

$$y = \frac{H(\text{kG})}{\hbar\omega(\text{eV})E(\text{GeV})}.$$

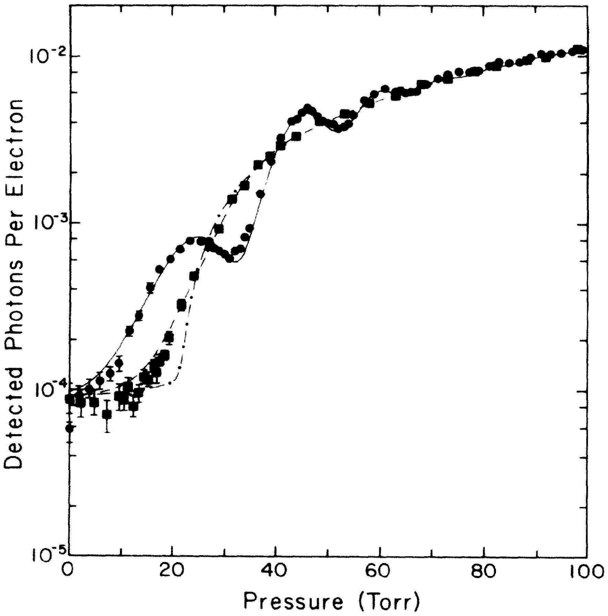

FIG. 3. The number of photons detected per 378-MeV electron. The solid circles (squares) represent photons with polarization perpendicular (parallel) to the 56.4-gauss magnetic field. The three smooth curves are from calculations described in the text.

The $P_j(x),\ j = 1, 2$ are related to Airy functions by

$$P_j(x) = \left(j - \tfrac{5}{2}\right)Ai'(-x) + \frac{x}{6} + \frac{x}{2}\int_0^x dt\, Ai(-t).$$

Subscript 1 (2) refers to polarization perpendicular (parallel) to the applied magnetic field. A plot of the functions $P_j(x)$ appears in ref. 4 and a table in ref. 5. When $\Delta n = 0$ the above expression reduces to the usual form for synchtrotron radiation, while at zero magnetic field it reduces to the previously stated form for Čerenkov radiation ($P_j(x) \to x/2$ at large x). Associated with the oscillatory behavior of the function P_1 is an oscillatory angular distribution of the radiation at fixed x,[6] that could not be explored in the present experiment.

It is interesting to note that this theory does not predict a simple interference effect of form $(A - B)^2$, where A and B are the amplitudes for the two kinds of radiation. Rather the formalism suggests that synchrotron and Čerenkov radiation are best described by a single theory, which yields the usual, and apparently distinct, results only in the limiting cases which have been investigated experimentally heretofore.

Photons were detected in our apparatus due to the simultaneous effects of Čerenkov, synchrotron and transition radiation. There is no detailed theory of this situation in the literature, so we have simply combined the calculation of the Čerenkov-synchrotron radiation described in the preceding paragraph, with the correction to Čerenkov radiation due to transition radiation according to Ref. 7. This is shown as the solid curve in Fig. 2. The predicted oscillations due to the Airy functions match the solid circles rather well.

The solid (dashed) curve in Fig. 3 is the prediction for Čerenkov-synchrotron + transition radiation for polarization perpendicular (parallel) to the magnetic field lines. Again the agreement with the data is quite good. The dash-dot curve shows the prediction for Čerenkov + transition radiation as would be observed for zero magnetic field. This is not in agreement with the data for polarization parallel to the magnetic field, which in fact oscillate about the expectations for zero field but without the appearance of local extremes. As synchrotron radiation is partially polarized parallel to the magnetic field lines, some interference with Čerenkov radiation of this polarization is to be expected, as found in the detailed calculations given above.

The interference effect observed in this experiment occurs very close the Čerenkov threshold, and only a single photon was detected in over 99% of the Čerenkov-coincidence events. This confirms the well-known fact,[8] that a classical analysis of optical interference actually predicts the behavior of individual photons. Also note that the deflection of the electrons in the magnetic field was only 1°, but the data agree well with an analysis of synchrotron radiation in terms of harmonics of the angular velocity of the electrons. The detector was long, however, compared to the 'formation length' $\sim R/\gamma$ for synchrotron radiation,[9] which was 34 cm for the present experiment.

The interference effect will have little impact on the use of Čerenkov counters as particle detectors, as these are normally operated at Čerenkov angles $\gg 1/\gamma$. The large enhancement of radiation below Čerenkov threshold could be important for a counter used as a beam-flux monitor,[10] if it were placed in a magnetic field. Note also that the position of, say, the first interference minimum is a known function of the index of refraction, but, unlike the Čerenkov threshold in zero magnetic field, is associated with a finite radiation rate. This feature may find application in precision studies of the indices of refraction of gases at ultraviolet frequencies.[4]

In a followup experiment we wish to study a closely related phenomenon. At high frequencies, any medium reponds like a plasma to an applied wave, and the index of refraction is less than 1. Clearly no Čerenkov radiation is possible at these frequencies, but there can still be a non-trivial snychrotron-Čerenkov effect.[3] At frequencies not too far above the plasma frequency there should be a very severe reduction in the synchrotron-radiation rate due to the presence of the medium. We might explore this by looking at radiation emitted at ~ 200 eV energy when 700-MeV electrons traverse a 1 T magnetic field, while varying the pressure of the helium gas. This effect may well have implications for astrophysical sources of synchrotron radiation.

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[1] K.D. Bonin *et al.*, Phys. Rev. Lett. **57**, 2264 (1986).
[2] J. Schwinger *et al.*, Ann. Phys. **96**, 303 (1976).
[3] T. Erber *et al.*, Ann. Phys. **102**, 405 (1976).
[4] T.M. Rynne, J. Appl. Phys. **52**, 6471 (1981).
[5] T.M. Rynne, Ph.D. dissertation, Ill. Inst. Tech. (1979), unpublished.
[6] T.M. Rynne *et al.*, J. Appl. Phys. **49**, 2233 (1978).
[7] L.L. DeRaad *et al.*, Phys. Rev. D **18**, 2152 (1978).
[8] G.I. Taylor, Proc. Camb. Phil. Soc. **15**, 114 (1909).
[9] R. Coïsson, Opt. Comm. **22**, 135 (1977); V.G. Bagrov *et al.*, Phys. Rev. D **28**, 2464 (1983).
[10] A. Bodek *et al.*, Z. Phys. C **18**, 289 (1983).