Observation of Interference between Čerenkov and Synchrotron Radiation

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The rate of radiated photons per electron passing through a helium-filled Čerenkov counter is observed to oscillate as a function of the gas pressure when a weak, static magnetic field is applied. The data agree well with an analysis which treats Čerenkov and synchrotron radiation as limiting manifestations of a unified process.

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Associated with the experimental discoveries of Čerenkov radiation[1] and synchrotron radiation[2], separate theoretical analyses were based on the different physical circumstances of the two phenomena.[3,4] In recent years, Erber and collaborators have noted in a series of papers[5−13] that when a relativistic charged particle traverses a dielectric medium subject to an applied magnetic field the resulting radiation is best considered as a single process which can be interpreted as Čerenkov radiation or synchrotron radiation only in limiting cases. Striking interference effects are predicted, which have been observed for the first time in the experiment described in this Letter.

Approximate conditions for interference can be deduced from the usual understanding of Čerenkov and synchrotron radiation. In both cases the energy for the radiation is derived from the charged particle, and not the surrounding medium. The small momentum transfers to the field and the medium are essentially unobservable. An interference effect might then be expected when the frequency and angular distributions of Čerenkov and synchrotron radiation are similar. As synchrotron radiation is emitted with angular spread $\sim 1/\gamma$, where $E = \gamma mc^2$ is the energy of the charged particle, interference can occur when the Čerenkov angle is also $\sim 1/\gamma$, which occurs close to the Čerenkov threshold for large γ. Then if the Čerenkov angle is varied the angular distribution of Čerenkov radiation sweeps through that of synchrotron radiation and the total radiation rate can be observed to oscillate. Čerenkov radiation is unpolarized on average, while synchrotron radiation is largely polarized perpendicular to the applied magnetic field, so the interference effect will be more pronounced for the latter polarization.

To observe this effect we have performed an experiment with 344- and 378-MeV electrons at the Bates Linear Accelerator Laboratory. The detector, sketched in Fig. 1, consisted of a 5.1-m-long Čerenkov counter which was filled with helium gas at various pressures so as to vary the index of refraction and hence the Čerenkov angle. A weak magnetic field was applied transverse to the axis of the counter by a pair of coils whose aspect ratio was chosen to provide maximum field uniformity. The resulting spectrum of synchrotron radiation peaked in the infrared, with a useful tail into optical frequencies.

The electron beam was deflected 7.5 cm horizontally by the magnetic field as it traversed the counter. To keep the electrons within the good-field region the downstream end of the counter was displaced by 3.75 cm from the field-off beam axis. The magnetic field was measured with a Rawson-Lush Model 784 digital gaussmeter to vary by no more than 1% within ±4 cm of the detector axis. The ends of the magnet coils were outside the active length of the counter to avoid the (interesting) complication of synchrotron radiation in a nonuniform magnetic field.[14]

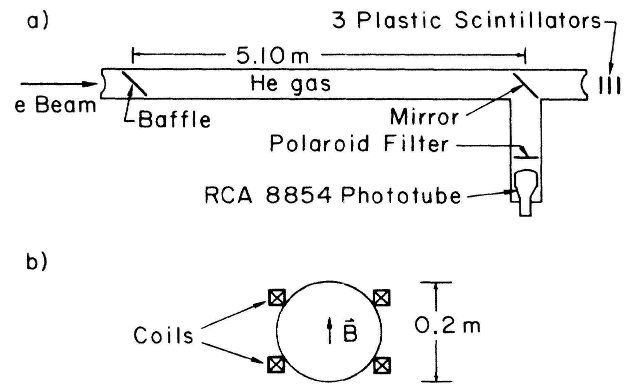

FIG. 1. **a)** Side view of the apparatus; **b)** view along the beam, showing the orientation of the magnetic field.

Radiation emitted at any point along the electron's trajectory was collected in a single photomultiplier tube (RCA 8854) with an active diameter of 14 cm. This phototube clearly resolved the signal from single photoelectrons. Light was reflected onto the face of the phototube by a plane mirror oriented at 45° to the beam. To insure that light emitted close to the mirror would be detected, the electron beam passed through the mirror. The upstream end of the radiator volume was defined by a baffle, also oriented at 45° to the beam. The interior of the detector was painted black to suppress the collection of light from scintillation of the helium gas[15] or from Čerenkov radiation in the quartz coating of the mirror.

A Polaroid filter (Type HN38S) was placed before the phototube, both of which were separated from the helium gas by a 12.5-mm-thick window of B270 glass. According to the manufacturers' specifications for the transmission of the Polaroid filter and spectral response of the phototube, the detector was sensitive to light of wavelengths from 390 to 520 nm (FWHM). This relatively narrow bandwidth served to minimize the effect of dispersion in the helium gas, enhancing the interference effect at some loss of rate.

The gas pressure was monitored with a Datametrics Model 590A-100T-2Q1 transducer, stabilized at 40 °C to maintain the stated accuracy of 0.05% of the reading. The temperature of the aluminum wall of the pressure vessel was monitored at several points with National Semiconductor LM135 temperature sensors, with accuracy about 0.3 °C. We assumed the temperature of the gas was the same as that of the wall, and have corrected all reported pressures to those at 20 °C that would have the same gas density.

Electrons passing through the detector were counted in a coincidence of three 2.5×2.5 cm^2 scintillation counters, which defined the *nominal* beam. The flux through these counters was typically 10^4 per second, corresponding to a peak flux of 10^6 per second, noting the 1% duty cycle of the accelerator. This low intensity was obtained in a parasite mode using those electrons deflected by a 25-μm wire placed in the main beam. The rate of double-electron triggers within our 25-ns coincidence window was monitored by means of a coincidence in which one of the three scintillator signals was delayed by 160 ns. The ratio of delayed to in-time coincidences was typically 3%, consistent with the stated flux and coincidence resolving time.

The transverse dimensions of the parasite electron beam were larger than those of the three beam-defining scintillators due to multiple scattering upstream of the detector. Typically only 20% of the total beam was within the nominal beam. Hence there was some probability that the Čerenkov signal was due to a second electron outside the nominal beam (or an electron ejected from the photocathode by thermionic emission), in accidental coincidence with an electron in the nominal beam. The rate of these events was monitored with a coincidence in which the Čerenkov signal was delayed with respect to the beam signal by 160 ns. At low pressures where the true signal was quite small, the rate of phototube-noise events was nearly equal that of the signal.

The reported photon signal was obtained by subtracting the number of Čerenkov-delayed coincidences from the in-time Čerenkov coincidences. The ratio of detected photons per electron, displayed in Figs. 2 and 3, was obtained by dividing the corrected photon rate by the rate of in-time beam electrons. No correction was made for the events with two electrons within the nominal beam, as this affects the numerator and denominator of the ratio in the same way. The errors shown are statistical, and are never less than about 3%.

The data were collected during a total of 24 hours of parasitic beam time. No on-line computer was used.

Fig. 2 compares the data collected with 344-MeV electrons for the magnet on (at 51.3 gauss) and off. In both cases the photon polarization was perpendicular to the direction of the magnetic field lines. The solid circles are seen to oscillate about the positions of the squares,[16] with three periods being resolved. The first minimum of the magnet-on data occurred at a pressure of 36 torr, at which the Čerenkov angle was $\sim 0.6/\gamma$.

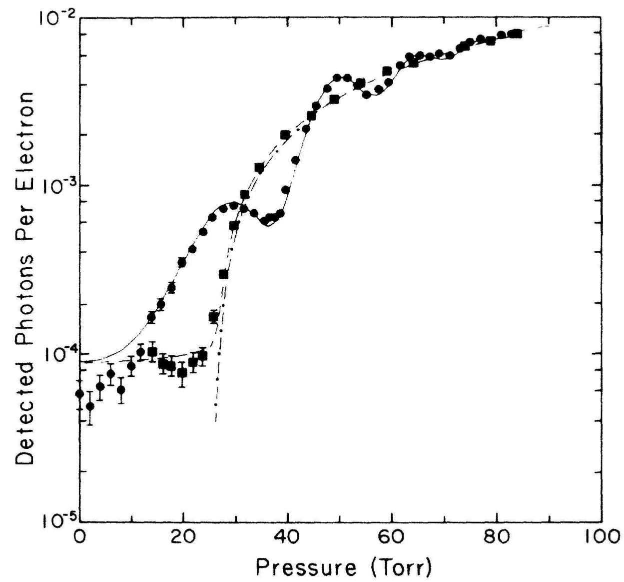

FIG. 2. The number of photons detected per 344-MeV electron. The solid circles (squares) represent data collected with a magnetic field of 51.3 (0) gauss. The Polaroid filter transmitted photons with polarization perpendicular to the direction of magnetic field lines. The three smooth curves are from calculations described in the text.

The features displayed in Fig. 2 are confirmed in the data shown in Fig. 3, which were taken with 378-MeV electrons. For the latter, the magnetic field was always 56.4 gauss, but the Polaroid filter was oriented both parallel and perpendicular to the field lines. Again an oscillatory behavior is observed as a function of pressure for the perpendicular polarization, but the size of the oscillations relative to the case of parallel polarization[17] is not as great as that relative to the magnet-off data shown in Fig. 2.

In the magnet-off data there appear to be about 8×10^{-5} detected photons per electron below Čerenkov threshold, independent of pressure. We attribute this to transition radiation at the surface of the baffle and the mirror which limit the region of light collection. The magnet-on data in figs. 2 and 3 all tend to a finite intercept of about 8×10^{-5} detected photons per electron at zero pressure. We again attribute this primarily to transition radiation, which is calculated below to be a factor of ten stronger than synchrotron radiation in the rather weak magnetic field. We estimate that scintillation of the helium gas,[15] which varies linearly with pressure, was about 1/4 as large as transition radiation just below Čerenkov threshold.

In all data taken with non-zero magnetic field there is a large enhancement of the radiation rate below Čerenkov threshold. This is reproduced by the detailed calculations described below, but can be understood qualitatively using the Huygens' construction familiar in discussions of Čerenkov radiation.[18] For an electron with a circular trajectory in a magnetic field, the secondary wavelets superimpose to form a sharp wavefront on the inside of the circle even for velocities slightly below c/n.

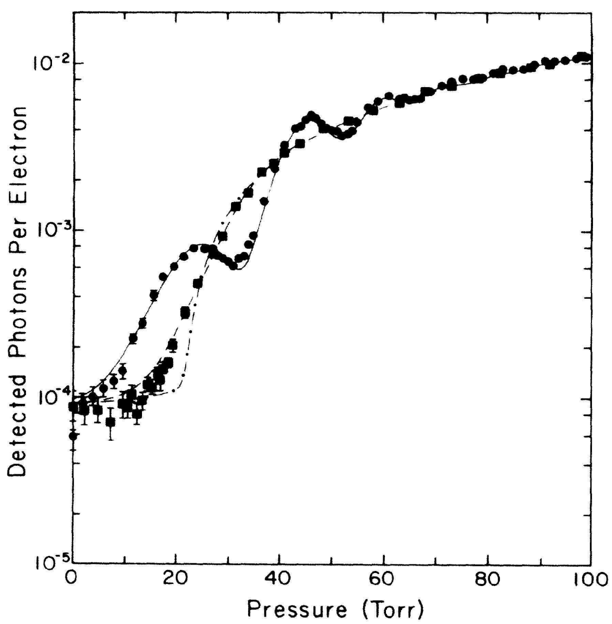

FIG. 3. The number of photons detected per 378-MeV electron. The solid circles (squares) represent photons with polarization perpendicular (parallel) to the 56.4-gauss magnetic field. The three smooth curves are from calculations described in the text.

The dash-dot curve in Fig. 2 shows the usual dependence of the Čerenkov radiation rate as a function of gas pressure:[7]

$$\frac{d^2N}{d\hbar\omega\, dL}\left(\frac{\text{photons}}{\text{eV}-\text{m}}\right) = 3.70\times 10^4\left(2\Delta n - \frac{1}{\gamma^2}\right),$$

where $\hbar\omega$ is the photon energy, L is the radiator length, and Δn is the deviation of the index of refraction from 1. The shape of this curve agrees with the data only for pressures well above the Čerenkov threshold.

This and the other curves are normalized to the data by the same scale factor, which accounts for the unknown detection efficiency as a function of photon energy. This factor is however 1/4 as large as would be inferred from the statements of the manufacturers as to the phototube quantum efficiency, filter transmissivity, *etc.* We also find best agreement with the data if we scale the measured pressure downwards by 2.5% from the calibration provided by the manufacturer of the transducer.

For the case with magnetic field off, a contribution from transition radiation cannot simply be added to the Čerenkov radiation, as the distinction between the two is unclear near Čerenkov threshold.[19] A detailed calculation[20] reveals two notable effects. First, extremely close to the Čerenkov threshold there is a logarithmic singularity in the radiation rate. This will be the subject of a future experiment. Second, at pressures below the nominal Čerenkov threshold there is an enhancement over the rate of transition radiation in vacuum, as illustrated by the dashed curve in Fig. 2. This can also be thought of as a diffraction effect arising when Čerenkov radiation is observed over a finite path length, and has been detected in two experiments.[21,22] The present experiment does not resolve these subtleties, but did detect radiation below Čerenkov threshold at a rate which is consistent with that expected from transition radiation.

The classical prediction[6,7] as to the combined effects of Čerenkov and synchrotron radiation can be cast into practical units as the number of photons of polarization j emitted by an electron per meter of flight path and per eV of optical bandwidth:[13]

$$\frac{d^2N_j}{d\hbar\omega\, dL}\left(\frac{\text{photons}}{\text{eV}-\text{m}}\right) = 0.121\, y^{2/3}\, P_j(x).$$

Here E is is electron energy, H is the magnetic field,

$$x = 3.06\times 10^5\, y^{-2/3}\left(2\Delta n - \frac{1}{\gamma^2}\right),$$

and

$$y = \frac{H(\text{kG})}{\hbar\omega(\text{eV})E(\text{GeV})}.$$

The $P_j(x),\ j = 1, 2$ are related to Airy functions by

$$P_j(x) = \left(j - \tfrac{5}{2}\right)Ai'(-x) + \frac{x}{6} + \frac{x}{2}\int_0^x dt\, Ai(-t).$$

Subscript 1 (2) refers to polarization perpendicular (parallel) to the applied magnetic field. A plot of the functions $P_j(x)$ appears in Ref. 13 and a table in Ref. 12. When $\Delta n = 0$ the above expression reduces to the usual form for synchtrotron radiation, while at zero magnetic field it reduces to the previously stated form for Čerenkov radiation ($P_j(x) \to x/2$ at large x). Associated with the oscillatory behavior of the function P_1 is an oscillatory angular distribution of the radiation at fixed x,[9] that could not be explored in the present experiment.

Photons were detected in our apparatus due to the simultaneous effects of Čerenkov, synchrotron and transition radiation. There is no detailed theory of this situation in the literature, so we have simply combined the calculation of the Čerenkov-synchrotron radiation described in the preceding paragraph, with the correction to Čerenkov radiation due to transition radiation according to ref. 20. This is shown as the solid curve in fig. 2. The predicted oscillations due to the Airy functions match the solid circles rather well.

The solid (dashed) curve in fig. 3 is the prediction for Čerenkov-synchrotron + transition radiation for polarization perpendicular (parallel) to the magnetic field lines. Again the agreement with the data is quite good. The dash-dot curve shows the prediction for Čerenkov + transition radiation as would be observed for zero magnetic field. This is not in agreement with the data for polarization parallel to the magnetic field, which in fact oscillate about the expectations for zero field but without the appearance of local extremes. As synchrotron radiation is partially polarized parallel to the magnetic field lines, some interference with Čerenkov radiation of this polarization is to be expected, as found in the detailed calculations given above.

The interference effect observed in this experiment occurs very close the Čerenkov threshold, and only a single photon was detected in over 99% of the Čerenkov-coincidence events. This confirms the well-known fact,[23] that a classical analysis of optical interference actually predicts the behavior of individual photons. Also note that the deflection of the electrons in the magnetic field was only 1°, but the data agree well with an analysis of synchrotron radiation in terms of harmonics of the angular velocity of the electrons. The detector was long, however, compared to the 'formation length' $\sim R/\gamma$ for synchrotron radiation,[14] which was 34 cm for the present experiment.

The interference effect will have little impact on the use of Čerenkov counters as particle detectors, as these are normally operated at Čerenkov angles $\gg 1/\gamma$. The large enhancement of radiation below Čerenkov threshold could be important for a counter used as a beam-flux monitor,[21] if it were placed in a magnetic field. Note also that the position of, say, the first interference minimum is a known function of the index of refraction, but, unlike the Čerenkov threshold in zero magnetic field, is associated with a finite radiation rate. This feature may find application in precision studies of the indices of refraction of gases at ultraviolet frequencies.[13] The experiment

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[16] The numbers of detected photons for the magnet off have been multiplied by a factor 1.05 so as to approach the magnet-on results at high pressures. This factor could arise from position dependence of the phototube efficiency.

[17] The numbers of detected photons with parallel polarizations have been multiplied by a factor 1.04 so as to approach the results for perpendicular polarization at high pressures. This factor is likely due to differing reflectivity of the quartz-coated 45° mirror for the two polarizations.

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