A Maxwellian Perspective on Particle Acceleration

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The Newtonian View

A charge e of mass m in fields $\mathbf{E}$ and $\mathbf{B}$ feels the Lorentz force:

$$\mathbf{F} = \gamma m\mathbf{a} = e\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right) \qquad \text{(Gaussian units).}$$

$\Rightarrow$ Both $\mathbf{E}$ and $\mathbf{B}$ change the particle's momentum, but only $\mathbf{E}$ can change it's energy.

$\Rightarrow$ Integrate equation of motion to find everything you want to know...

Why do we need another perspective?

- Integration sometimes difficult.
- Forces sometimes obscure (Čerenkov radiation....).
- Useful to have a cross check.
- Useful to have a method for order-of magnitude estimation.

Personal motivation: Paradoxes of laser acceleration.

A Maxwellian Perspective

The key feature of particle acceleration is energy transfer between a charged particle and an electromagnetic field.

In Maxwell's view, the electromagnetic field stores energy:

$$U_{\text{field}} = \int \frac{\mathbf{E}^2 + \mathbf{B}^2}{8\pi} d\text{Vol},$$

and momentum:

$$\mathbf{P}_{\text{field}} = \int \frac{\mathbf{E} \times \mathbf{B}}{4\pi c} d\text{Vol},$$

and angular momentum:

$$\mathbf{L}_{\text{field}} = \int \frac{\mathbf{r} \times (\mathbf{E} \times \mathbf{B})}{4\pi c} d\text{Vol}.$$

Conservation laws $\Rightarrow$ if a particle gains energy (or $\mathbf{P}$ or $\mathbf{L}$) from the field, the field must lose an equal amount of energy (or $\mathbf{P}$ or $\mathbf{L}$).

$$\Delta U_{\text{gained by particle}} = -\Delta U_{\text{lost by field}}.$$

$\Rightarrow$ Understand particle acceleration by analysis of loss of field energy.

Separation of the Fields

We separate the electromagnetic fields into two parts:

- The **external** (or applied) fields. We suppose the sources of these field are not perturbed by the accelerating charge:

$$\Rightarrow U_{\text{ext}} = \int \frac{\mathbf{E}_{\text{ext}}^2 + \mathbf{B}_{\text{ext}}^2}{8\pi} d\text{Vol} = \text{constant}.$$

- The fields of the accelerating **charge**. These are characterized as near zone (quasistatic, Coulombic), induction zone, and far zone (radiation).

$$U_{\text{charge}} = \int \frac{\mathbf{E}_{\text{charge}}^2 + \mathbf{B}_{\text{charge}}^2}{8\pi} d\text{Vol} \to \infty,$$

as the classical radius of the charge goes to zero.

We perform a "classical renormalization", and declare $U_{\text{particle}} = U_{\text{mechanical}} + U_{\text{charge}}$ to be $\gamma m c^2$, and ignore the issue of changes in U_{charge}.

This ignores the radiated energy, and hence also the radiation reaction force.

The Interference Term

The part of the electromagnetic energy that changes to balance the energy gain of an accelerating particle is therefore the interference term:

$$U_{\rm int} = \int \frac{\mathbf{E}_{\rm ext} \cdot \mathbf{E}_{\rm charge} + \mathbf{B}_{\rm ext} \cdot \mathbf{B}_{\rm charge}}{4\pi} d{\rm Vol} = -\Delta U_{\rm gain}.$$

For a charged particle to gain energy from an external field, this interference term must be negative.

The interference term (and energy gain) will be zero if the polarization of the external fields is orthogonal to that of the fields of the charge.

In general, the interference term (and energy gain) will be zero unless the fields of the charge include a component at the same frequency as that of the external fields.

We can often evaluate the interference term after the charge has left the confined region of the external fields. Then the interference is due to the radiation fields of the charge that were trapped in the confined region.

“Spontaneous” and “Stimulated” Fields

The **spontaneous** fields of the charge are those that exist when the external fields are set to zero. These fields differ from those of an isolated charge due to the presence of walls or other media.

Important examples of nontrivial spontaneous fields are image fields, transition radiation and Čerenkov radiation.

The **stimulated** fields of the charge are the additional fields that arise due to the response of the charge to the external fields.

In the first approximation, the strength of the stimulated fields is proportional to the strength of the external fields.

“Linear” Acceleration and “Quadratic” Acceleration

In the view of most people, “particle acceleration” is an effect that is **linear** in the strength of the external fields.

But the interference energy,

$$U_{\text{int}} = \int \frac{\mathbf{E}_{\text{ext}} \cdot \mathbf{E}_{\text{charge}} + \mathbf{B}_{\text{ext}} \cdot \mathbf{B}_{\text{charge}}}{4\pi} d\text{Vol} = -\Delta U_{\text{gain}}.$$

will be linear in the external fields only if the fields of the charge are independent of the external fields.

Hence **linear** acceleration depends on the **spontaneous** part of the fields of the accelerating charge!

The interference between the external fields and the **stimulated** fields of the charge leads to **quadratic** acceleration (energy gain proportional to the square of the external field strength) and is usually ignored.

The Maxwellian Perspective

The Maxwellian perspective on particle acceleration emphasizes the spontaneous fields of the charge as it passes through the accelerating structure.

Detailed prescription: Calculate the particle's trajectory in the presence of the external fields. Then evaluate the spontaneous fields on that trajectory, but with external fields set to zero.

Impulse approximation: Evaluate the spontaneous fields for the particle's trajectory in the absence of the external fields.

The Lawson-Woodward "Theorem"

If there is no accelerating "structure", there is no **linear** acceleration.

More precisely, a charged particle cannot gain net energy that is proportional to the strength of electromagnetic fields with which it interacts in vacuum over a finite path length, if all other matter is so remote that the spontaneous fields of the charged particle are negligible.

This is "obviously" true in the impulse approximation of the Maxwellian perspective.

We may be able to give a subtle counterexample when we go beyond the impulse approximation.

Applications

- Acceleration by a static field is at the expense of the field energy of the charge + image charge.
- "Linear" acceleration in an RF cavity is due to interference between transition radiation (aperture radiation) and the cavity fields.
- The mechanical momentum and energy of an electron in a plane was is compensated by the interference between the near field of the electron and the plane wave.

  Once the electron leaves the wave the interference dies out, and no net acceleration is possible.
- Acceleration by a laser beam is most favorable via an "axicon" mode which has a nonzero longitudinal electric field on axis.

  However, in vacuum, with all mirrors, lenses, and gases far away, no acceleration is possible.

- An axicon laser beam in a "cavity" defined by mirrors can provide "linear" acceleration, due to interference between transition radiation and the laser beam.

  However, when cavity length $\gg \lambda$, the energy gain varies as $1/\gamma$, and any apertures in the mirrors must be smaller than $\gamma\lambda_{\rm laser}$.

- If a gas is added to the axicon laser cavity, "linear" acceleration occurs due to interfernce between Čerenkov radiation and the laser beam.

  The energy gain varies as $1/\gamma$, but apertures in the mirrors can be big.

- "Quadratic" acceleration by a transversely polarized laser beam is possible under very limited conditions, and is ineffective for relativistic particles.