Comment on "Coherent Acceleration by Subcycle Laser Pulses" by Bernhard Rau, T. Tajima, and H. Hojo, Phys. Rev. Lett. 78, 3310 (1997)

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Rau *et al.* remark that if a "unipolarlike" electromagnetic pulse existed, it could be used to transfer large amounts of energy to a charged particle that it overtakes in vacuum and far from other any material. An earlier work by Lai [1] made essentially the same point. Rau *et al.* then argue that there exists a solution to Maxwell's equations that has the desired unipolarlike form, at least in an interesting limit.

However, this cannot be, according to a rather simple argument. See also [2].

Any physical source of electromagnetic waves is bounded, so that source appears pointlike when viewed from a great enough distance. Then, energy conservation requires that the pulse energy density fall off as $1/r^2$, for distance r measured from some characteristic point within the source. Since the energy density is proportional to the square of the electromagnetic fields, we have the well-known result that the radiation fields from a bounded source fall off as $1/r$ far from the source.

This is in contrast to the static fields, which must fall off at least as quickly at $1/r^2$, far from a bounded source.

Now, consider the possibility of a unipolarlike pulse, *i.e.*, one for which the electric field components $E_i(\mathbf{r}, t)$ have only one sign at a fixed observation point $\mathbf{r}$ far from the source. Then, the time integral of at least one component of such a pulse would be nonzero:

$$\int E_i(\mathbf{r}, t)\, dt \neq 0. \tag{1}$$

A Fourier analysis of this component,

$$E_i(\mathbf{r}, \omega) = \int E_i(\mathbf{r}, t) e^{i\omega t} dt, \tag{2}$$

would have a nonzero value at zero frequency, $E_i(\mathbf{r}, \omega = 0) \neq 0$.

However, the quantity $E_i(\mathbf{r}, \omega = 0)$ would then be a static solution to Maxwell's equation, and so must fall off like $1/r^2$. This contradicts the hypothesis that $E_i(\mathbf{r}, t)$ represented an electromagnetic pulse from a bounded source, which must fall off as $1/r$.

Thus, a bounded source cannot emit a unipolar electromagnetic pulse.

This result does not hold for 1-dimensional electromagnetic waves, such as plane waves, which, strictly speaking, can only be generated by an unbounded source. A well-known pedagogic example [3] is based on this case.

If a charged particle were accelerated in only one direction, the associated radiation would be unipolar. However, unless the time integral of the acceleration is zero, the particle eventually moves arbitrarily far, and cannot be called a bounded source.

One can produce an electromagnetic pulse that consists almost entirely of a single central pulse of one sign. But, this pulse must include long tails of the opposite sign so that the time integral of the fields vanish at any point far from the source. This behavior is shown in the data of refs. [3-5] of Rau *et al.*, as well as in [4,5].

The fields discussed by Rau *et al.* are not unipolarlike. An error appears to have resulted in their neglect of the small, long tails that cancel the energy transfer from the peak region of the pulse. To be specific, the time integral of the expression for E_x given in eq. (2) of Rau *et al.* evidently vanishes, while the time integral of their expression (7) does not, although their (7) is claimed to be a special case of their (2). The factor $\exp[-k_0^2\sigma^2/4(1+\rho^2)]$ that appears in eq. (7) of Rau *et al.* actually should be $\exp[-k_0^2\sigma^2\rho^2/2(1+\rho^2)]$. With this correction, the time integral of the second, negative, and small but broad term in their eq. (7) cancels that of the first, positive, and narrow term. Then, their quantity A is also zero, and, by their argument, the energy gain of a particle in the proposed laser beam would be zero.

[1] H.M. Lai, Phys. Fluids **23**, 2373 (1980).
[2] G.G. Stokes, Phil. Mag. **34**, 52 (1849).
[3] R.P. Feynman, R.B. Leighton and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1964), sec. II-20.
[4] B.I. Greene *et al.*, Appl. Phys. Lett. **59**, 893 (1991).
[5] D. You *et al.*, Opt. Lett. **18**, 290 (1993).