FINAL EXAM 1980-81

PHYSICISTS ANIMAL = BOX TERRIER.

1) a) SURVIVAL ON DESERT $\Leftrightarrow$ WATER SUPPLY

TOTAL SUPPLY $\sim L^3$

4) RATE OF LOSS TO EVAPORATION $\sim$ SURFACE AREA $\sim L^2$

TIME OF SURVIVAL $\sim L^3/L^2 \sim \underline{L}$

b) STORED ENERGY $\sim L^3$

BUT 'HORSEPOWER' = RATE OF CONVERSION OF STORED ENERGY

TO MECHANICAL WORK. CONCEIVABLY MUSCLE MASS $\sim L^3$,

4) BUT THERE'S MORE TO POWER THAN JUST MUSCLES. FOR A

SUSTAINED EFFORT, YOU MUST EXHAUST SOME ENERGY

(YOU PANT & SWEAT). THE RATE OF THE EXHAUST LIMITS

THE RATE OF ENERGY CONVERSION $\Rightarrow$ RATE $\sim$ SURFACE AREA $\sim \underline{\underline{L^2}}$

[A HORSE HAS $L \sim 2 \cdot L_{MAN}$. $M_{HORSE} \sim 8 M_{MAN}$ IS WELL SATISFIED

THE HORSEPOWER OF A MAN IS CLOSER TO $1/4$ H.P THAN $1/8$ H.P]

c) IN OVERCOMING AIR RESISTANCE, YOU NEED POWER = Fv

AND $F \sim L^2 v$ (OR $L^2 v^2$)

8) SO $L^2 v^2 \sim$ POWER $\sim L^2 \Rightarrow \underline{v \sim L^0}$ INDEPENDENT OF L

(IF CLAIM POWER $\sim L^3$, $v \sim \frac{1}{\sqrt{L}}$ OR $\frac{1}{\sqrt[3]{L}}$

RUNNING UPHILL POWER $\sim Mgv \sim L^3 v$

$\Rightarrow v \sim 1/L$ (OR $v \sim L^0$ IF CLAIM POWER $\sim L^3$

d) IN A JUMP, SUSTAINED POWER IS NOT SO RELEVANT.

4) RATHER THE MAXIMUM FORCE EXERTED AGAINST THE GROUND

DETERMINES THE MAXIMUM HEIGHT. THE FORCE CAN BE

EXERTED WHILE THE C.M. MOVES THRU A DISTANCE $\sim L$

$\Rightarrow$ WORK DONE $\sim FL = Mgh \sim L^3 h$ h = HEIGHT OF JUMP.

THE MAXIMUM FORCE IS THAT WHICH WOULD ALMOST BREAK THE BONES. BONE STRENGTH $\sim$ CROSS-SECTIONAL AREA $\sim L^2$ SO $L^3 \sim L^3 h$

$\Longrightarrow$ h IS INDEPENDENT OF L

CATS, PEOPLE & HORSES ALL JUMP ABOUT THE SAME HEIGHT!

2 a)

6

$$\Delta P_{cm} = I = 2M V_{cm} \rightarrow \boxed{V_{cm} = \frac{I}{2M}}$$

$$\Delta L_{cm} = I\ell = \omega \cdot I_{cm}$$

$$I_{cm} = \frac{1}{12} 2M(2\ell)^2 = \frac{2}{3} M\ell^2 \Longrightarrow \boxed{\omega = \frac{3}{2}\frac{I}{M\ell}}$$

14 b) IN SOME SENSE, THE 'AVERAGE' BEHAVIOR OF THE JOINTED STICK MUST BE THE SAME AS IN PART a).

CERTAINLY BOTH STICKS WILL MOVE.

LAGRANGE'S METHOD IS HELPFUL.

SHORTER SOLUTION ON P 49

GENERALISED IMPULSE = Δ GEN. MOM.

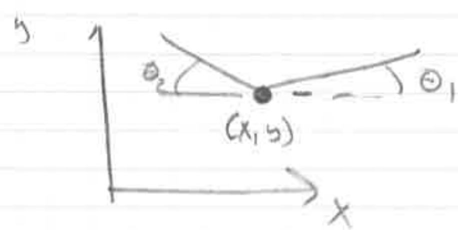
$$G_j = \frac{\partial T}{\partial \dot{q}_j}$$

$$G_j = \bar{I} \cdot \frac{\partial \bar{r}}{\partial q_j}$$

CHOOSE COORDS: 4 ARE NEEDED. A SIMPLE CHOICE IS

x, y OF PIVOT,

Θ_1, Θ_2 OF THE 2 RODS



$$T_1 = \frac{1}{2} m V_1^2 + \frac{1}{2} I_1 \dot{\theta}_1^2$$

$$v_1^2 = \dot{x}_1^2 + \dot{y}_1^2 \qquad x_1 = x + \frac{l}{2}\cos\theta_1 \qquad y_1 = y + \frac{l}{2}\sin\theta_1$$

$$\dot{x}_1 = \dot{x} - \frac{l}{2}\sin\theta_1\,\dot{\theta}_1 \qquad \dot{y}_1 = \dot{y} + \frac{l}{2}\cos\theta_1\,\dot{\theta}_1$$

$$v_1^2 = \dot{x}^2 + \dot{y}^2 + l\dot{\theta}_1(\dot{y}\cos\theta_1 - \dot{x}\sin\theta_1) + \frac{l^2}{4}\dot{\theta}_1^2$$

$$I_1 = \frac{1}{12} M l^2$$

$$T_1 = \frac{M}{2}\left(\dot{x}^2 + \dot{y}^2 + l\dot{\theta}_1(\dot{y}\cos\theta_1 - \dot{x}\sin\theta_1)\right) + \frac{1}{6} m l^2 \dot{\theta}_1^2$$

$$T_2 = \frac{M}{2}\left(\dot{x}^2 + \dot{y}^2 + l\dot{\theta}_2(-\dot{y}\cos\theta_2 + \dot{x}\sin\theta_1)\right) + \frac{1}{6} m l^2 \dot{\theta}_2^2$$

$$T = T_1 + T_2$$

THE GENERALISED MOMENTA ARE (SET $\theta_1 = \theta_2 = 0$ INITIALLY

$$P_x = \frac{\partial T}{\partial \dot{x}} = 2M\dot{x}$$

$$P_y = \frac{\partial T}{\partial \dot{y}} = 2M\dot{y} + \frac{Ml}{2}(\dot{\theta}_1 + \dot{\theta}_2)$$

$$P_{\theta_1} = \frac{\partial T}{\partial \dot{\theta}_1} = \frac{Ml}{2}\dot{y} + \frac{1}{3} M l^2 (\dot{\theta}_1)$$

$$P_{\theta_2} = \frac{\partial T}{\partial \dot{\theta}_2} = \frac{Ml}{2}\dot{y} + \frac{1}{3} M l^2 \dot{\theta}_2$$

THE GENERALISED IMPULSES ARE

$I_x = 0$, $I_y = I$, $r = (x + l\cos\theta_1, y + l\sin\theta_1)$ = END OF ROD 1

$$G_x = 0 = G_{\theta_2}$$

$$G_y = I\frac{\partial r_y}{\partial y} = I \qquad G_{\theta_1} = I\frac{\partial r_y}{\partial \theta_1} = Il$$

INITIALLY $\dot{x} = \dot{y} = \dot{\theta}_1 = \dot{\theta}_2 = 0$

SO THE FINAL VELOCITIES ARE

$$\underline{\dot{x} = 0}$$

$$2M\dot{y} + \frac{Ml}{2}(\dot{\theta}_1 + \dot{\theta}_2) = I$$

$$\frac{Ml}{2}\dot{y} + \frac{1}{3}Ml\dot{\theta}_1 = Il$$

$$\frac{Ml}{2}\dot{y} + \frac{1}{3}Ml^2\dot{\theta}_2 = 0$$

$$\Rightarrow \dot{\theta}_2 = -\frac{3\dot{y}}{2l}$$

$$\text{so } \frac{5}{4}\dot{y} + \frac{l\dot{\theta}_1}{2} = I/M$$

$$\frac{\dot{y}}{2} + \frac{1}{3}l\dot{\theta}_1 = I/M$$

$$\Delta = \frac{5}{12} - \frac{1}{4} = \frac{1}{6}$$

$$\dot{y} = \frac{\frac{1}{3} - \frac{1}{2}}{1/6}\frac{I}{M} = \boxed{-I/M = \dot{y}}$$

$$l\dot{\theta}_1 = \frac{\frac{5}{4} - \frac{1}{2}}{1/6} = \frac{3}{4}6 = \frac{9}{2}\frac{I}{M} \Rightarrow \boxed{\dot{\theta}_1 = \frac{9}{2}\frac{I}{Ml}}$$

$$\boxed{\dot{\theta}_2 = \frac{3}{2}\frac{I}{Ml}}$$

SINCE θ_2 DEFINED OPP TO θ_1

$$\dot{\theta}_{AVE} = \frac{\dot{\theta}_1 - \dot{\theta}_2}{2} = \frac{1}{2}\left(\frac{9}{2} - \frac{3}{2}\right)\frac{I}{Ml} = \underline{\underline{\frac{3}{2} I/Ml}}$$

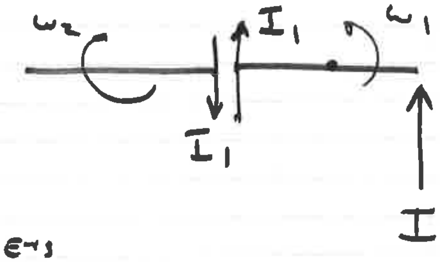
$$V_{cm} = \frac{1}{2}(\dot{y}_{1cm} + \dot{y}_{2cm}) = \frac{1}{2}\left(2\dot{y} + \frac{l}{2}(\dot{\theta}_1 + \dot{\theta}_2)\right) = \frac{1}{2}\left(-2\frac{I}{M} + \frac{l}{2}\frac{12}{2}\frac{I}{Ml}\right)$$

$$= \underline{\underline{\frac{I}{2M}}}$$

$$L_{PIVOT} = I_1\dot{\theta}_1 - I_2\dot{\theta}_2 = \frac{1}{3}Ml^2(\dot{\theta}_1 - \dot{\theta}_2) = \underline{Il}$$

$$\frac{\dot{\theta}_1 - \dot{\theta}_2}{2} = \frac{3I}{2Ml} = \omega$$

② b) URCIOLI METHOD



CM OF EACH ROD OBEYS

$$v_1 = \frac{I + I_1}{m} \qquad v_2 = -\frac{I_1}{m}$$

$$\omega_1 = \frac{\ell}{2}\frac{I - I_1}{N} \qquad \omega_2 = \frac{\ell}{2}\frac{I_1}{N} \qquad N = \frac{1}{12} m \ell^2$$

= MOMENT OF INERTIA OF ROD ABOUT ITS C.M.

CONSTRAINT : $V_{cm} = \frac{I}{2m} = \frac{m v_1 + m v_2}{2m} = \frac{v_1 + v_2}{2} = \frac{I}{2m}$ SATISFIED

ALSO PIVOT JOINS THE 2 ROD

$$\therefore\ v_{\text{PIVOT}} = v_1 - \frac{\ell}{2}\omega_1 = v_2 - \frac{\ell}{2}\omega_2$$

$$v_1 - v_2 = (\omega_1 - \omega_2)\,\ell/2$$

$$\frac{I + 2I_1}{m} = \frac{\ell^2}{4}\frac{(I - 2I_1)}{N}\ ;\ I + 2I_1 = \frac{m\ell^2}{4}\frac{(I - 2I_1)}{\frac{1}{12} m \ell^2}$$

$$I + 2I_1 = 3(I - 2I_1)$$

$$8 I_1 = 2I$$

$$\boxed{I_1 = \frac{I}{4}} \qquad v_1 = \frac{5}{4}\frac{I}{m} \qquad v_2 = -\frac{I}{4m}$$

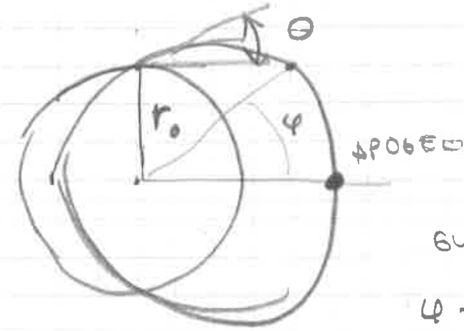
$$\boxed{\omega_1 = \frac{\ell}{2}\frac{\frac{3}{4} I}{\frac{1}{12} m \ell^2} = \frac{36}{8}\frac{I}{m\ell^2},\ \frac{9}{2}\frac{I}{m\ell}}$$

$$\boxed{\omega_2 = \frac{\ell}{2}\frac{I}{4N} = \frac{\omega_1}{3} = \frac{3}{2}\frac{I}{m\ell}}$$

$$\boxed{v_{\text{PIVOT}} = v_1 - \frac{\ell}{2}\omega_1 = \frac{5}{4}\frac{I}{m} - \frac{\ell}{2}\frac{9}{2}\frac{I}{m\ell} = -\frac{I}{m}} = -2V_{cm}$$

NOTE THAT THE PIVOT IS <u>NOT</u> THE CM ONCE THE RODS HAVE ROTATED.

3 a)



ORBIT IS ELLIPSE

$$\frac{1}{r} = \frac{1-\epsilon\cos\varphi}{A}$$

GUESS: TO 1ST ORDER IN θ

$\varphi = 90^\circ$ WHEN $r = r_0$

IE $A = \frac{1}{r_0}$ SO AT APOGEE $r = \frac{r_0}{1-\epsilon} \sim r_0(1+\epsilon)$

RELATE ϵ TO θ $\quad r \sim r_0(1+\epsilon\cos\varphi)$

$$dr = -r_0\,\epsilon\sin\varphi\, d\varphi$$

AT $\varphi = 90^\circ$, $\frac{dr}{r_0\,d\varphi} = -\epsilon =$ ANGLE BETWEEN ORBIT & THE CIRCLE $= -\theta$

IE $\epsilon = \theta$

APOGEE $\boxed{r = r_0(1+\theta)}$ ✓

MORE EXACT SOLUTION:

$$\frac{1}{r} = \frac{1-\epsilon\cos\varphi}{a(1-\epsilon^2)} \qquad \text{APOGEE} \Rightarrow r = a(1+\epsilon)$$

FACT: $a = -\frac{\alpha}{2E}$

WHEN $r = r_0$ v IS CORRECT FOR A CIRCULAR ORBIT

$$\frac{mv^2}{r_0} = \frac{GMm}{r_0^2} = \frac{\alpha}{r_0^2} \qquad mv^2 = \frac{\alpha}{r_0}$$

$$E = KE + PE = \tfrac{1}{2}mv^2 - \frac{GMm}{r_0} = \frac{\alpha}{2r_0} - \frac{\alpha}{r_0} = -\frac{\alpha}{2r_0}$$

$\therefore a = r_0 \Rightarrow$ APOGEE $= r_0(1+\epsilon)$

FACT: $e = \sqrt{1 + \frac{2EL^2}{m\alpha^2}}$

$L = m v r_0 \cos\theta$ BY DEFINITION OF θ

$$\epsilon = \sqrt{1 + \frac{2}{m\alpha^2}\left(-\frac{\alpha}{2r_0}\right) \cdot m^2 v^2 r_0^2 \cos^2\theta} = \sqrt{1 - \frac{2}{m\alpha^2}\left(\frac{\alpha}{2r_0}\right) m \left(\frac{\alpha}{r_0}\right) r_0^2 \cos^2\theta}$$

$= \sin\theta \Rightarrow$ APOGEE $= \boxed{r_0(1+\sin\theta)}$ EXACT!

b) $r = a(1+e)$

$$a = -\frac{\alpha}{2E} \qquad E = \frac{1}{2}mv^2 - \frac{GMm}{r_0}$$

$$E = \frac{1}{2}m(v_0+\epsilon)^2 - \frac{GMm}{r_0} \qquad v_0^2 =$$

$$// \approx \frac{1}{2}mv_0^2\left(1+\frac{2\epsilon}{v_0}\right) - \frac{\alpha}{r_0} \qquad mv_0^2 = \frac{\alpha}{r_0}$$

$$\frac{\alpha}{r_0}\left(\frac{(1+\epsilon/v_0)^2}{2} - 1\right)$$

$$\approx -\frac{\alpha}{2r_0}\left(1-\frac{2\epsilon}{v_0}\right) \implies a = r_0\left(1+\frac{2\epsilon}{v_0}\right)$$

$$L = m(v_0+\epsilon)r_0 = m r_0 v_0\left(1+\frac{\epsilon}{v_0}\right)$$

$$L^2 = m^2 r_0^2\left(1+\frac{2\epsilon}{v_0}\right)v_0^2 = m r_0 \alpha\left(1+\frac{2\epsilon}{v_0}\right)$$

$$e = \sqrt{1+\frac{2EL^2}{\mu\alpha^2}} = \sqrt{1+\frac{2}{m\alpha^2}\left(-\frac{\alpha}{2r_0}\right)\left(1-\frac{2\epsilon}{v_0}\right)mr_0\alpha\left(1+\frac{\epsilon}{v_0}\right)^2}$$

$$= \sqrt{1 - \cancel{\ }\left(1-\frac{2\epsilon}{v_0}\right)\left(\right.}$$

AGAIN, MUST KEEP HIGHER ORDER

$$e = \sqrt{1+\frac{2}{m\alpha^2}\frac{\alpha}{r_0}\left(\frac{(1+\epsilon/v_0)^2}{2}-1\right)mr_0\alpha\left(1+\frac{\epsilon}{v_0}\right)^2}$$

$$\sqrt{1 - \left[2-\left(1+\frac{\epsilon}{v_0}\right)^2\right]\left[1+\frac{\epsilon}{v_0}\right)^2}$$

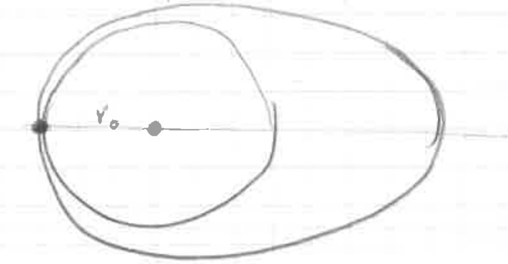
$$\sqrt{1 - 2\left(1+\frac{\epsilon}{v_0}\right)^2 + \left(1+\frac{\epsilon}{v_0}\right)^4}$$

$$\sqrt{1 - 2 - 4\frac{\epsilon}{v_0} - 2\frac{\epsilon^2}{v_0^2} + 1 + 4\frac{\epsilon}{v_0} + 6\frac{\epsilon^2}{v_0^2} \dots}$$

$$\sqrt{\frac{4\epsilon^2}{v_0^2}} \qquad e = \frac{2\epsilon}{v_0}$$

$$A = a(1+e) = r_0\left(1+\frac{2\epsilon}{v_0}\right)\left(1+\frac{2\epsilon}{v_0}\right) = \boxed{r_0\left(1+\frac{4\epsilon}{v_0}\right)}$$

b)



DIRECTION PROPER, WRONG $\Rightarrow$ INITIAL POSITION IS THE

PERIGEE $\quad \frac{1}{r} = \frac{1 - \epsilon \cos\varphi}{a(1-\epsilon^2)}$ IN GENERAL

PERIGEE $\Rightarrow \varphi = 180^\circ \quad \frac{1}{r_0} = \frac{1+\epsilon}{a(1-\epsilon^2)}$ or $r_0 = a(1-\epsilon)$

APOGEE $= a(1+\epsilon) = A$

L IS CONSERVED $\Rightarrow (v_0 + \epsilon) r_0 = v' A$

E IS CONSERVED $\Rightarrow \frac{1}{2}(v_0+\epsilon)^2 - \frac{GM}{r_0} = \frac{1}{2} v'^2 - \frac{GM}{A}$

AGAIN $\frac{m v_0^2}{r_0} = \frac{GMm}{r_0^2}$ OR $v_0^2 = \frac{GM}{r_0}$

$$\frac{1}{2}(v_0+\epsilon)^2 - v_0^2 = \frac{1}{2}(v_0+\epsilon)^2 \frac{r_0^2}{A^2} - \frac{v_0^2 r_0}{A}$$

$$A^2\left(\frac{\epsilon^2}{2} + \epsilon v_0 - \frac{v_0^2}{2}\right) + v_0^2 r_0 A - \frac{1}{2}(v_0+\epsilon)^2 r_0^2 = 0$$

$$A = \frac{-v_0^2 r_0 \pm \sqrt{v_0^4 r_0^2 + (v_0+\epsilon)^2 r_0^2 (\epsilon^2 + 2\epsilon v_0 - v_0^2)}}{(\epsilon^2 + 2\epsilon v_0 - v_0^2)}$$

TO 1ST ORDER IN ϵ

$$A = \frac{-v_0^2 r_0 \pm \overbrace{\sqrt{v_0^4 r_0^2 - (v_0^2 + 2\epsilon v_0) r_0^2 (v_0^2 - 2\epsilon v_0)}}^{0 \text{ TO ORDER } \epsilon}}{-(v_0^2 - 2\epsilon v_0)}$$

$$= \frac{-v_0^2 r_0 \pm r_0}{-v_0^2\left(1 - \frac{2\epsilon}{v_0}\right)} \sim \boxed{r_0\left(1 + \frac{2\epsilon}{v_0}\right) = A}$$

IF IGNORE HIGHER TERMS INSIDE $\sqrt{\ }$

MUST GO TO ORDER ϵ^2 INSIDE $\sqrt{\ }$!

$$A \cong \frac{-v_0^2 r_0 \left(1 \pm \sqrt{1 - \left(1 + \frac{2\epsilon}{v_0} + \frac{\epsilon^2}{v_0^2}\right)\left(1 - \frac{2\epsilon}{v_0} - \frac{\epsilon^2}{v_0^2}\right)}\right)}{-v_0^2(1 - 2\epsilon/v_0)}$$

$$= r_0\left(1 + \frac{2\epsilon}{v_0}\right)\left(1 \pm \sqrt{1 - \left[1 - \frac{4\epsilon^2}{v_0^2} - \epsilon^3 \cdot \epsilon^4\right.}\right) \sim r_0\left(1 + \frac{2\epsilon}{v_0}\right)\left(1 \pm \frac{2\epsilon}{v_0}\right) = \left\{\begin{array}{l} r_0 \\ \boxed{r_0\left(1 + \frac{4\epsilon}{v_0}\right)} \end{array}\right.$$

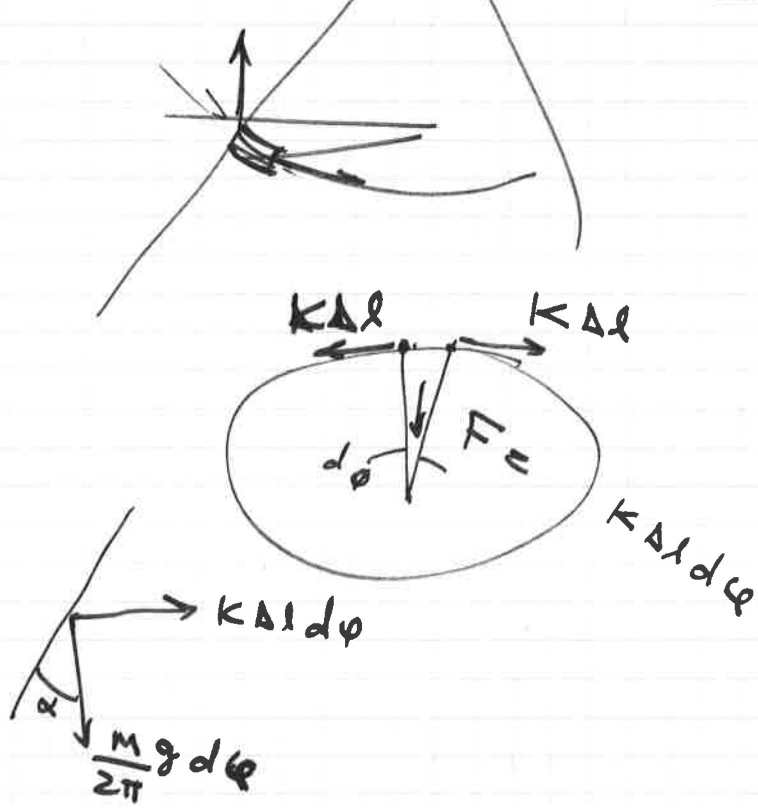
4

P.1

$\cancel{(l-l_0)\,K\cos\alpha\sin\alpha = mg\cos^2\alpha}$

$\cancel{l = l_0 + \frac{mg}{K}\cot\alpha = 2\pi r\sin\alpha}$

$\boxed{\text{F = MA METHOD}}$



TANGENTIAL COMP $\quad \frac{Mg\,d\varphi}{2\pi}\cos\alpha = K\,\Delta l\,d\varphi\,\sin\alpha$

$$\Delta l = \frac{Mg}{2\pi K}\cot\alpha = l - l_0$$

$$l = 2\pi r\sin\alpha = l_0 + \frac{Mg}{2\pi K}\cot\alpha$$

$$\boxed{r = \frac{l_0}{2\pi\sin\alpha} + \frac{Mg\cot\alpha}{4\pi^2 K\sin\alpha}}$$

OSCILLATION

$$\underbrace{\frac{m}{2\pi}d\varphi}_{dm}\ \ddot{r} = F_{\text{TANGENTIAL}} = \frac{mg\,d\varphi\cos\varphi}{2\pi} - K\,\Delta l\,d\varphi\sin\alpha$$

$$\Delta l = l - l_0 = (2\pi r\sin\alpha - l_0)$$

$$m\ddot{r}\frac{d\varphi}{2\pi} = -2\pi K r\sin^2\alpha\,d\varphi + \text{const.}$$

$$\ddot{r} = \frac{-4\pi^2 K\sin^2\alpha}{m} + \text{const}$$

$$\boxed{\omega = 2\pi\sin\alpha\sqrt{\frac{K}{m}}}$$

RAYLEIGH'S METHOD

$$\langle T \rangle = \langle V_{spring} \rangle$$

$$r = r_0 + A \cos \omega t$$

$$\dot{r} = -A\omega \sin \omega t$$

$$\langle T \rangle = \frac{1}{2}\frac{1}{2} M A^2 \omega^2$$

$$l = 2\pi \sin \alpha \, r$$

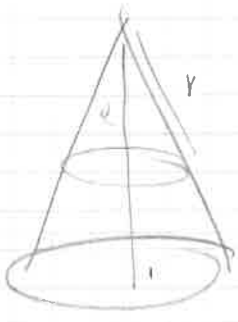
$$\Delta l = 2\pi \sin \alpha \, A \cos \omega t$$

$$\langle V \rangle = \frac{1}{2} \cdot \frac{1}{2} k \Delta l^2 = \frac{1}{4} (2\pi \sin \alpha)^2 A^2 k$$

$$\therefore \quad \omega^2 = (2\pi \sin \alpha)^2 \frac{k}{m}$$

\7

(4)



a) PRINCIPLE OF VIRTUAL WORK

$$PE_{GRAV} = -Mg\,r\cos\alpha$$

$$PE_{SPRING} = \frac{1}{2}K\left(2\pi r\sin\alpha - l_0\right)^2$$

$$\left.\frac{\partial PE}{\partial r}\right|_{r_0} = 0 \qquad \frac{\partial PE}{\partial r} = -Mg\cos\alpha + K\left(2\pi r_0\sin\alpha - l_0\right)2\pi\sin\alpha$$

$$\Rightarrow \quad 2\pi r_0\sin\alpha = l_0 + \frac{Mg\cos\alpha}{2\pi K\sin\alpha}$$

$$\boxed{r_0 = \frac{l_0}{2\pi\sin\alpha} + \frac{Mg\cos\alpha}{4\pi^2 K\sin^2\alpha}}$$

b) OSCILLATIONS LOWEST MODE $\Rightarrow$ WHOLE STRING MOVES VERTICALLY. USE LAGRANGE

$$T = \frac{1}{2}M\dot{r}^2 = \cancel{\frac{1}{2}M\dot{r}^2\cos^2\alpha}$$

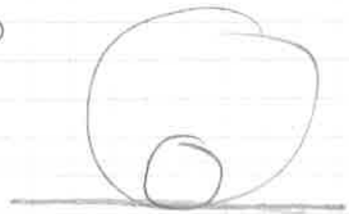
$$V = -Mg\,r\cos\alpha + \frac{1}{2}K\left(2\pi r\sin\alpha - l_0\right)^2$$

$$M\ddot{r} = Mg\cos\alpha - 2\pi K\sin\alpha\left(2\pi r\sin\alpha - l_0\right)$$

OSC. $\quad \ddot{r} + \dfrac{4\pi^2 K\sin^2\alpha}{M}\,r = \text{CONST}$

$$\boxed{\omega = 2\pi\sin\alpha\sqrt{\frac{K}{M}}}$$

(5)



ALL ROLLING WITHOUT SLIPPING

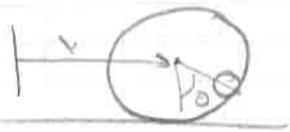
SINCE CONTACT IS ALONG A LINE

NO ROTATIONS ABOUT A VERTICAL AXIS

ARE POSSIBLE. ONLY ROTATION ABOUT HORIZONTAL AXIS

⇒ CONSTRAINTS ARE HOLONOMOUS

⇒ NEED ONLY 2 COORDS TO DESCRIBE THE SYSTEM.



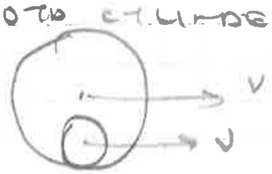
x = CM POS. OF LARGE CYLINDER.

θ = ANGLE OF CM OF SMALL CYLINDER TO VERTICAL THRU C.M. OF LARGE CYLINDER

⇒ 2 MODES.

BUT 1 MODE IS A ROLLING MODE $\omega_{osc} = 0$

BOTH CYLINDERS ROLL WITH THE SAME C.M. VELOCITY



$V_A = V_B$ $\quad \omega_A = \frac{V_A}{A}$ $\quad \omega_B = \frac{V_B}{B}$

OR $\omega_A = \frac{b}{a}\omega_B$

THE 2ND MODE IS OSCILLATORY

THE C.M. STAYS FIXED.

TO FIND THE FREQUENCY, WE USE LAGRANGE'S METHOD

$$T_A = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} I_A \omega_A^2 = M\dot{x}^2$$

$I_A = M a^2$, $\quad \omega_A = \dot{x}/a$

$V_A = 0$

$$T_B = \frac{1}{2} m v_B^2 + \frac{1}{2} I_B \omega_B^2$$

$x_B = x + (a-b)\sin\theta \qquad y_B = a - (a-b)\cos\theta$

$\dot{x}_B = \dot{x} + (a-b)\cos\theta\,\dot{\theta} \qquad \dot{y}_B = (a-b)\sin\theta\,\dot{\theta}$

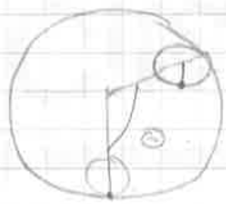
$$v_B^2 = \dot{x}^2 + 2(a-b)\,\dot{x}\cos\theta\,\dot{\theta} + (a-b)^2\dot{\theta}^2$$

$$I_B = m b^2$$

HARDEST PART IS $\omega_B = f(\dot{x}, \dot{\theta})$

SUPPOSE $\dot{\theta} = 0$ THEN $\omega_B = \frac{a}{b}\omega_A = \frac{b}{a}\frac{\dot{x}}{b}$ AS NOTED ABOVE

SUPPOSE $\dot{x} = 0$



$$a\theta = b(\theta_B + \theta)$$

$$\theta_B = \frac{a-b}{b}\theta$$

$$\omega_B = \frac{a-b}{b}\dot{\theta}$$

ALTOGETHER $\omega_B = \frac{b}{a}\frac{\dot{x}}{b} + \frac{a-b}{b}\dot{\theta}$

SO $T_B = \frac{1}{2}m\left(\dot{x}^2 + 2(a-b)\,\dot{x}\cos\theta\,\dot{\theta} + (a-b)^2\dot{\theta}^2\right) + \frac{mb^2}{2}\left[\frac{\dot{x}^2}{b^2} + \frac{2(a-b)\dot{x}\dot{\theta}}{b^2} + \frac{(a-b)^2\dot{\theta}^2}{b^2}\right]$

$$= m\dot{x}^2 + m(a-b)\,\dot{x}\dot{\theta}(1+\cos\theta) + m(a-b)^2\dot{\theta}^2$$

$$V_B = mg\,y_B = mg\left(a - (a-b)\cos\theta\right)$$

$$L = (M+m)\dot{x}^2 + m(a-b)\,\dot{x}\dot{\theta}(1+\cos\theta) + m(a-b)^2\dot{\theta}^2 + mg(a-b)\cos\theta + \text{const}$$

WE WANT SMALL OSCILLATIONS ABOUT $\theta = 0$

$$L \sim (M+m)\dot{x}^2 + 2m(a-b)\,\dot{x}\dot{\theta} + m(a-b)^2\dot{\theta}^2 + mg(a-b)\frac{\theta^2}{2} + K$$

$$\frac{\partial L}{\partial \dot{x}} = 2(M+m)\dot{x} + 2m(a-b)\dot{\theta}$$

SO $(M+m)\ddot{x} + m(a-b)\ddot{\theta} = 0$

$$\frac{\partial L}{\partial \dot{\theta}} = 2m(a-b)\dot{x} + 2m(a-b)^2\dot{\theta}$$

$$2m(a-b)\ddot{x} + 2m(a-b)^2\ddot{\theta} = \frac{\partial L}{\partial \theta} = -mg(a-b)\theta$$

$$\ddot{x} = -\frac{m}{M+m}(a-b)\ddot{\theta}$$

$$(a-b)^2\left(1-\frac{m}{M+m}\right)\ddot{\theta} = -\frac{g(a-b)}{2}\theta$$

$$\ddot{\theta} = \frac{g}{a-b}\,\frac{M+m}{2M}\,\theta$$

$$\boxed{\omega = \sqrt{\frac{g}{a-b}\,\frac{M+m}{2M}}}$$

$$X_{MAX} = -\frac{m}{M+m}(a-b)\,\theta_{MAX}$$

$$\theta_{A\,MAX} = \frac{X_{MAX}}{a} = -\frac{m}{M+m}\left(\frac{a-b}{a}\right)\theta_{MAX}$$

etc ...