Ph 205 Problem Set I

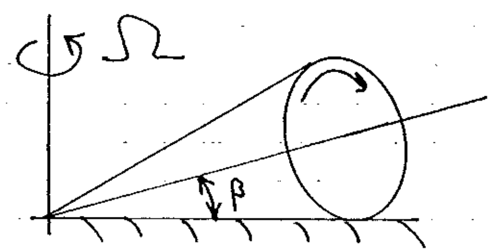
DUE: TUES. SEPT 27, 1988

- THIS COURSE IS ORIENTED TOWARDS PROBLEM SOLVING, AS WILL BE REFLECTED IN THE COURSE GRADE:

PROBLEM SETS	50%
FINAL EXAM	40%
LABORATORY	10%

- EACH PROBLEM WILL BE GRADED ON A BASIS OF 10 POINTS. ON THIS SET, A MAXIMUM SCORE OF <u>70 POINTS</u> WILL BE RECORDED. ANY EXTRA POINTS EARNED WILL <u>NOT</u> BE CARRIED OVER TO THE NEXT SET. YOU MAY EARN 70 POINTS VIA PARTIAL CREDIT ON ALL 10 PROBLEMS....

- ALTHOUGH SOME OF THE PROBLEMS ON THIS SET ARE DIFFICULT, THEY MAY ALL BE SOLVED BY ELEMENTARY METHODS.



(1) A CONE OF HALF ANGLE β ROLLS WITHOUT SLIPPING ON A PLANE. THE ANGULAR VELOCITY ABOUT AN AXIS PERPENDICULAR TO THE PLANE THRU THE POINT OF THE CONE IS Ω. WHAT IS THE ANGULAR VELOCITY ω OF THE CONE ABOUT THE INSTANTANEOUS AXIS OF ROTATION WHICH PASSES THRU THE POINT OF THE CONE? WHERE IS THE INSTANTANEOUS AXIS? ANS: $\omega = \Omega \cot \beta$.

(2) PROB 2 p 28 B&O. SUPPOSE THE CAR HAS MASS M, AND ITS CENTER OF MASS IS HEIGHT h ABOVE THE ROAD. LET ℓ_1 = <u>HORIZONTAL</u> DISTANCE FROM THE C.M. TO THE FRONT AXLE; ℓ_2 = DISTANCE FROM C.M. TO REAR AXLE. UNDER WHAT CONDITIONS CAN WHEELS LEAVE THE ROAD WHILE BRAKING ON A HORIZONTAL ROAD?

(3) PROB 14 p 81 B&O

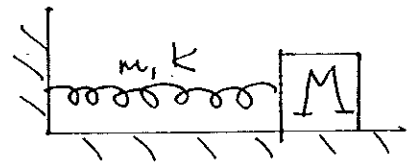
[SEE P. 5 FOR THE 'B&O' PROBLEMS]

(4) a) PROB 8 p 29 B&O

b) SUPPOSE THE CABLE IS BUNCHED UP NEAR THE EDGE OF THE TABLE, AGAIN WITH LENGTH ℓ_0 HANGING OVER THE EDGE INITIALLY. AS THE CABLE FALLS, IN EFFECT ONLY THE

PART HANGING OVER THE EDGE IS IN MOTION. FIND THE VELOCITY AS A FUNCTION OF THE LENGTH ℓ OVER THE EDGE. VERIFY THAT K.E. + P.E. IS NOT CONSTANT. WHERE HAS THE MISSING ENERGY GONE?

(5) A SPRING WITH SPRING CONSTANT K HAS MASS m. A BLOCK OF MASS M IS ATTACHED TO ONE END OF THE SPRING. THE OTHER END IS FIXED. WHAT IS THE PERIOD OF OSCILLATION? IGNORE FRICTION. HINT: CONSIDER THE ENERGY.



LATER IN THE COURSE WE WILL CONSIDER WAVES ON A MASSIVE SPRING AND FIND THAT THE SIMPLE RESULT OF THIS PROBLEM IS AN EXCELLENT APPROXIMATION.

(6) PROB 15, p196 B&O.

(7) VARIATION ON PROB 15, p109 B&O.

A SPHERE OF CROSS SECTIONAL AREA A MOVES THRU THE AIR WITH VELOCITY v. SUPPOSE THE DENSITY OF THE AIR IS ρ, AND THAT ALL MOLECULES HAVE THE SAME SPEED S. ASSUME ALL COLLISIONS BETWEEN AIR MOLECULES AND THE SPHERE ARE COMPLETELY INELASTIC.

a) IF $S \ll v$, SHOW $F_{DRAG} = \rho A v^2$

b) IF $S \gg v$ SHOW $F_{DRAG} \approx \rho A v S$. AS A SIMPLIFICATION, SUPPOSE 50% OF THE MOLECULES MOVE IN THE SAME DIRECTION AS THE SPHERE, AND 50% MOVE TOWARDS IT, HEAD ON.

c) REPEAT a) AND b) FOR ARBITRARY S, SUPPOSING THE MOLECULAR DIRECTIONS ARE ISOTROPICALLY DISTRIBUTED.

SHOW

IF $S \leq v$, $F_{DRAG} = \rho A\left(v^2 + \frac{2}{3}S^2 - \frac{1}{15}S^4/v^2\right)$

IF $S \geq v$, $F_{DRAG} = \rho A\left(\frac{4}{3}vS + \frac{4}{15}v^3/S\right)$

IN REAL LIFE, $F \sim v^2$ IS A GOOD APPROXIMATION EVEN WHEN $v < S$

(8) A SPHERICAL RAINDROP FALLS VERTICALLY DUE TO GRAVITY, (g = CONSTANT HERE), BUT EXPERIENCES A DRAG FORCE $\bar{F} = -K r^2 \bar{v}$, WHERE r = RADIUS OF DROP, $\bar{v}$ = VELOCITY.

a) IF THE RADIUS IS CONSTANT, FIND $v(t)$ OF THE DROP IF $v(0) = 0$. HOW DOES $v(t)$ BEHAVE FOR SMALL TIMES? I.e., IF $v(t) = g t(1-\epsilon)$, WHAT IS ϵ? FOR LARGE TIMES, HOW DOES THE VELOCITY DEPEND ON THE RADIUS OF THE DROP? I.e. WHAT IS $v_{TERMINAL}(r)$?

(8) b) SUPPOSE AT $t=0$ THE DROP HAS VELOCITY v_0 AND RADIUS r_0. IT THEN ENTERS A CLOUD AND THE DROP GAINS MASS ACCORDING TO $dm/dt = \alpha r^2 \sim$ SURFACE AREA.

DENSITY ρ REMAINS CONSTANT. NOW WHAT IS $v(t)$?

AS A SPECIAL CASE, SHOW THAT IF $r_0 \to 0$ AND $v_0 \to 0$, THEN

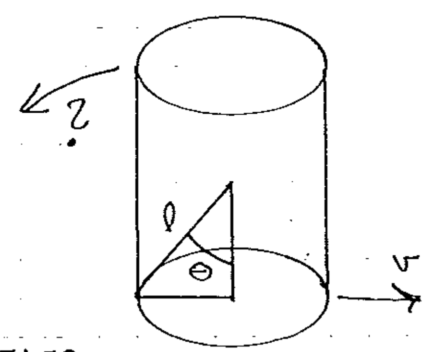
$$v = \frac{4\pi\rho g r}{4\alpha + 3K} = \frac{gt}{4 + 3K/\alpha} = \frac{v_{TERMINAL}(r)}{1 + 4\alpha/3K} < v_{TERMINAL}(r)$$

HINT: FIRST SHOW $dr/dt =$ CONSTANT, THEN REPLACE dv/dt BY dv/dr...

(9) GREEK TEMPLE SEISMOGRAPH. VARIATION ON PROB 5 p194 B&O.
IN THE 1850'S A STRONG EARTHQUAKE KNOCKED DOWN MANY COLUMNS OF GREEK TEMPLES. IN 1857 ONE R. MALLET SUGGESTED THAT THE VIOLENCE OF THE EARTHQUAKE COULD BE JUDGED FROM THE HEIGHTS OF THE COLUMNS LEFT STANDING.

CONSIDER A CYLINDER RESTING ON A FLAT, BUT PERFECTLY ROUGH SURFACE. SUDDENLY THE SURFACE IS JERKED SIDEWAYS (HORIZONTALLY) WITH VELOCITY v. SHOW THAT THE CYLINDER WILL FALL OVER IF

$$v^2 \geq \frac{g\ell}{6} \frac{(15 + \cos^2\theta)(1 - \cos\theta)}{\cos^2\theta}$$



ℓ = HALF DIAGONAL LENGTH.
ESTIMATE v_{MIN} TO TIP OVER A PERSON.

TO GET STARTED YOU MIGHT CONSIDER WHETHER THE CYLINDER WOULD FALL OVER IF A BULLET HIT IT, RATHER THAN AN EARTHQUAKE.

IF THE CYLINDER IS TALL ENOUGH WE MIGHT EXPECT THE BASE OF THE CYLINDER TO LEAVE THE GROUND WHILE IT'S FALLING OVER. SHOW THAT THIS IS TRUE IF

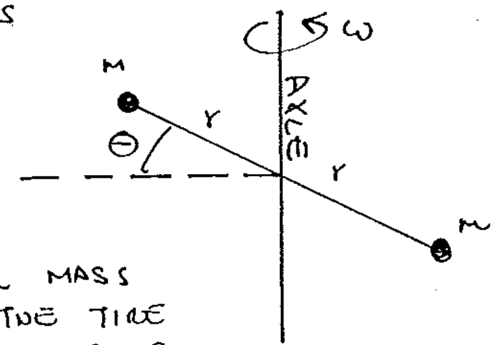
$$\cos\theta \geq \tfrac{1}{3} + \tfrac{1}{6}\sin\theta$$

WHEN $v = v_{MIN}$ IN THE ABOVE RELATION FOR v^2.

HINT: FIND A CONDITION THAT THE FORCE ON THE BASE VANISHES SOMETIME DURING THE MOTION. YOU SHOULD EVENTUALLY GET A QUADRATIC EQUATION IN $\cos\phi$, $\phi \equiv$ ANGLE OF THE DIAGONAL TO THE VERTICAL...
WE THEN FIND $\theta_{CRITICAL} \sim 61.3°$, SO THAT IF HEIGHT > 0.55 × DIAMETER THE BASE WILL LEAVE THE GROUND.

Ph 205 SET 1

(10) <u>UNBALANCED TIRE</u> SUPPOSE A TIRE IS "BALANCED" EXCEPT FOR 2 MASSES M EACH A DISTANCE r FROM THE CENTER ALONG A COMMON LINE WHICH MAKES ANGLE $90° - \Theta$ TO THE AXLE.



A GAS STATION ATTENDANT USING A STATIC BALANCE WOULD CLAIM THIS TIRE IS BALANCED.

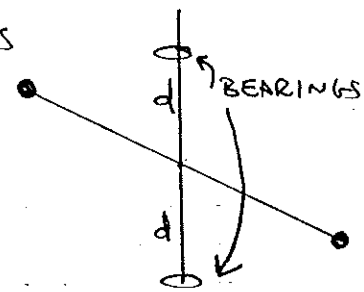
IN THIS PROBLEM IGNORE ALL OTHER MASS OF THE TIRE EXCEPT THE 2M. SUPPOSE THE TIRE IS <u>NOT</u> ROLLING, BUT IS IN A SPECIAL SET-UP WITH THE AXIS VERTICAL & THE C.M. FIXED – THE "BALANCING" DEVICE.

a) THE TIRE ROTATES ABOUT THE VERTICAL AXLE WITH CONSTANT ANGULAR VELOCITY ω. USE THE DEFINITIONS

$$\bar{L} = \sum_i \bar{r}_i \times \bar{p}_i \quad \text{AND} \quad \bar{N} = \frac{d\bar{L}}{dt}$$

TO FIND $\bar{L}$ AND $\bar{N}$.

b) SUPPOSE THE AXLE IS SUPPORTED BY TWO BEARINGS EACH A DISTANCE d FROM THE CM. WHAT FORCE IS EXERTED BY THE BEARINGS? IN WHAT DIRECTION? IGNORE GRAVITY.



c) SUPPOSE AT SOME MOMENT THE WHEEL BREAKS FREE FROM THE BEARINGS. DESCRIBE THE SUBSEQUENT MOTION RELATIVE TO THE C.M. IMAGINE THE C.M. REMAINS FIXED. IGNORE GRAVITY. WHAT IS THE PERIOD OF THE MOTION?

d) SUPPOSE THE WHEEL BOLTS ARE LOOSE SO THE ANGLE Θ IS FREE TO VARY, BUT THE WHEEL IS STILL FORCED TO ROTATE ABOUT THE AXLE WITH ANGULAR VELOCITY ω = CONSTANT. SHOW THAT Θ OSCILLATES ABOUT $\Theta = 0$ – THE WHEEL WOBBLES!

WHAT IS THE FREQUENCY OF OSCILLATION, SUPPOSING Θ_{MAX} IS SMALL? YOU MAY WISH TO CONSIDER THE FORCE IN THE 'SPOKE' JOINING THE MASS M TO THE CENTER OF THE WHEEL. SINCE Θ IS FREE TO VARY, NO FORCE CAN BE "TRANSMITTED" FROM THE AXLE TO M WITH COMPONENT IN THE Θ DIRECTION ...

15

Ph 205 Set 1

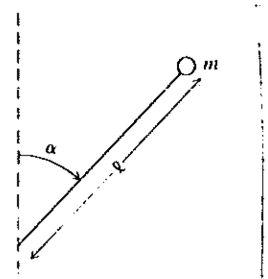
Problems From B & O

2

2. A vehicle has brakes on all four wheels. Find the deceleration which corresponds to maximum possible braking. Indicate why disk brakes are usually put on only the front wheels. (*Hint:* Calculate the normal forces on front and back wheels.)

3

14. A mass m is attached at one end of a massless rigid rod of length l, and the rod is suspended at its other end by a frictionless pivot, as illustrated. The rod is released from rest at an angle $\alpha_0 < \pi/2$ with the vertical. At what angle α does the force in the rod change from compression to tension?



4 9)

8. A perfectly flexible cable has length l. Initially, a length l_0 of the cable hangs at rest over the edge of a table. Neglecting friction, compute the length hanging over the edge after a time t.

6

15. A thin, uniform rod of mass M is supported by two vertical strings, as shown. Find the tension in the remaining string immediately after one of the strings is severed.