Box Toss

Kirk T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544
(October 17, 2011)

1 Problem

A person and a box are initially on a moving platform that has horizontal speed v_0. If the person throws the box off the back of the platform, the box is observed to have final horizontal speed $v_b = v_0/2$ with respect to the ground, in the same direction as the motion of the platform. If instead the person were to throw the box off the front of the platform, in the direction of its motion, what would be the final speed v_f of the box with respect to the ground? Ignore air resistance and friction between the platform and the ground, and suppose the person always ends up at rest with respect to the platform.

2 Solution

The key to this problem is that the person gives the box the same speed relative $\Delta v > 0$ to the final speed of the platform whether the box is thrown backwards or forwards.

Four frames of reference are relevant to this problem: the lab frame, the initial frame of the platform (= center-of-mass frame), and the final frames of the platform in the two cases of throwing of the box. I label the speed of the box as v and the speed of the platform plus person as V, adding the subscript b for final quantities when the box is thrown backwards, and adding subscript f for final quantities when the box is thrown forwards. Quantities measured in the frame(s) of the platform are indicated with a $'$. Thus, the signed final velocities of the box and platform in the final frame(s) of the platform are (ignoring air resistance),

$$v'_b = -\Delta v, \qquad v'_f = \Delta v, \qquad V'_b = 0 = V'_f. \tag{1}$$

The velocity transformations (Galilean relativity) from the lab frame (where the initial velocity of the box is v_0) to the final frames of the platforms are,

$$v'_b = v_b - V_b, \qquad v'_f = v_f - V_f. \tag{2}$$

Combining eqs. (1) and (2) we find that,

$$V_b = v_b - v'_b = v_b + \Delta v, \qquad V_f = v_f - v'_f = v_f - \Delta v. \tag{3}$$

However, we need more relations to solve the problem. For this, we note that momentum is conserved whenever the box is thrown, which implies that,

$$(m + M)v_0 = mv_b + MV_b = mv_f + MV_f. \tag{4}$$

Hence,

$$v_b = \frac{m+M}{m}v_0 - \frac{M}{m}V_b = \frac{m+M}{m}v_0 - \frac{M}{m}v_b - \frac{M}{m}\Delta v, \qquad \frac{m+M}{m}v_b = \frac{m+M}{m}v_0 - \frac{M}{m}\Delta v, \tag{5}$$

$$v_b = v_0 - \frac{M}{m+M}\Delta v, \tag{6}$$

and similarly,

$$v_f = \frac{m+M}{m}v_0 - \frac{M}{m}V_f = v_0 + \frac{M}{m+M}\Delta v, \tag{7}$$

From eq. (6) we obtain,

$$\Delta v = \frac{m+M}{m}(v_0 - v_b), \tag{8}$$

and then eq. (7) tells us that,

$$v_f = 2v_0 - v_b, \tag{9}$$

no matter what are the values of m and M.

In particular, if $v_b = v_0/2$, then $v_b = 3v_0/2$.

The velocity v_b could be negative in the lab frame, if the box is thrown hard. The transition case is $v_b = 0$, for which $v_f = 2v_0$.

Since v_0 is the center-of-mass velocity in this problem, eq. (9) has the same form as the relation between initial and final velocity of either mass in a one-dimensional elastic *collision, $v_{\mathrm{final}} = 2v_{\mathrm{cm}} - v_{\mathrm{initial}}$. It is peculiar that we obtain the same form in the present problem, which involves throwing, a kind of inelastic "collision/explosion", and the quantities v_b and v_f are the final velocities in two different "explosions".*