Magnetic Field in a Time-Dependent Capacitor

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1 Problem

Reconsider the classic example of the use of Maxwell's displacement current to calculate the magnetic field in the midplane of a capacitor with circular plates of radius R while the capacitor is being charged by a time-dependent current $I(t)$. In particular, consider the displacement current density, $\epsilon_0 \partial \mathbf{E}/\partial t$ in MKSA units for vacuum between the plates, to consist of a collection of small, close-packed "wires" that extend from one plate of the capacitor to the other. Sum the magnetic field from this set of virtual wires to find the total field.[1]

2 Solution

Aspects of this problem are considered in [1]-[28].

From the circular symmetry of the capacitor, and Ampère/Maxwell's law that,

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \left(\mathbf{J}_{\text{conduction}} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right), \tag{1}$$

in vacuum, we infer that the magnetic field is purely azimuthal, $\mathbf{B} = B(r,z)\hat{\phi}$, and independent of azimuth in a cylindrical coordinate system (r, θ, z) whose axis is that of the capacitor (and feed wires).

We suppose that the thickness of the capacitor is small compared to its radius R, so that we may approximate the electric field as being uniform and in the $\hat{\mathbf{z}}$ direction inside the capacitor, and zero outside. Then, we have $\mathbf{E} = E\,\hat{\mathbf{z}}$, and,

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\pi R^2 \epsilon_0} \qquad (r < R), \tag{2}$$

where the charge Q on the capacitor is related to the charging current according to $I = dQ/dt$. Hence, on the midplane of the capacitor eq. (1) becomes,

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \, \mathbf{J}_{\text{displacement}}, \tag{3}$$

where,

$$\mathbf{J}_{\text{displacement}} = \frac{I}{\pi R^2} \begin{cases} \hat{\mathbf{z}} & (r < r), \\ 0 & (r > R). \end{cases} \tag{4}$$

This is a quasi-static approximation that ignores the radiation of the accelerating charges in the circuit.

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[1] The "moral" of this problem is summed up in sec. A.7 below.

By considering a loop of radius r, centered on the origin and in the midplane of the capacitor, the integral form of eq. (3) immediately tells us that,

$$B(r) = \frac{\mu_0 I}{2\pi} \begin{cases} \frac{r}{R^2} & (r < r), \\ \frac{1}{r} & (r > R). \end{cases} \tag{5}$$

Outside the capacitor, the magnetic field has the same form as that of a wire which carries current I. Maxwell invented the concept of displacement current to insure that eq. (1) would lead to such results.

The particular challenge of the present problem is to imagine that the displacement current consists of small filaments, the sum of whose fields should give eq. (5).

The displacement current in a virtual wire of radius $a \ll R$ that runs between the plates of the capacitor is,

$$I_{\text{displacement}} = \pi a^2 J_{\text{displacement}} = I \frac{a^2}{R^2}. \tag{6}$$

The magnetic field due to this virtual wire circulates around the wire, and has magnitude,

$$B_{\text{virtual}} = \frac{\mu_0 I_{\text{displacement}}}{2\pi s} = \frac{\mu_0 I a^2}{2\pi R^2 s}, \tag{7}$$

where s is the perpendicular distance from the observer to the virtual wire. Strictly speaking, this form holds only for distance s small compared to the length of the virtual wire. However, having accepted that the electric field is uniform within the capacitor, we can use form (7) without further error.

We now must add up the contributions from all the virtual wires to find the total magnetic field at an arbitrary point in the midplane. Because of the azimuthal symmetry, we can take this point to be $(r, 0, 0)$, which lies on the x axis, without loss of generality. We expect the total magnetic at this point to be along the y axis, so we sum up only the y components of the fields (7) of the virtual wires.

The center of an arbitrary virtual wire is written as $(r', \theta, 0)$. The distance s from the observation point to the virtual wire is,

$$s = \sqrt{r^2 + r'^2 - 2rr' \cos\theta}, \tag{8}$$

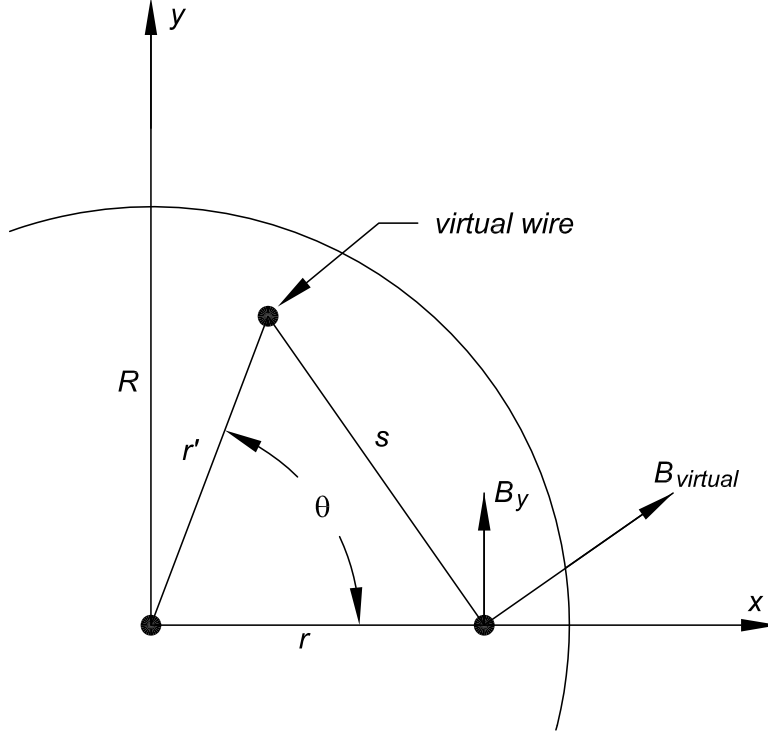as shown in the figure. The unit vector along the direction of s has components,

$$\hat{\mathbf{s}} = \frac{(r - r' \cos\theta, -r' \sin\theta)}{s}, \tag{9}$$

so that a unit vector perpendicular to $\hat{\mathbf{s}}$, which is in the direction of the magnetic field of the virtual wire, has components,

$$\hat{\mathbf{B}}_{\text{virtual}} = \frac{(r' \sin\theta, r - r' \cos\theta, 0)}{s}, \tag{10}$$

Combining eqs. (7) and (10), the y-component of the magnetic field of the virtual wire is,

$$B_{\text{virtual},y} = \frac{\mu_0 I a^2}{2\pi R^2 s} \frac{r - r' \cos\theta}{s} = \frac{\mu_0 I a^2 (r - r' \cos\theta)}{2\pi R^2 (r^2 + r'^2 - 2rr' \cos\theta)}. \tag{11}$$

We integrate over the area of the midplane, noting that an area element $r'dr'd\theta$ contains $r'dr'd\theta/\pi a^2$ virtual wires of radius a,

$$B(r,0,0) = \int_0^R dr' \int_0^{2\pi} \frac{r'd\theta}{\pi a^2} B_{\text{virtual},y}(s) = \frac{\mu_0 I}{2\pi^2 R^2} \int_0^R r'dr' \int_0^{2\pi} d\theta \frac{r - r'\cos\theta}{r^2 + r'^2 - 2rr'\cos\theta}.$$
(12)

The θ integral must be evaluated separately for the cases $r > r'$ and $r < r'$. For $r > r'$ we have,

$$\int_0^{2\pi} d\theta \frac{r - r'\cos\theta}{r^2 + r'^2 - 2rr'\cos\theta} = \frac{1}{r} \int_0^{2\pi} d\theta \frac{1 - \frac{r'}{r}\cos\theta}{1 + \frac{r'^2}{r^2} - 2\frac{r'}{r}\cos\theta} = \frac{2\pi}{r} \qquad (r > r'),$$
(13)

using Dwight 859.100, 859.101, and 859.122. Thus, for an observer outside the capacitor plates, we find,

$$B(r,0,0) = \frac{\mu_0 I}{\pi R^2 r} \int_0^R r'dr' = \frac{\mu_0 I}{2\pi r} \qquad (r > R),$$
(14)

in agreement with eq. (5).

For $r < r'$, the angular integral is,

$$\int_0^{2\pi} d\theta \frac{r - r'\cos\theta}{r^2 + r'^2 - 2rr'\cos\theta} = \frac{1}{r'} \int_0^{2\pi} d\theta \frac{\frac{r}{r'} - \cos\theta}{1 + \frac{r^2}{r'^2} - 2\frac{r}{r'}\cos\theta} = 0 \qquad (r < r').$$
(15)

That is, there is no contribution to the total magnetic field at radius r due to the virtual wires at radius $r' > r$, as could be anticipated from Ampère's law. Hence, for an observer at $r < R$ we integrate over the virtual wires with $r' < r$ to find,

$$B(r,0,0) = \frac{\mu_0 I}{\pi R^2 r} \int_0^r r'dr' = \frac{\mu_0 I r}{2\pi R^2} \qquad (r < R),$$
(16)

which is also in agreement with eq. (5).

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A Appendix. The Effects of Currents in the Capacitor Plates

Since the charge on the capacitor plates is time dependent, there must be currents flowing on those plates. What contribution, if any, do these "radial" currents make to the magnetic field?

A.1 Magnetic Field in the Plane of the Capacitor, but Outside It

One way to address this question is via Ampère's law, as illustrated in the figure below.
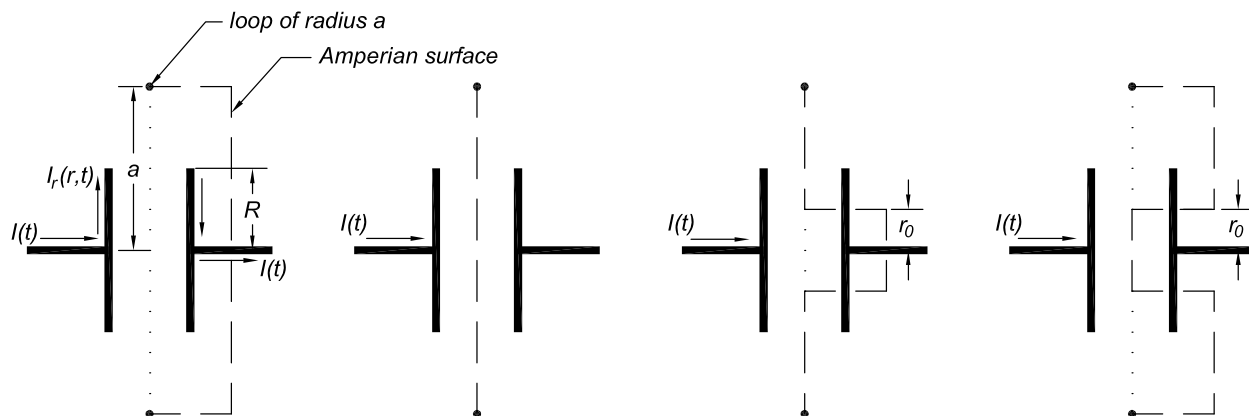
Ampère's law in integral form states that the integral of the tangential component of the magnetic field around a loop is equal to (μ_0 times) the current through the loop. To determine the current "through the loop", we consider a surface that is bounded by the loop and use the current that crosses that surface.

The left figure above shows a section through a capacitor with circular plates of radius R that is fed by a time-dependent current $I(t)$. We desire the magnetic field at radius $a > R$ on a circular loop located in the midplane of the capacitor. We first take the Ampèrian surface to be a kind of cylindrical "cup" such that the base of the cup is pierced by the righthand lead wire of the capacitor (left figure). Then the only current through the surface is the current I in the wire. The tangential component $B(a)$ of the magnetic field at radius a is independent of the position around the loop, so,

$$\oint B(a,t)\,dl = 2\pi a B(a,t) = \mu_0 I(t), \tag{17}$$

or,

$$B(a,t) = \frac{\mu_0 I(t)}{2\pi a}, \tag{18}$$

as noted earlier in eq. (5).

If we choose the Ampèrian surface bounded by the loop of radius a to be the disc in the plane of the loop (second figure), then no "real" (*i.e.*, conduction) current passes through the loop. However, since there is a time-dependent electric field at the disc, there is a non-zero displacement current across it, of, magnitude

$$I_{\mathrm{displ}}(t) = \int \mathbf{J}_{\mathrm{displ}} \cdot d\mathbf{Area} = \int \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{Area} = \epsilon_0 \frac{d}{dt}\frac{Q}{\pi R^2 \epsilon_0}\pi R^2 = \frac{dQ}{dt} = I(t), \tag{19}$$

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recalling eq. (2) for the electric field in a capacitor. Hence, use of the Ampèrian surface in the middle figure also leads to eq. (18) for the magnetic field at radius a.

We could also choose the Ampèrian surface to be that shown in the third figure on p. 4, which consists of the outer portion of the disc in the plane of the loop, plus a "cup" of radius r_0 whose bottom is pierced by the righthand lead of the capacitor. In this case, there are 3 contributions to the current through the surface:

1. The displacement current for radii $r > r_0$,

$$I_{\text{displ}}(r > r_0, t) = \int_{r > r_0} \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{Area} = \epsilon_0 \frac{d}{dt} \frac{Q}{\pi R^2 \epsilon_0} \pi (R^2 - r_0^2) = \left(1 - \frac{r_0^2}{R^2}\right) I(t). \quad (20)$$

2. The radial current $I_r(r_0, t)$ at radius r_0 on the capacitor plate.

3. The current $I(t)$ in the lead of the capacitor.

For Ampère's law to calculate the same magnetic field no matter which Ampèrian surface is used, Maxwell noted that the total current across any surface that is bounded by the same loop must be the same. Applying this result to the first and third surfaces, we have,

$$I(t) = I_{\text{displ}}(r > r_0, t) - I_r(r_0, t) + I(t), \quad (21)$$

where we use a minus sign for the term $I_r(r_0, t)$ since the radial current crosses the Ampèrian surface in the opposite sense to that of the current $I(t)$ in the lead wire. Thus, we conclude that,

$$I_r(r_0, t) = I_{\text{displ}}(r > r_0, t) = \left(1 - \frac{r_0^2}{R^2}\right) I(t). \quad (22)$$

When we consider the Ampèrian surface in the fourth part of the figure on p. 4, we see that,

$$I(t) = I_{\text{displ}}(r < r_0, t) + I_r(r_0, t), \quad (23)$$

and we infer that,

$$I_r(r_0, t) = I(t) - I_{\text{displ}}(r < r_0, t) = I(t) - \frac{\pi r_0^2}{\pi R^2} I(t) = \left(1 - \frac{r_0^2}{R^2}\right) I(t), \quad (24)$$

as found in eq. (22).

A.2   Direct Calculation of the Radial Current

We digress slightly to verify eq. (24) for the radial current by an approach that does not involve the displacement current.

Consider a ring of inner radius r and outer radius $r + dr < R$ on the righthand capacitor plate of the figure on p. 4. The area of this ring is $dA = 2\pi r\ dr$. Since the charge $Q(t)$ on the plate is uniformly distributed (in our approximation), the charge δQ on the ring is,

$$\delta Q(r, t) = Q \frac{dA}{A} = \frac{2rQ}{R^2} dr. \quad (25)$$

Conservation of charge tells us that the time rate of change of charge on the ring is equal to the difference of the radial currents at r and $r + dr$,

$$I_r(r + dr, t) - I_r(r, t) = \frac{d\delta Q}{dt} ,\qquad (26)$$

noting that the radial current is inwards on the righthand capacitor plate. Hence,

$$\frac{dI_r(r, t)}{dr} dr = \frac{2r\ dr}{R^2} \frac{dQ}{dt} = -\frac{2I(t) dr}{r} ,\qquad (27)$$

using eq. (25), and noting that when $I(t) > 0$ the charge on the righthand plate is decreasing with time. Thus,

$$\frac{dI_r(r, t)}{dr} = -\frac{2r I(t)}{R^2} ,\qquad (28)$$

which integrates to,

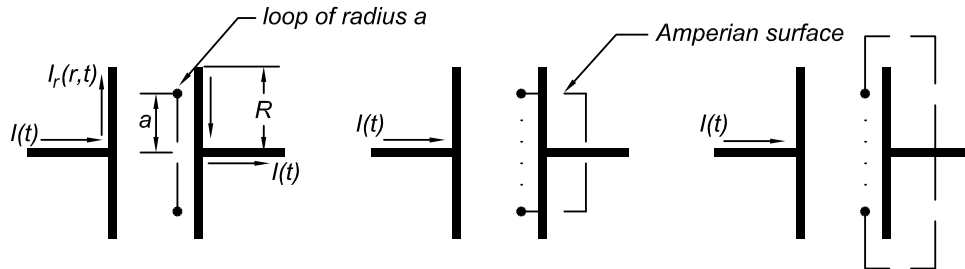$$I_r(r, t) = A - \frac{r^2 I(t)}{R^2} .\qquad (29)$$

Since the radial current vanishes when $r = R$ we see that $A = I$, and that,

$$I_r(r, t) = \left(1 - \frac{r^2}{R^2}\right) I(t),\qquad (30)$$

as found previously (for $r = r_0$).[2]

A.3   Magnetic Field Inside the Capacitor

We seek the value of the magnetic field at a radius $a < R$ inside the capacitor, as shown in the figure below.

[2]If the frequency of the sinusoidal current is high enough, the current flows only on the exterior of the conductors. In this case we can speak of the radial current $I_{r,\text{out}}(r)$ that flows outwards from the lead wire on the outside of a capacitor plate, and the current $I_{r,\text{in}}(r)$ that flows inwards from the outer edge onto the inside of the plate. If we continue to ignore fringe-field effects then there is no charge accumulation on the outside of the plate, while the charge distribution on the inside of the plate is uniform. Therefore, $I_{r,\text{out}}(r) = I$ (no charge accumulation on the outside of the plate). The inside current has the general form of eq. (29) (so as to provide a uniform charge accumulation on the inside of the plate), while at the outer rim of the plate we have $I_{r,\text{in}}(r) = A - I = -I_{r,\text{out}}(R) = -I$. Hence, we have $A = 0$, and,

$$I_{r,\text{out}}(r) = I, \qquad I_{r,\text{in}}(r) = -I\frac{r^2}{R^2} , \qquad I_{r,\text{total}}(r) = I\left(1 - \frac{r^2}{R^2}\right) .\qquad (31)$$

The magnetic field due to the currents on the (thin) capacitor plates is that due to $I_{r,\text{total}}(r)$, and hence is the same as that deduced by our other approaches.

We calculate the field three ways, using the three Ampèrian surfaces illustrated above.

First, we take the Ampèrian surface to be the disc bounded by the loop. Then, no conduction current passes through the surface, and we consider only the displacement current. Ampère's law tells us that,

$$\oint B(a,t)\, dl = 2\pi a B(a,t) = \mu_0 I_{\text{through}} = \mu_0 I_{\text{displ}}(r < a, t) = \mu_0 I(t) \frac{a^2}{R^2}. \qquad (32)$$

Thus,

$$B(a,t) = \mu_0 I(t) \frac{a}{2\pi R^2}, \qquad (33)$$

in agreement with eq. (5).

If we wish to avoid use of the displacement current, we can use the Ampèrian surface shown in the middle of the figure on p. 6, for which Ampère's law becomes,

$$2\pi a B(a,t) = \mu_0 I_{\text{through}} = \mu_0 [I(t) - I_r(a,t)] = \mu_0 I(t) \left[1 - \left(1 - \frac{a^2}{R^2}\right)\right] = \mu_0 I(t) \frac{a^2}{R^2}, \quad (34)$$

so the magnetic field is again given by eq. (33).

We could avoid use of the radial current by considering the Ampèrian surface in the right figure on p. 6. In this case Ampère's law tells us that,

$$2\pi a B(a,t) = \mu_0 I_{\text{through}} = \mu_0 [I(t) - I_{\text{displ}}(r > a, t)] = \mu_0 I(t) \left[1 - \left(1 - \frac{a^2}{R^2}\right)\right] = \mu_0 I(t) \frac{a^2}{R^2}, \qquad (35)$$

and again the magnetic field is given by eq. (33).

The magnetic field eq.(33) inside the capacitor is not equal to the magnetic field that exists at the same radius a about a wire when no capacitor is present. However, we cannot give a unique answer to the question as to why the field is different.

We can say that the field is due to the current in the wire plus the radial current in the capacitor plates. Or, we can say that the field is due to the displacement current at radius $r < a$. Or, we can say that the field is due to the current in the wire plus (actually minus) the displacement current at radius $r > a$.[3]

A.4 Magnetic Field Outside the Capacitor at $a < R$

We seek the value of the magnetic field at a radius $a < R$, but outside the capacitor, as shown in the figure below.

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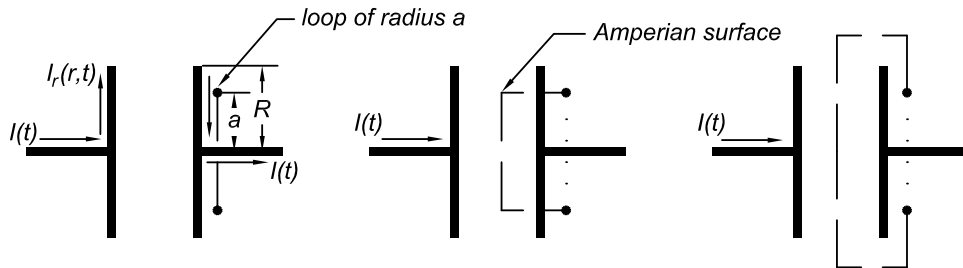
[3]The fact that multiple approaches to calculating the magnetic field give the same answer is a strength rather than a defect of the approach of Ampère and Maxwell, and should not be taken as a sign that the displacement current is fictitious.

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We calculate the field three ways, using the three Ampèrian surfaces illustrated above.

First, we take the Ampèrian surface to be the disc bounded by the loop. Then, the only current through the surface is the current in the wire lead. Hence, Ampère's law tells us that,

$$\oint B(a,t)\, dl = 2\pi a B(a,t) = \mu_0 I_{\text{through}} = \mu_0 I(t). \tag{36}$$

Thus,

$$B(a,t) = \mu_0 \frac{I(t)}{2\pi a}, \tag{37}$$

in agreement with eq. (5).

But, we can avoid direct use of the current in the capacitor lead by either of the second or third Ampèrian surfaces in the figure on p. 7. For the second surface, Ampère's law becomes,

$$
\begin{aligned}
2\pi a B(a,t) &= \mu_0 I_{\text{through}} = \mu_0 [I_{\text{displ}}(r<a,t) + I_r(a,t)] = \mu_0 I(t)\left[\frac{a^2}{R^2} + \left(1 - \frac{a^2}{R^2}\right)\right] \\
&= \mu_0 I(t),
\end{aligned} \tag{38}
$$

so the magnetic field is again given by eq. (37).

We could avoid use of the radial current by considering the Ampèrian surface in the right figure on p. 7. In this case, Ampère's law tells us that,

$$2\pi a B(a,t) = \mu_0 I_{\text{through}} = \mu_0 I_{\text{displ}}(r<R,t) = \mu_0 I(t), \tag{39}$$

and again the magnetic field is given by eq. (37).

A.5   Comments

Use of the third figure on p. 7 permits us to calculate the magnetic field anywhere around the capacitor without direct use of any conduction current. However, this should not be construed as evidence that time-dependent magnetic fields are only due to displacement currents, since we can equally well perform calculations that do not use the displacement currents. Rather, we follow Maxwell in noting that both conduction and displacement currents exist, and either or both types of currents may be useful in analyzing particular physical situations.

The calculations in secs. A.1 and A.3-4 have all been based on Ampère's law, which gives quick results in situation with high symmetry. This approach is not, however, practical in general. An alternative approach is to use what is sometimes called the retarded Biot-Savart law (see, for example, [29]),

$$\mathbf{B}(\mathbf{x},t) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}_{\text{cond}}(\mathbf{x}',t'=t-s/c)\times \hat{\mathbf{s}}}{s^2}\, d^3\mathbf{x}' + \frac{\mu_0}{4\pi c}\int \frac{\dot{\mathbf{J}}_{\text{cond}}(\mathbf{x}',t'=t-s/c)\times \hat{\mathbf{s}}}{s}\, d^3\mathbf{x}', \tag{40}$$

where $\mathbf{s} = \mathbf{x} - \mathbf{x}'$, $s = |\mathbf{s}|$, $\hat{\mathbf{s}} = \mathbf{s}/s$, and $\mathbf{J}_{\text{cond}}$ is the conduction current.[4]

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[4]This prescription is not evidence that the displacement current is nonexistent, but only that there are methods of calculation that don't use it.

A.6 Magnetic Field Inside a Narrow-Gap Capacitor Via the Biot-Savart Law

In the present example of a capacitor with a low-frequency current, retardation and time variation of the current can be ignored. In this case eq. (40) reduces to the familiar form of the Biot-Savart law. A detailed calculation via the Biot Savart law of the magnetic field in the gap between the capacitor plates requires use of the radial currents as well as the currents in the lead wires. The approximation of the present problem is that the electric field inside the capacitor is uniform, which corresponds to the gap between the plates being much smaller than their radius. In this approximation, the contributions to the magnetic fields from the radial currents in the two capacitor plates cancel for points outside the capacitor and the magnetic field is the same as that due to the wire leads, which value is unaffected by the small gap in the leads at the location of the capacitor.

The calculation of the field at a point inside a narrow-gap capacitor via the Bio-Savart law can be made as follows. The lead wires run along the z axis, and the capacitor plates lie in the planes $z = \pm\epsilon$ where $\epsilon \ll R$. We desire the field at the point $(x, y, z) = (a, 0, 0)$ where $a < R$. Then the Biot-Savart calculation is,

$$\mathbf{B}(a, 0, 0) = \frac{\mu_0}{4\pi} \int_{\text{leads}} I \, dz \, \frac{\hat{\mathbf{z}} \times \mathbf{s}}{s^3} + \frac{\mu_0}{4\pi} \int_{\text{plates}} \frac{I_r}{2\pi r} \, d\text{Area} \, \frac{\hat{\mathbf{r}} \times \mathbf{t}}{t^3}, \tag{41}$$

where $\mathbf{s} = (a, 0, z)$ is the vector distance between the observation point and a point on the lead wire, $\mathbf{t}$ is the vector distance between the observation point and a point on the capacitor plates, and $I_r \hat{\mathbf{r}}/2\pi r$ is the radial current density on the plates, where I_r is given by eq. (30).

We first evaluate the integral over the leads. Here, $\hat{\mathbf{z}} \times \mathbf{s} = a \, \hat{\mathbf{y}}$, and $s = (a^2 + z^2)^{1/2}$. Thus,

$$\mathbf{B}_{\text{leads}}(a, 0, 0) = 2\frac{\mu_0}{4\pi} aI \, \hat{\mathbf{y}} \int_\epsilon^\infty \frac{dz}{(a^2 + z^2)^{3/2}} = \frac{\mu_0 I \, \hat{\mathbf{y}}}{2\pi a} \left(1 - \frac{\epsilon}{a}\right) \approx \frac{\mu_0 I}{2\pi a} \, \hat{\mathbf{y}}, \tag{42}$$

just as if the capacitor wasn't there.

In the integral over the plates, the main contribution comes from points $(r, \theta, z) \approx (a + \delta x, \delta y \pm \epsilon)$ on the plate that are very close to the observation point, where $|\delta x| \ll \epsilon$, $|\delta y| \ll \epsilon$. For this region the vector distance is approximately parallel to the z axis, and we use the approximation $\mathbf{t} = \pm\epsilon \, \mathbf{z}$. Also, $\hat{\mathbf{r}} \approx \hat{\mathbf{x}}$ in this region, so $\hat{\mathbf{r}} \times \mathbf{t} \approx \pm\epsilon \, \hat{\mathbf{y}}$. Since $r \approx a$ in this region, we have $I_r(r) \approx I_r(a) = \mp I(1 - a^2/R^2)$.

We set up a new, polar coordinate system (ρ, ϕ) in the x-y plane, centered on the point $(x, y) = (a, 0)$. Then, $t = (\epsilon^2 + \rho^2)^{1/2}$. The integrand is now independent of angle ϕ, so we write the area element as $d\text{Area} = 2\pi\rho \, d\rho$. The magnetic field due to the currents in the plates is therefore,

$$\begin{aligned} \mathbf{B}_{\text{plates}}(a, 0, 0) &\approx -2\frac{\mu_0}{4\pi} \frac{I}{2\pi a} \left(1 - \frac{a^2}{R^2}\right) \pi\epsilon\hat{\mathbf{y}} \int_0^\infty \frac{2\rho \, d\rho}{(\epsilon^2 + \rho^2)^{3/2}} \\ &= -\frac{\mu_0 \epsilon I}{4\pi a} \left(1 - \frac{a^2}{R^2}\right) \hat{\mathbf{y}} \frac{2}{\epsilon} = -\frac{\mu_0 I}{2\pi a} \left(1 - \frac{a^2}{R^2}\right) \hat{\mathbf{y}}. \end{aligned} \tag{43}$$

The total magnetic field at the observation point inside the capacitor is the sum of

eqs. (42) and (43),

$$\mathbf{B}(a,0,0) = \frac{\mu_0 I}{2\pi a}\left[1 - \left(1 - \frac{a^2}{R^2}\right)\right]\hat{\mathbf{y}} = \frac{\mu_0 I a}{2\pi R^2}\,\hat{\mathbf{y}}, \tag{44}$$

as found previously via Ampère's law.

A.7   A Final Comment

Ampère's law and the Biot-Savart law represent somewhat different ways of thinking about the origin of the magnetic fields. In both views we say that the magnetic field is due to currents. Ampère's law (as stated by Maxwell) implements the idea that the magnetic field at a point can be determined only from knowledge of the total currents (conduction plus displacement) on some surface that encloses that point. In contrast, the Bio-Savart law requires knowledge only of the conduction currents, but throughout all space.

Thus, Ampère's law is more economical in its requirements for information about the physical system. But, as Maxwell was the first to note, the more elegant approach of Ampère requires a more sophisticated conception of the meaning of the term "current". The reward for mastery of these enlargements of the notion of "current" was our understanding of electromagnetic wave phenomena.

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