What is a "Rocket Force"?

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We consider a "rocket" to be a variable-mass device in which energy initially stored in the system is transfered to some "working fluid" (matter, or even light) in a "combustion" chamber. The "fluid" thereafter exits the "rocket" as an "exhaust" through an aperture/nozzle with a velocity $\mathbf{u}$ different than velociity $\mathbf{v}$ of (the center of mass of) the "rocket", which action increases the "forward" momentum of the "rocket".[1,2,3]

In the absence of any external force, the total momentum, $\mathbf{p}_{\rm rocket} + \mathbf{p}_{\rm exhaust}$, is constant, so $d\mathbf{p}_{\rm rocket}/dt = -d\mathbf{p}_{\rm exhaust}/dt$, and $-d\mathbf{p}_{\rm exhaust}/dt$ can be interpreted as associated with a propulsive force, which we call a "rocket force".

The quantity $d\mathbf{p}_{\rm exhaust}/dt$ is not associated with a force on the (imaginary) surface of the aperture/nozzle. Rather, the propulsive force, $-d\mathbf{p}_{\rm exhaut}/dt$, is due to the backscattering of exhaust particles off the wall of the "combustion" chamber opposite to the aperture. The propulsive force can be called a "recoil force" as well as a "rocket force". In addition, the propulsive force can also be called a "reaction force", which generally describes any force $-\mathbf{F}$ that is equal and opposite to some force $\mathbf{F}$.

We now have three terms that might be used to describe the propulsive force. Of these terms, "reaction force" is the most general, with "recoil force" being less so, and "rocket force" being the most restrictive of the three terms. For example, all pressure forces can be called "recoil forces", but not all pressure forces are "rocket forces".

We can consider a block in uniform motion with velocity $\mathbf{v}$ to be a variable-mass system with respect to a plane (at rest in the observer's frame) perpendicular to $\mathbf{v}$. While $d\mathbf{p}/dt$ across this plane is nonzero when the plane passes through the block, there is no release of energy associated with this quantity, and we should not call $-d\mathbf{p}/dt$ a "rocket force" or a "recoil force" or even a "reaction force" in this case.

The quantity $d\mathbf{p}/dt$ is also nonzero across a plane perpendicular to fluid flow with steady velocity $\mathbf{v}$. In particular, water flow out of a fire hose [1] involves no transformation of internal energy, and hence no "rocket force". Furthermore, there is no "reaction/recoil force" on the hose unless it is bent (so $\mathbf{v}$ changes direction and is not steady everywhere) such that a "reaction force" acts at a bend in the hose. This force (if present) is related to the water pressure in the hose, and could be called a "recoil force".[4]

A famous example, dating back to Torricelli, is the case of water falling out of a tank through an aperture in, say, its bottom surface. Here, gravity drives the flow of water, rather than a release of internal energy in the water of the tank, so no "rocket force" is involved. Momentum flows out of the aperture at a rate $d\mathbf{p}_{\rm leaving}/dt$, but there is no "recoil

[1]We consider the exhaust to be a separate entity from the "rocket".

[2]A gasoline automobile is not a "rocket" in that the exhaust fumes play negligible role in the propulsion of the automobile.

[3]The "combustion" chamber could simply be a chamber that is initially pressurized above the ambient, outside pressure, as in a "bottle rocket".

[4]The (small) viscosity of the water leads to frictional forces that oppose the flow. The "reaction force" associated with viscosity should not be called a "rocket force", but could be called a "recoil force".

force" at the aperture, nor on the upper surface of the water in the tank. Rather, ignoring viscosity and surface tension, the total force on the water in the tank is simply related by $\mathbf{F} = m\mathbf{g} + \mathbf{N} + \mathbf{F}_{\rm tensile} = d\mathbf{p}/dt = \partial\mathbf{p}/\partial t + d\mathbf{p}_{\rm leaving}/dt$,[5] where m is the mass of the water in the tank, $\mathbf{g}$ is the acceleration due to gravity (of the Earth), $\mathbf{N}$ is the (upward) normal force of the tank on the water, $\mathbf{F}_{\rm tensile}$ is the (very small) downward pull of the water outside the tank on the water still inside,[6] and $\mathbf{p}$ is the momentum of the water in the tank.[7] The "reaction force", $-\mathbf{F}$, is the sum of: the force of gravity of the water in the tank on all other mass, the normal force $-\mathbf{N}$ of the water on the tank,[8] and a (small) upward force $-\mathbf{F}_{\rm tensile}$ acting at the surface of the outlet. However, it is not appropriate to call the "reaction force", $-\mathbf{F}$, a "recoil force".

Unfortunately, misleading claims about "rocket" and/or "recoil" forces on a "leaky" water tank have been made in [4, 5]. The latter paper presents experimental results for the apparent weight ($|\mathbf{F}| = F = mg - N + F_{\rm tensile}$ with $N > 0$) of the water in a "leaky" tank that are consistent a computation of dp/dt, while strangely calling F a "recoil force" (in eq. (8) of [5]). See also Sec. 1.2 of [2].

References

[1] F. Vera and C. Núñez, *Backward Reaction Force in a Firehose*, Fire Tech. **54**, 811-818 (2018). https://doi.org/10.1007/s10694-018-0719-x
http://kirkmcd.princeton.edu/examples/fluids/vera_ft_54_811_18.pdf

[2] J. Otto and K.T. McDonald, *Torricelli's Law for Large Holes* (May 15, 2018),
http://kirkmcd.princeton.edu/examples/leaky_tank.pdf
J. Otto is an author of [4], but he did not discuss that paper with the present author before its publication.

[3] K.T. McDonald, *Forces on a Siphon* (Oct. 21, 2024),
http://kirkmcd.princeton.edu/examples/siphon

[4] J. Otto and C.E. Mungan, *Flow of water out of a funnel*, Eur. J. Phys. **45**, 055007 (2024). See the Abstract and Sec. 3.1. https://doi.org/10.1088/1361-6404/ad7107
http://kirkmcd.princeton.edu/examples/fluids/otto_ejp_45_055007_24.pdf

[5] A. Marchewka and Y. Navot, *The Paradox of the Recoil Force Acting on a Leaking Water Tank* (June 24, 2024), https://arxiv.org/abs/2406.13330
http://kirkmcd.princeton.edu/examples/fluids/marchewka_2406.13330.pdf

[5]Recall that for a rocket (with no external force), $\mathbf{F}_{\rm rocket} = d\mathbf{p}_{\rm rocket}/dt = -d\mathbf{p}_{\rm exhaust}/dt = -d\mathbf{p}_{\rm leaving}/dt$, so the term $d\mathbf{p}_{\rm leaving}/dt$ enters the force equations with opposite signs in the cases of a rocket and a leaky tank.

[6]This tensile force is important in the operation of a siphon [3], which is a variant of a leaky tank.

[7]In this example, it is easier to compute $d\mathbf{p}/dt$ than $\mathbf{N} + \mathbf{F}_{\rm tensile}$, as discussed in Sec. 1.2 of [2].

[8]The quantity $-\mathbf{N}$ is a (downward) pressure force, so could be called a "recoil force".