

Could Electrons Be Black Holes?

Kirk T. McDonald

Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

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In classical physics it is often convenient to suppose that electrons can be regarded as “point” particles.

In Einstein’s (classical) theory of general relativity a “point” particle of mass m would actually be a black hole with apparent radius,

$$R = \frac{2Gm}{c^2}, \quad (1)$$

the well-known Schwarzschild radius, where G is Newton’s gravitational constant, and c is the speed of light in vacuum (and in flat spacetime).

Hawking [1] added that quantum effects lead to radiation by a “black” hole, such that it has a surface temperature,

$$T = \frac{\hbar g}{2\pi ck}, \quad (2)$$

where g is the acceleration due to gravity at the surface of the black hole, and k is Boltzmann’s constant.

Here, we estimate the lifetime of a black hole due to Hawking radiation, a quantum effect, supposing the black hole to be a thermodynamic black body, with surface gravity,

$$g = \frac{Gm}{R^2} = \frac{c^4}{4Gm}. \quad (3)$$

Then, the surface temperature (2) is,

$$T = \frac{\hbar c^3}{16\pi k G m}, \quad (4)$$

and according to the Stefan-Boltzmann law, the rate of radiation of energy/mass is,

$$\frac{dm}{dt} = \frac{1}{c^2} \frac{dU}{dt} = -\frac{4\pi\sigma R^2 T^4}{c^2} = -\frac{\sigma c^6 \hbar^4}{(16\pi)^3 k^4 G^2 m^2} = -\frac{c^4 \hbar}{960\pi G^2 m^2} \equiv -\frac{A}{m^2}, \quad (5)$$

noting that $\sigma = \pi^2 k^4 / 60 \hbar^3 c^2$. Then, $m^2 dm = -A dt$, $m^3 = m_0^3 - 3A t$, and the lifetime τ is,

$$\tau = \frac{m_0^3}{3A} = \frac{320\pi G^2}{c^4 \hbar} m_0^3 = 5.3 \times 10^{-18} m_0^3 \text{ s} = 1.3 \times 10^{66} \left(\frac{m_0}{m_\odot}\right)^3 \text{ yr}, \quad (6)$$

for m_0 in kg, with the mass of the Sun being $m_\odot = 2 \times 10^{30}$ kg.¹

For an electron of mass $m_0 = 9 \times 10^{-31}$ kg, its lifetime would be only about 4×10^{-108} s if it were a black hole.

That is, electrons are **not** black holes.

The related question of whether photons could be black holes is discussed in sec. 4.3 of [3]. Conceivably, photons of wavelength less than the Planck length could be black holes.

¹This estimate is similar to that in [2].

References

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