Does Space Charge or the Dielectric Constant Affect Induced Charge in a Liquid Argon Detector?

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1 Problem

A liquid-argon detector is a type of ionization chamber, basically a parallel-plate capacitor filled with a (liquid) dielectric. When passing charged particles create electron-ion pairs in the liquid, which drift apart in the electric field of the capacitor, charge is induced on the (conductive) capacitor plates, which charge constitutes the signal.

Is this signal affected by the presence of additional (space) charge in the liquid, or by the its dielectric constant?

2 Solution

In brief, the answer is no.

A quick argument is that charge is induced on the conductors due to a free "test" charge in the chamber liquid so as to terminate the field lines from the test charge which are intercepted by the conductors. The electric field obeys a linear superposition principle, such that if several free charges are present in the liquid, the total induced charge is the sum of the charge induced by each of the free charges. That is, the induced signal of one free charge (at a given location) in the liquid is not affected by other free charges.[1]

Also, if the dielectric constant ϵ is uniform (outside the conductors), the pattern of field lines from the test charge to the conductors is independent of ϵ, and hence the induced free charge is independent of the dielectric constant.[2,3]

In the above, we have not needed to know the charge induced on a particular conductor, although this is of great interest in practice. A method for this was developed by Shockley [1] and by Ramo [2], assuming only a single test charge, and $\epsilon = 1$. Their argument is complicated enough that it is not self evident what happens in case of multiple free charges

[1] The motion of the test charge in the liquid depends on the electric field due to all other charges, including possible additional free charges (space charge) in the liquid. But, for any particular time/place of the test charge, its induced signal is not directly affected by the other (space) charges in the liquid.

[2] The total induced charge includes bound charge at the surface of the liquid next to the conductors, but this bound charge is not part of the signal charge on the conductors. The induced free charge flows onto the conductors from a "battery", and the resulting (free) current is amplified to become the nominal signal of the detector.

[3] In a bit more detail, since $\mathbf{D} = \epsilon\epsilon_0\mathbf{E}$ and ϵ is uniform outside the conductors, $\mathbf{D}$ and $\mathbf{E}$ have the same pattern of field lines, and this pattern can be determined for $\mathbf{D}$ from only the free charges, according to $\boldsymbol{\nabla}\cdot\mathbf{D} = \rho_{\rm free}$ and $\boldsymbol{\nabla}\times\mathbf{D} \approx 0$, with no knowledge of ϵ. Hence, the free charge induced on the conductors is independent of the dielectric constant ϵ, if the latter is uniform. *If, say, the detector were only half filled with liquid argon, the induced signal would depend on ϵ.*

and $\epsilon \neq 1$. This has led to several discussions of the Shockley-Ramo "weighting" method[4] for various assumptions, with the general conclusion that the induced charge from a single test charge is independent of the possible presence of other charges in the liquid (or gas or solid) dielectric medium, and is also independent of the dielectric constant. A representative discussion is [4].

Acknowledgment

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References

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[2] S. Ramo, *Currents Induced by Electron Motion*, Proc. I.R.E. **27**, 584 (1939), http://kirkmcd.princeton.edu/examples/EM/ramo_pire_27_584_39.pdf

[3] C. Lu and K.T. McDonald, *The Charge Distribution on the Cathode of a Straw Tube Chamber* (Oct. 1, 1998), http://kirkmcd.princeton.edu/examples/straw.pdf

[4] L.A. Hamel and M. Julien, *Generalized demonstration of Ramo's theorem with space charge and polarization effects*, Nucl. Instr. Meth. A **597**, 207 (2008), http://kirkmcd.princeton.edu/examples/EM/hamel_nim_a597_207_08.pdf

[4]For a simple illustration of the weighting method, see [3].