Does Induced $\mathcal{EMF}$ Occur Instantaneously?

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(May 18, 2024)

1 Problem

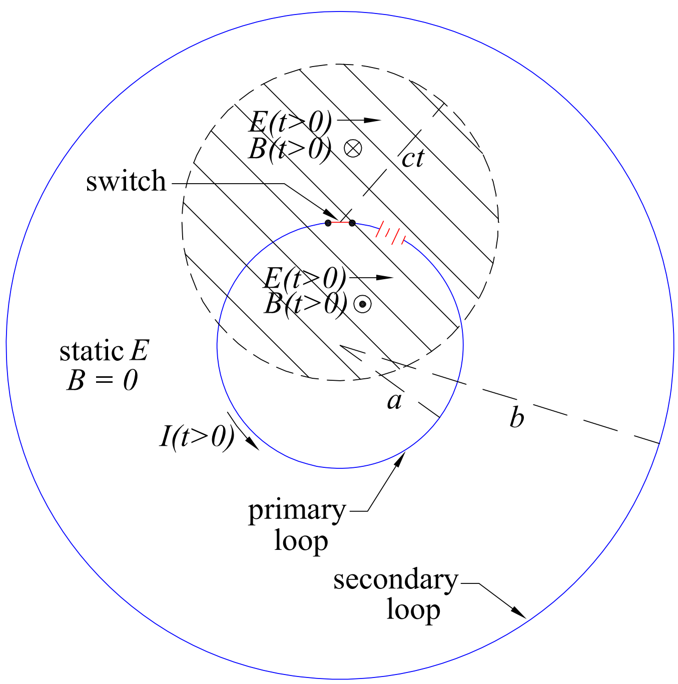
The integral form of Faraday's "flux rule",[1] $\mathcal{EMF} = -d\Phi_B/dt$, where $\Phi_B = \oint_{\text{loop}} \mathbf{B} \cdot d\mathbf{Area}$ is the magnetic flux through some closed loop, might imply that the electromotive force, $\mathcal{EMF}$ (and the resulting electrical current in the closed loop if that loop is a resistive conductor), occurs instantaneously once the electrical current that generates the magnetic field $\mathbf{B}$ starts to change, no matter how far the closed loop is from the source currents of the magnetic field. If so, this would be instantaneous action at a distance in electromagnetism.[2]

Can this be so?

2 Solution

NO!

We consider the example in the figure below to illustrate an argument for this conclusion.



A conducting loop of radius a (the primary loop) contains a battery of potential difference V, and a switch that is initially open. Initially there is no electrical current, and no magnetic field $\mathbf{B}$, anywhere, although a static electric field $\mathbf{E}$ due to the battery exists everywhere.

[1] Discussion by the author of Faraday's "flux rule" is given in [1, 2].

[2] See [3] for discussion of the related issue of the time delay of effects in long circuits.

At time $t = 0$ the switch is closed, such that a counterclockwise source current $I(t > 0)$ flows in the primary loop. The current I increases with time before stabilizing at the value $I_{\rm steady} = V/R$, where R is the electrical resistance of the conducting loop of radius a.

Starting at time $t = 0$, transient electromagnetic fields flow in this system, and are nonzero at time t within a sphere of radius ct about the switch.[3,4]

If the expanding sphere of nonzero magnetic field does not intersect a secondary loop, such as the circle of radius b in the figure above, the magnetic flux through the secondary loop is zero (no matter what is the shape of that loop). This follows from Gauss' divergence theorem and the fact that $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$, which implies that the magnetic flux through a loop is the same when computed over any surface bounded by the loop.[5] Hence, we can compute the flux through the secondary loop using a surface that lies entirely outside the sphere of nonzero $\mathbf{B}$ due to the primary loop, on which surface the magnetic field is everywhere zero, and find the magnetic flux is zero in the secondary loop.[6]

According to Faraday's "flux rule" there is no $\mathcal{EMF}$ induced in a secondary loop that does not intersect the expanding sphere of nonzero magnetic field. Hence, no current is induced in the secondary loop prior to the time when a "signal" arrives (at the speed of light) from the primary loop that the current is changing there.

The induced current is NOT instantaneous action at a distance.[7]

In the example shown in the figure above the (time dependent) magnetic flux through a (conducting) loop of radius $b < a + ct$ can be nonzero, such that a nonzero $\mathcal{EMF}$, and a nonzero electric current current can be induced in the loop at time $t > (b - a)/c$.[8]

This delay in the start of the induced current is very small in ordinary circuits, and is negligible in practice.

[3]Discussions of such transient fields for source currents in long/infinite wires and solenoids have been given by the author in [4, 5].

[4]For $t < 0$, the static (dipole) electric field associated with the open switch (which is a kind of a capacitor) points to the left in the figure, and falls off as $1/r^3$ from the switch. After the switch is closed at $t = 0$, the electric field between the contacts of the switch decreases with time, so the transient/radiation field in the vicinity of the switch points to the right. This transient/radiation field falls off as $1/r$ from the switch, so the electric field away from the switch for $t = 0$ points to the right, as shown in the figure above.

[5]For any two surfaces bounded by the same loop, consider the closed surface defined by these two surfaces (1 and 2), and the volume enclosed by these two surfaces. The flux across the closed surface is the difference between the fluxes across surfaces 1 and 2,
$\int_{\rm closed\ surface} \mathbf{B} \cdot d\mathbf{Area} = \int_{\rm surface\ 1} \mathbf{B} \cdot d\mathbf{Area} - \int_{\rm surface\ 2} \mathbf{B} \cdot d\mathbf{Area}$.
Then, Gauss tells us that $\int_{\rm closed\ surface} \mathbf{B} \cdot d\mathbf{Area} = \int_{\rm enlosed\ volume} \boldsymbol{\nabla} \cdot \mathbf{B}\, d{\rm Vol} = 0$, and so the fluxes across the surfaces 1 and 2 are equal.

[6]For the example in the figure above, the transient magnetic field inside the circle of radius a points out of the page, while the magnetic field outside this circle points into the page. Then, for a secondary loop of radius b larger than $a + ct$, the portions of the flux inside and outside radius a cancel.

[7]An interesting video, somewhat related to the present problem, was pointed out by David Griffiths, https://www.youtube.com/watch?v=bHIhgxav9LY. For me, the most interesting feature of this video is around time 10:40, when the transatlantic cable appears to leap off the deck of the ship as it flows into the ocean below. This is may be the first recorded example of a "cable/chain fountain", which topic is reviewed by the author in [6].

[8]The electric field $\mathbf{E}(t > (b - a)/c)$ induces a clockwise current in the secondary loop, which leads to a magnetic field inside the primary loop pointed into the page. This magnetic field is opposite to that inside inside the primary loop due to its own current, in accordance with Lenz' law.

In contrast, the delay between the emission of a signal by one antenna and the reception of that signal by a second, distant antenna is well known, and can be regarded as evidence of the time delay associated with induced $\mathcal{EMF}$.

References

[1] K.T. McDonald, *What is the "Flux Rule"?* (May 16, 2023),
http://kirkmcd.princeton.edu/examples/flux_rule.pdf

[2] K.T. McDonald, *Is Faraday's Disk Dynamo a Flux-Rule Exception?* (July 27, 2019),
kirkmcd.princeton.edu/examples/faradaydisk.pdf

[3] K.T. McDonald, *Transient Voltage in a Long Circuit* (May 25, 2024),
kirkmcd.princeton.edu/examples/transient.pdf

[4] K.T. McDonald, *The Electromagnetic Fields Outside a Wire That Carries a Linearly Rising Current* (Nov. 28, 1996), kirkmcd.princeton.edu/examples/wirefield.pdf

[5] K.T. McDonald, *The Fields Outside a Solenoid with a Time-Dependent Current*, Am. J. Phys. **65**, 1176 (1997), http://kirkmcd.princeton.edu/examples/EM/mcdonald_ajp_65_1176_97.pdf
kirkmcd.princeton.edu/examples/solenoid.pdf

[6] K.T. McDonald, *"Hidden" Momentum in a Leaping Beaded Chain?* (Aug. 13, 2013),
kirkmcd.princeton.edu/examples/chain.pdf