

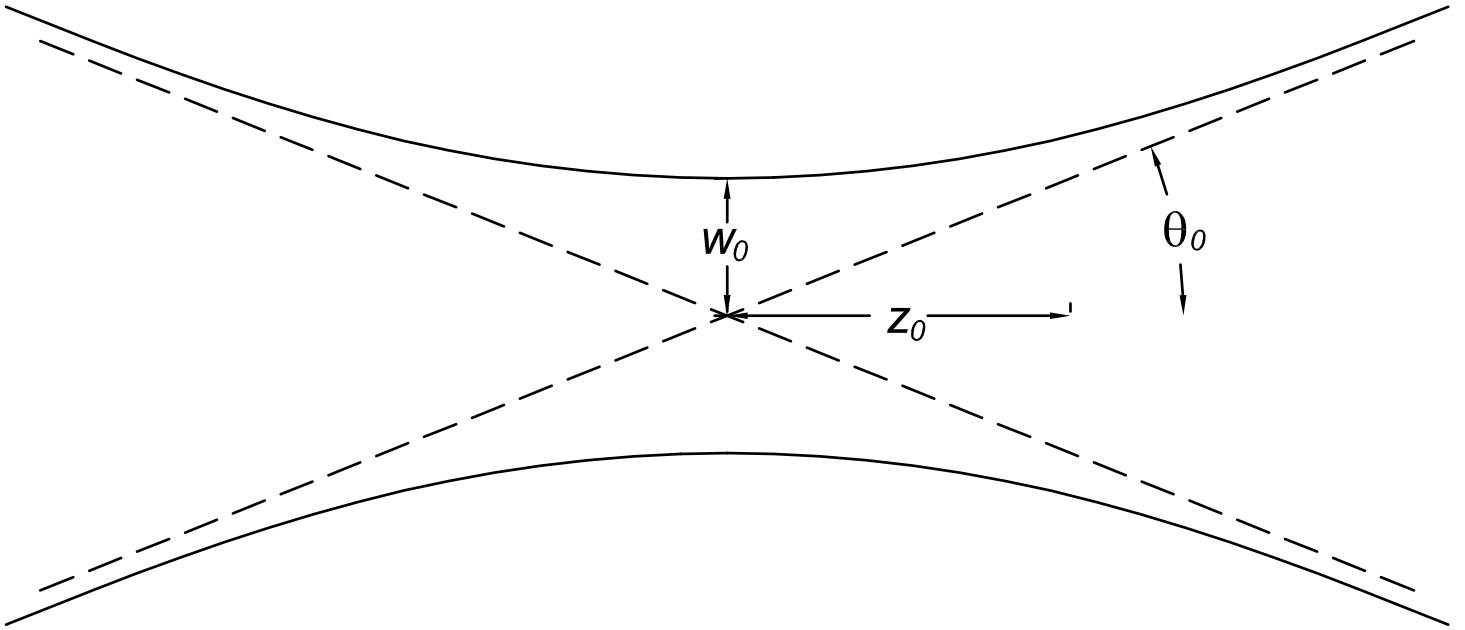
## Parameters of a Laser Focus

Diffraction angle  $\theta_0$ ,

Waist  $w_0$ ,

Depth of focus (Rayleigh range)  $z_0$ :

$$\theta_0 = \frac{w_0}{z_0} = \frac{2}{kw_0}, \quad \text{and} \quad z_0 = \frac{kw_0^2}{2} = \frac{2}{k\theta_0^2}$$

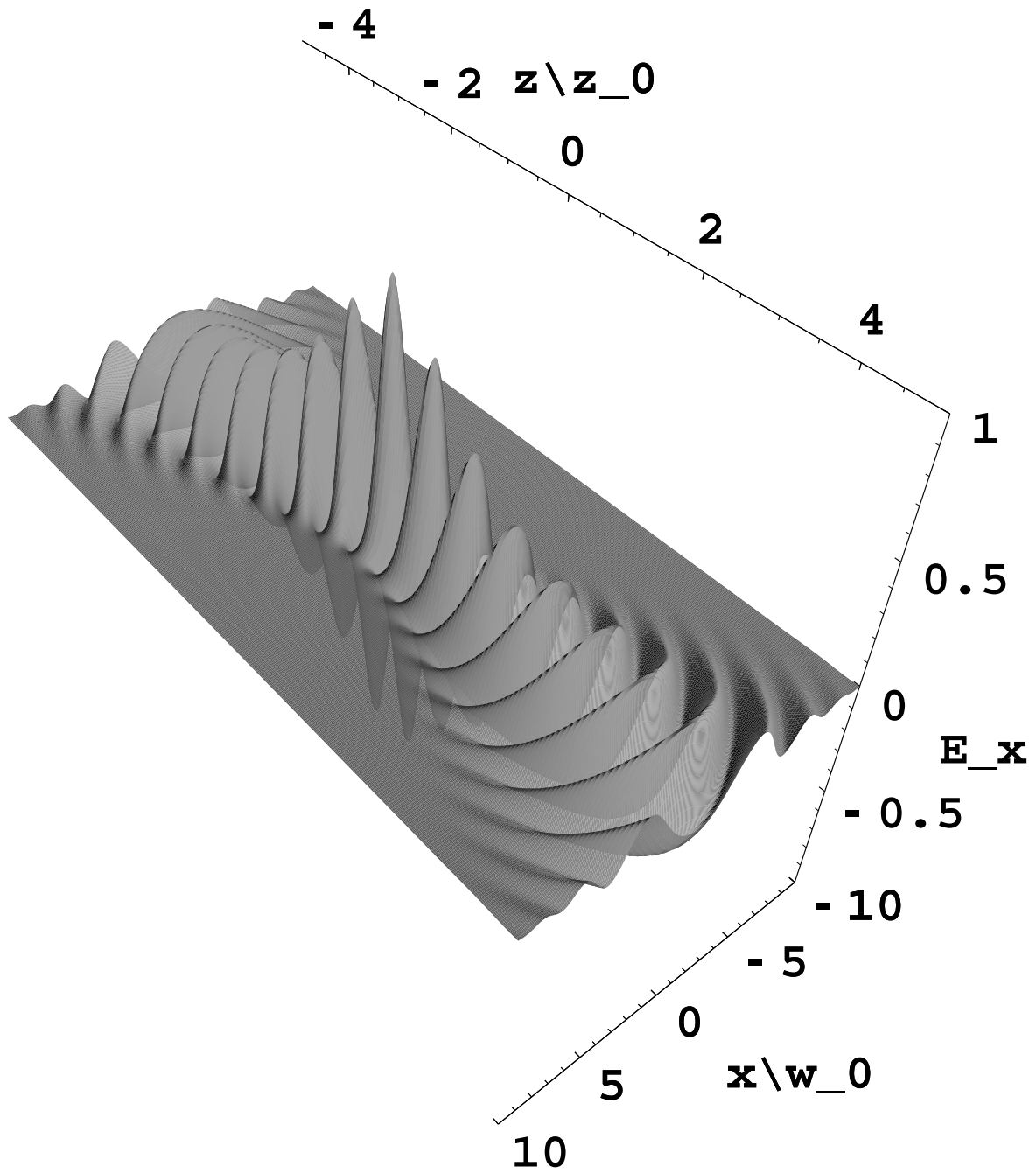


$$\xi = \frac{x}{w_0}, \quad v = \frac{y}{w_0}, \quad \rho = \frac{r_{\perp}}{w_0}, \quad \varsigma = \frac{z}{z_0}$$

Paraxial wave equation:  $\nabla_{\perp}^2 \psi + 4i \frac{\partial \psi}{\partial \varsigma} = 0$

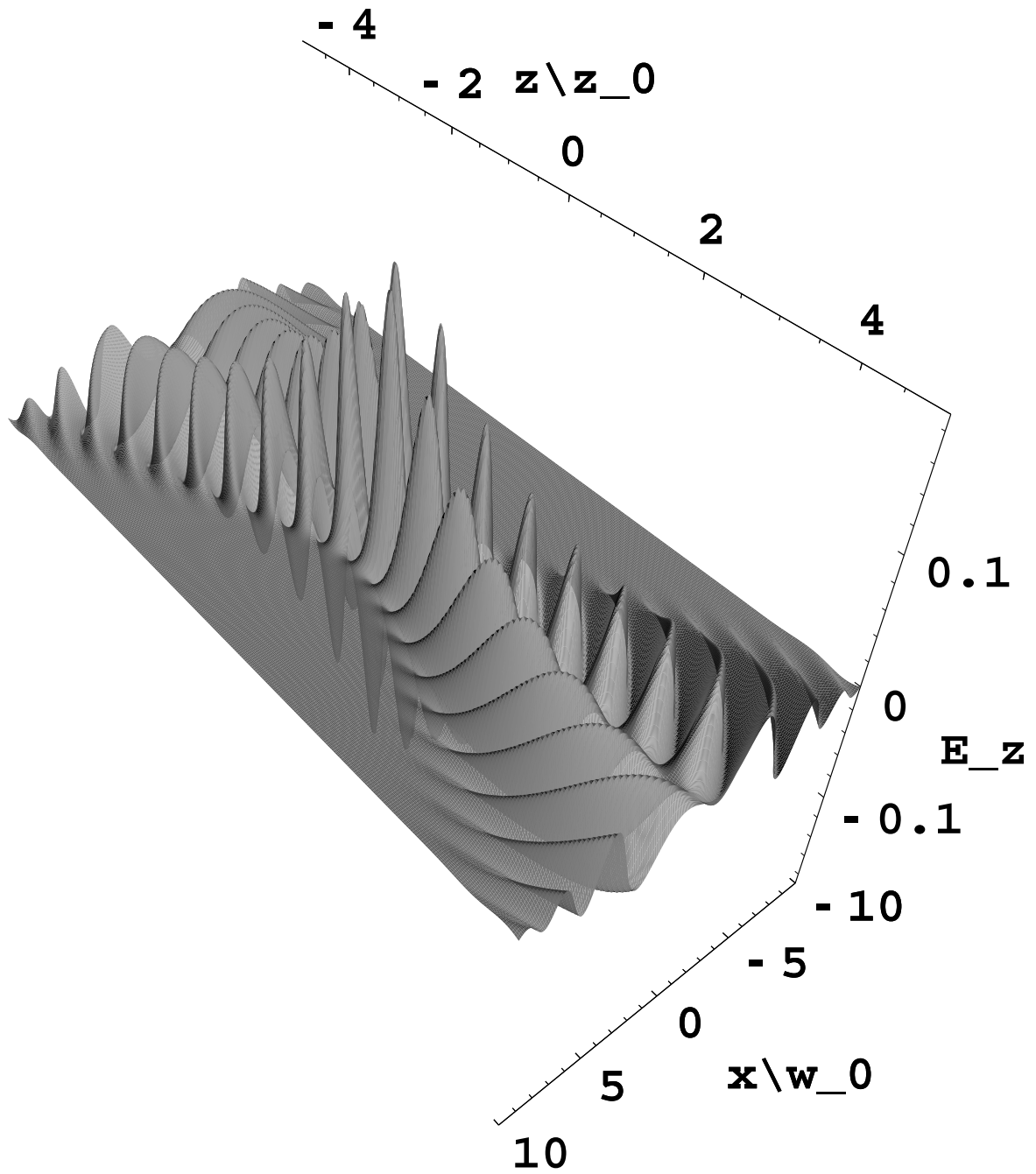
$$\psi = f e^{-f\rho^2}, \quad f = \frac{1}{1+i\varsigma} = \frac{1-i\varsigma}{1+\varsigma^2} = \frac{e^{-i \tan^{-1} \varsigma}}{\sqrt{1+\varsigma^2}}$$

# Linearly Polarized Gaussian Laser Beam, $A \propto \psi \hat{x}$



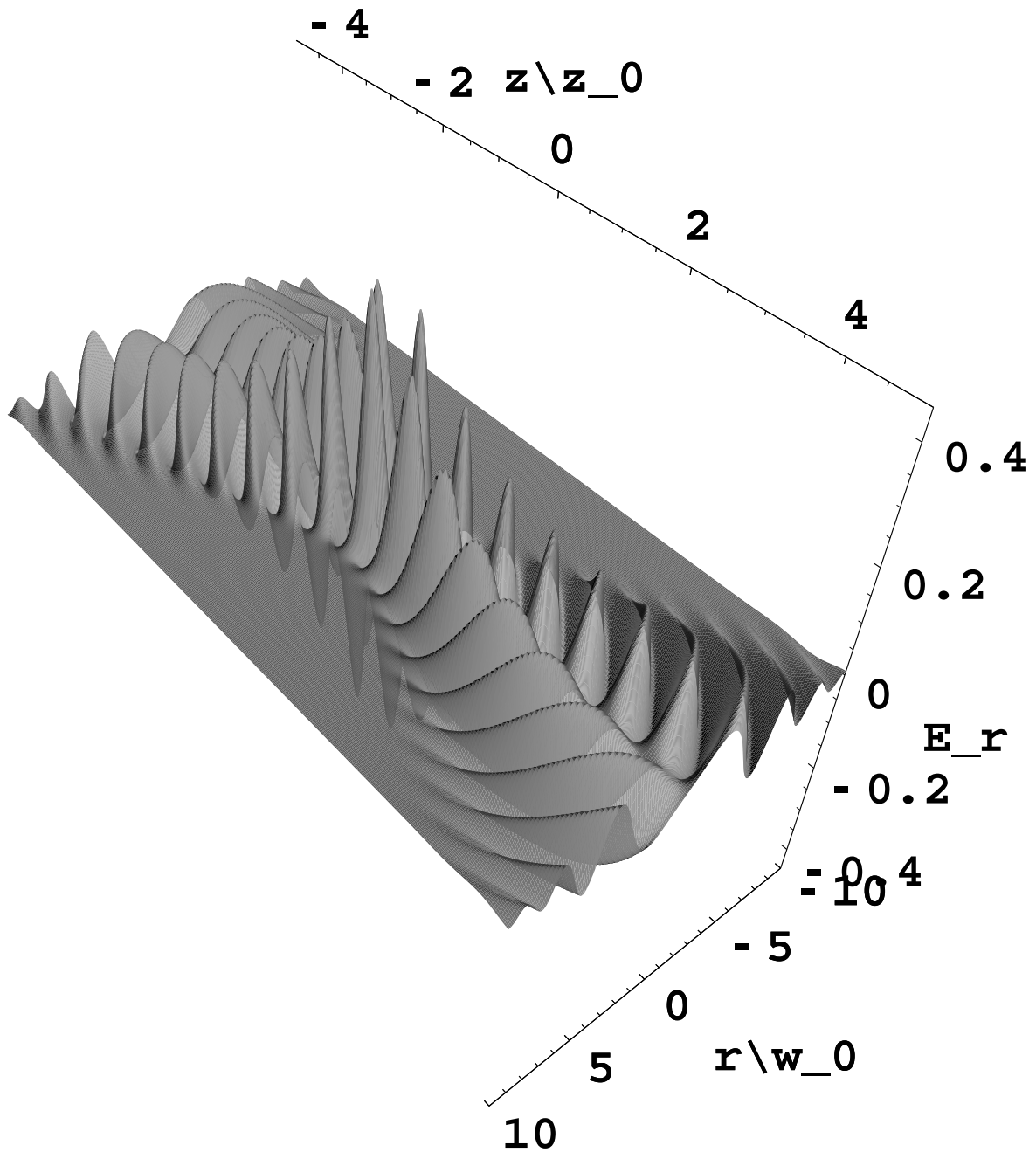
$$\begin{aligned}
 E_x &= E_0 f e^{-f\rho^2} e^{i(kz - \omega t)} \\
 &= \frac{E_0 e^{-r_{\perp}^2/w_0^2(1+z^2/z_0^2)}}{\sqrt{1+z^2/z_0^2}} e^{i\{kz[1+r_{\perp}^2/2(z^2+z_0^2)] - \omega t - \tan^{-1}(z/z_0)\}}
 \end{aligned}$$

# Linearly Polarized Gaussian Laser Beam, $A \propto \psi \hat{x}$



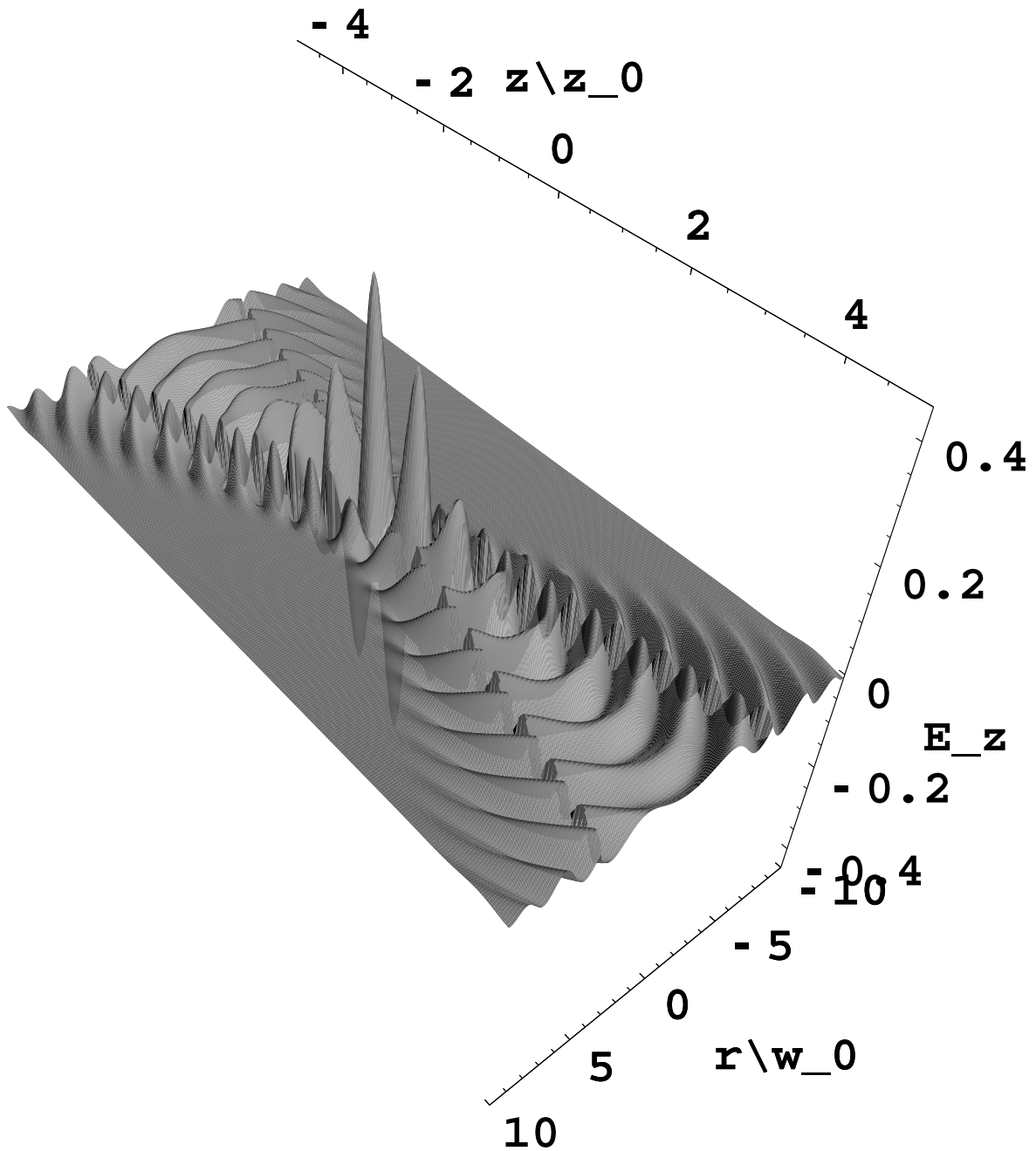
$$\begin{aligned}
 E_z &= -i\theta_0\xi E_0 f^2 e^{-f\rho^2} e^{i(kz-\omega t)} \\
 &= -i \frac{x e^{-i \tan^{-1}(z/z_0)}}{\sqrt{z^2 + z_0^2}} E_x
 \end{aligned}$$

## Axicon Gaussian Laser Beam, $A \propto \psi \hat{z}$



$$\begin{aligned}
 E_{r_{\perp}} &= \rho E_0 f^2 e^{-f\rho^2} e^{i(kz - \omega t)} \\
 &= \frac{r_{\perp} E_0 e^{-r_{\perp}^2/w_0^2(1+z^2/z_0^2)}}{1 + z^2/z_0^2} e^{i\{kz[1+r_{\perp}^2/2(z^2+z_0^2)] - \omega t - 2 \tan^{-1}(z/z_0)\}}
 \end{aligned}$$

## Axicon Gaussian Laser Beam, $A \propto \psi \hat{z}$



$$\begin{aligned}
 E_z &= i\theta_0 E_0 f^2 (1 - f\rho^2) e^{-f\rho^2} e^{i(kz - \omega t)} \\
 &= i \frac{\lambda}{\pi r_\perp} \left( 1 - \frac{r_\perp^2}{z^2 + z_0^2} \frac{e^{-i \tan^{-1}(z/z_0)}}{\theta_0^2} \right) E_{r_\perp}
 \end{aligned}$$