# McClymer's Electromagnetic Spaceship

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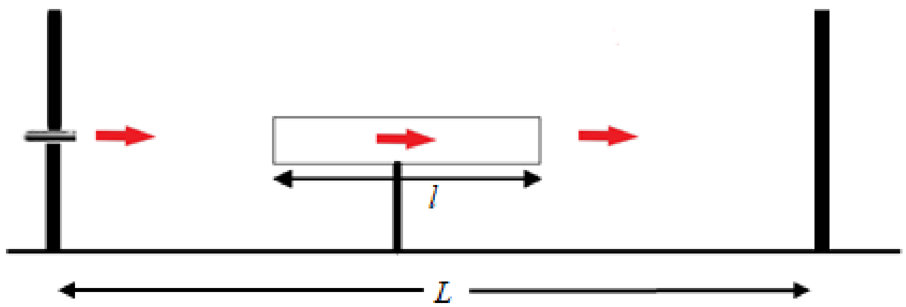
## 1 Problem

McClymer has recently claimed [1] that the apparatus sketched below constitutes a "bootstrap spaceship" with a "reactionless drive". A laser pulse emitted at the left end of an isolated, "rigid" box/platform (of length $L$) passes through a "rigid" block of (dispersionless) glass (of length $l$ and index of refraction $n$) which is "rigidly" attached to the box, and is absorbed at the right end of the box. Supposedly, the center of mass of the system at the end of this process is different from its initial location (in the inertial lab frame).




Can this be so?

## 2 Solution

If the claim were true, it would be a violation of the "center-of-energy theorem", that the center of mass/energy of an isolation system remains at rest if initially at rest. See, for example, sec. 1.1 and Appendix A of [2].

The present example is part of the larger issue that electromagnetic fields can exert a force on a charged particle, but a charged particle does not exert a force on the field, such that a "bootstrap" electromagnetic spaceship might seem possible.

The general resolution of such issues is that the electromagnetic field carries momentum (and energy and angular momentum), and that a careful accounting of the field momentum as well as of the "mechanical" momentum provides consistency with the center-of-energy theorem, *i.e.*, no "bootstrap spaceships."[1]

In examples like the present in which electromagnetic waves propagate inside a medium, an additional complication is that people often use the "electromagnetic" fields $\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$ and $\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M}$, in Gaussian units, where $\mathbf{P}$ and $\mathbf{M}$ are the (macroscopic) densities of electric and magnetic dipole moments, which are "mechanical" entities. That is, $\mathbf{D}$ and $\mathbf{H}$ would better be called "electromechanical" fields, and the name "electromagnetic" fields be

[1]Discussions by the author of possible and impossible electromagnetic spaceships include [3]-[12].

reserved for $\mathbf{E}$ and $\mathbf{B}$. In particular, the density of electromagnetic field momentum should be considered as,

$$\mathbf{p}_{\rm EM} = \frac{\mathbf{E} \times \mathbf{B}}{4\pi c}\,, \tag{1}$$

where $c$ is the speed of light in vacuum, while the so-called Abraham [13] and Minkowski [14] momentum densities,

$$\mathbf{p}^{(\rm A)} = \frac{\mathbf{E} \times \mathbf{H}}{4\pi c}\,, \qquad \text{and} \qquad \mathbf{p}^{(\rm M)} = \frac{\mathbf{D} \times \mathbf{B}}{4\pi c}\,, \tag{2}$$

describe electromechanical momenta, leading to confusion when trying to separate momenta into "electromagnetic" and "mechanical" components.[2]

McClymer's analysis [1] is based on use of the Minkowski momentum for the wave inside the glass block, which (assuming no reflection as the wave enters the block) is larger than the wave momentum in vacuum by a factor of the relative permittivity $\epsilon_{\rm rel}$, leading to his inference that the glass block, if otherwise isolated, would move opposite to the direction of the wave. However, it has been generally consider since an argument of Balazs in 1953 [16] that the Abraham, rather than the Minkowski, momentum is relevant to considerations of the bulk momentum of the medium that supports the wave.

Since that time, considerations in quantum optics have identified a role for the Minkowski momentum in experiments where quantities other than bulk velocities are measured. In particular, the so-called "photon drag effect" [17, 18] on charge carriers in the conduction band of a semiconductor is associated with the Minkowski momentum [19, 20, 21]. Nonetheless, a recent paper [22] claimed that the Minkowski, not the Abraham, momentum is the one to be used when considering the bulk velocity, apparently based on the spurious view that the rest mass of all photons must be zero (in contrast to the usual view that ("kinetic") photons of angular frequency $\omega$ inside a medium of index $n$ have rest mass $m = \sqrt{(\hbar\omega)^2 - (\hbar\omega/c\,n)^2 c^2}/c^2 = \hbar\omega\sqrt{n^2-1}/nc^2$).[3] This misunderstanding is repeated in [1], and leads to the claim there that the center-of-energy theorem is incorrect.

## 2.1 The Nonexistence of "Rigid" Objects

McClymer's spaceship invokes a "rigid" platform/box, and a "rigid" glass block, both of which are supposed to start moving instantaneously as a whole after emission or absorption of a laser pulse at one of their ends. This is impossible, as it implies infinite speed of sound in these objects. While the context of this device is unphysical, it is entertaining to see to what extent its behavior could be, within the doubtful premise of "rigid" objects..

The rest of sec. 2 of this note is largely equivalent to the analysis in [16], but without use of either the Abraham or Minkowski momenta. It is desirable to be able to neglect reflections of waves at interfaces, which we arrange by considering only indices $n$ slightly greater than unity (rather than invoking idealized antireflection coatings as in most of the quantum-optics literature).

[2] A bibliography of the Abraham-Minkowski debate, including large numbers of papers that claim to have resolved this "perpetual problem", is at [15].

[3] For another example of a misuse of the Minkowski momentum, see [23].

## 2.2 Analysis without the Glass Block

The discussion below of McClymer's example builds on an argument due to Einstein [24], with which he first deduced that $E = mc^2$.[4] Here, we follow sec. 27-6 of [31].

A laser pulse of energy $U$ and spatial length small compared to length $L$ of an isolated, "rigid" box is emitted (into vacuum) at time $t = 0$ from the left end, defined to be at $x = 0$, of the box, initially at rest in the inertial lab frame, and later absorbed at its right end. While the pulse is in motion, in the positive $x$-direction, the mass of the box is $M$ (*i.e.*, the initial mass/energy of the system was $M' = M + U/c^2$).[5]

The initial center of mass of the system, $x_{\rm cm,i}$, is related by,

$$M' x_{\rm cm,i} = \frac{U}{c^2} 0 + M \frac{L}{2} = \frac{ML}{2} \, . \tag{3}$$

The laser pulse (in vacuum) of energy $U$ carries momentum $U\,\hat{\mathbf{x}}/c$, so the box recoils to the left[6] with equal and opposite momentum, and moves with speed,

$$V = \frac{U}{Mc} \, . \tag{4}$$

When the laser pulse is absorbed at the right end of the box at time $T$, the box has moved to the left by distance $D = VT$, and the pulse has moved to the right by distance $L - D$. That is,

$$T = \frac{L-D}{c} \, , \qquad D = VT = \frac{UL}{Mc^2} - \frac{UD}{Mc^2} \, , \qquad D = \frac{UL}{Mc^2 + U} = \frac{UL}{M'c^2} \, . \tag{5}$$

The final center of mass of the system, $x_{\rm cm,f}$, is related by,

$$M' x_{\rm cm,f} = \frac{U}{c^2}(L - D) + M\left(\frac{L}{2} - D\right) = \frac{UL}{c^2}\left(1 - \frac{U}{Mc^2 + U}\right) + \frac{ML}{2} - \frac{MUL}{Mc^2 + U} = \frac{ML}{2} \, , \tag{6}$$

in agreement with the initial position, $x_{\rm cm,i}$ of eq. (3), of the center of mass of the system (which, of course, was the premise of Einstein's original argument).

## 2.3 Analysis with the Glass Block Unattached to the Box

Turning at last to McClymer's example, we suppose that the glass block is at the center of the box and has length $l < L$, while the laser pulse has spatial length small compared to $l$. However, it is simpler to suppose that the glass block is not attached to the (isolated) box,

[4]Einstein was aware of conceptual weaknesses in his original argument, and made many revisions thereof [25]-[28], as reviewed, for example, in [29, 30].

[5]This neglects the "relativistic" mass of the moving box, as this differs from the rest mass of the box by a (tiny) term of order $V^2/c^2 \propto 1/c^4$.

We also ignore the tiny Lorentz contraction of the box while in most with respect to the lab frame.

[6]Note also that Maxwell stress tensor of the laser pulse is diagonal, with negative $\mathsf{T}_{xx}$ $(= -(E_\perp^2 + B_\perp^2)/8\pi = -E_\perp^2/4\pi)$, which indicates that the force, $F_x = \int \mathsf{T}_{xx} \hat{n}_x \, d{\rm Area}$ of the laser pulse on the left end of the box (where $\hat{\mathbf{n}} = \hat{\mathbf{x}}$) is in the negative-$x$ direction when it is emitted, and in the positive-$x$ direction when it is later absorbed at the right end of the box (where $\hat{\mathbf{n}} = -\hat{\mathbf{x}}$).

but is free to move inside the box (unless it encounters a wall of the box).[7] In sec. 2.4 below we consider the case of a glass block attached to the box.

Again, the laser pulse of energy $U$ is emitted at time $t = 0$ from the left end of the box, and again the box recoils to the left with momentum $-(U/c)\,\hat{\mathbf{x}}$, and speed $V$ to the left given by eq. (4). The mass of the box when the laser pulse is in flight is again $M$, and we denote the mass of the glass block by $m$.[8] The total mass of the system is $M' = M + m + U/c^2$, and the initial location $x_{\rm cm,i}$ of its center of mass is related by,

$$M' x_{\rm cm,i} = \frac{U}{c^2} 0 + (M + m)\frac{L}{2} = \frac{(M + m)L}{2}\,. \tag{7}$$

### 2.3.1 The Approximation of No Reflected Wave

When the pulse enters the glass block (from vacuum) it is partly transmitted and partly reflected, which latter complicates the analysis. To simplify things, we will use the results for an electromagnetic wave crossing an interface that is at rest,[9] ignoring the tiny velocity of the glass in the lab frame in this regard. We also suppose that the relative permeability of the glass is unity $\mu_{\rm rel} = 1$,[10] and that the index of refraction is only slightly larger than 1, *i.e.*, $n = 1 + \alpha$ with $\alpha \ll 1$. Then, the reflected Poynting vector has magnitude $(n - 1)^2/(n + 1)^2$ times that of the incident wave, which is negligible for small $\alpha$. We restrict further discussion to this case, in which essentially all of the energy $U$ of the laser pulse is transmitted into the electromagnetic wave inside the glass.[11]

### 2.3.2 Momentum and Energy of the Electromagnetic Wave inside the Glass

Whenever $\mu_{\rm rel} = 1$, the transmitted electric field $\mathbf{E}_{\rm gl}$ (inside the glass block) and the transmitted magnetic field $\mathbf{B}_{\rm gl}$ are related to quantities in vacuum by,

$$\mathbf{E}_{\rm gl} = \frac{2}{n + 1}\mathbf{E}_{\rm vac}, \qquad \mathbf{B}_{\rm gl} = \frac{2n}{n + 1}\mathbf{B}_{\rm vac}. \tag{8}$$

The density of transmitted electromagnetic-field momentum is then, noting that $E_{\rm vac} = B_{\rm vac}$,

$$\mathbf{p}_{\rm gl} \;=\; \frac{\mathbf{E}_{\rm gl} \times \mathbf{B}_{\rm gl}}{4\pi c} = \frac{4n}{(n + 1)^2}\frac{\mathbf{E}_{\rm vac} \times \mathbf{B}_{\rm vac}}{4\pi c} = \frac{4n}{(n + 1)^2}\mathbf{p}_{\rm vac}. \tag{9}$$

The spatial length of the pulse when inside glass is $1/n$ times that when in vacuum (because the temporal length is the same but its speed is $1/n$ times that in vacuum), so the total electromagnetic-field momentum of the pulse when inside the glass is,

$$\mathbf{P}_{\rm EM,gl} = \int \mathbf{p}_{\rm EM,gl}\, d{\rm Vol}_{\rm gl} = \frac{4n}{n(n + 1)^2}\int \mathbf{p}_{\rm EM,vac}\, d{\rm Vol}_{\rm vac} \approx \frac{\mathbf{P}_{\rm EM,vac}}{n} = \frac{U}{c\,n}\,\hat{\mathbf{x}}, \tag{10}$$

[7] This is the scenario considered in [16].

[8] We again ignore the "relativistic" mass of the box and the glass block when in motion with respect to the lab frame.

[9] See, for example, sec. 7.2 of [32], particularly eq. (7.42).

[10] Note that because we have assumed that $\mu_{\rm rel} = 1$, then $\mathbf{B} = \mathbf{H}$, and the Abraham momentum (2) is the same as the electromagnetic field momentum (1).

[11] This contrasts with [1, 22], where the energy of the laser pulse when inside the glass is $n^2U$.

noting that for $n = 1 + \alpha$ with small $\alpha$,

$$\frac{4}{(n+1)^2} = \frac{4}{(2+\alpha)^2} \approx \frac{4}{4+4\alpha} = \frac{1}{n}\,. \tag{11}$$

If we think of the laser pulse inside the glass as a particle, with energy $U$ and momentum $\mathbf{P} = U\,\hat{\mathbf{x}}/cn$, it has invariant (rest) mass,

$$m_{\rm pulse} = \frac{\sqrt{U^2 - c^2P^2}}{c^2} = \frac{\sqrt{n^2-1}}{n}\frac{U}{c^2} \approx \frac{\alpha U}{c^2}. \tag{12}$$

A single photon of such a pulse would have energy $U = \hbar\omega$, where $\omega$ is the angular frequency of the laser, such that $m_{\rm photon} \approx \alpha\hbar\omega/c^2$.[12]

### 2.3.3 Motion of the Glass Block

The electromagnetic-field momentum (10) is smaller than the momentum of the pulse in vacuum, so momentum conservation implies that the glass has mechanical momentum,

$$\mathbf{P}_{\rm mech,gl} = \frac{U}{c}\,\hat{\mathbf{x}} - \frac{U}{c\,n}\,\hat{\mathbf{x}} = \frac{U(n-1)}{c\,n}\,\hat{\mathbf{x}}. \tag{13}$$

That is, the pulse gives a push to the glass, in the $+x$ direction when it enters the glass and in the $-x$ direction when it later leaves the glass.[13,14]

If the mechanical momentum (13) is shared by the entire glass block, then it takes on velocity,

$$\mathbf{V}_{\rm gl} = \frac{U(n-1)}{mc\,n}\,\hat{\mathbf{x}}, \tag{14}$$

while the pulse is inside the glass.[15]

After the pulse leaves the glass, its velocity would return to zero.

However, motion of a mechanical object in response to a force at one end propagates across that object at the speed of sound, which is very small compared to the speed of light, so it not possible that the glass block takes on the velocity (14) as a whole before the laser pulse leaves the block. In particular, the exit face of the glass block will not have moved

[12]In [1, 22] is it claimed (incorrectly) that no "model" exists in which photons inside that glass have nonzero rest mass.

[13]This agrees with inferences from the Maxwell stress tensor of the laser pulse when it just outside the glass block (and still in vacuum). As in sec. 2.1 above, the stress tensor is diagonal, with negative $\mathsf{T}_{xx}$ $(= -(E_\perp^2 + B_\perp^2)/8\pi = -E_\perp^2/4\pi)$, which indicates that the force, $F_x = \int \mathsf{T}_{xx}\hat{n}_x\,d$Area of the laser pulse on the left end of the glass (where $\hat{\mathbf{n}} = -\hat{\mathbf{x}}$) is in the $+x$ direction when it enters, and in the $-x$ direction when it later leaves the right end of the glass(where $\hat{\mathbf{n}} = \hat{\mathbf{x}}$).

[14]In contrast, Fig. 1 of [22], and McClymer (private communication), claims that the block is pulled in the $-x$ direction when the laser pulse enters it (and pulled in the $+x$ direction when the pulse leaves the block).

[15]Strictly, the block takes on kinetic energy $mv_{\rm gl}^2/2 = U(n-1)^2/2mc^2n \approx U\alpha^2/2c^2$, which is of the same order as the energy of the reflected laser pulse. In this note we neglect both of these effects.

before the pulse arrives at that face, so the time $T_{\rm glass}$ during which the pulse was inside the glass is given by,[16]

$$T_{\rm gl} = \frac{l}{c/n} = \frac{nl}{c}\,. \tag{15}$$

As the leading edge of the pulse moves across the block, it exerts a local force in the $+x$ direction that can set the local portion of the block in motion. However, as the trailing edge of the pulse passes any particular point inside the block, it exerts a local force in the $-x$ direction the tends to bring the local portion of the block back to rest. As such, the block can well experience a net displacement $D_{\rm gl}$ in the $+x$ direction, although this is unlikely to be related by the naïve expression,

$$D_{\rm gl} = V_{\rm gl} T_{\rm gl} = \frac{U(n-1)}{mc\,n}\frac{nl}{c} = \frac{Ul(n-1)}{mc^2}\,. \tag{16}$$

*There exists another possibility, that the mechanical momentum (13) is somehow "hidden" inside that glass, and the glass block does not move.*

We continue the analysis based on the very doubtful relation (16).

### 2.3.4 Motion of the Box

We are now alerted that the analysis in sec. 2.1 is very doubtful as it assumed that the entire box started moving as a "rigid" body immediately after the laser pulse was emitted. But the apparent success of that analysis in being consistent with the center-of-energy theorem (*i.e.*, that the center of mass should not move in that example), encourages us to proceed with the case of a "rigid" glass block inside a "rigid" box.

The total flight time $T$ of the pulse inside the box is no longer given by eq. (5). Instead, we note that in addition to the time $T_{\rm gl}$ that the laser spends inside the glass, it spends time $T_{\rm vac}$ in the vacuum inside the box but outside the glass. We again have two relations involving the time $T_{\rm vac}$ and the displacement $D$ of the ("rigid") box after the laser pulse is reabsorbed, which are now,

$$D = VT = \frac{U}{Mc}(T_{\rm vac} + T_{\rm gl}) = \frac{U}{Mc^3}(cT_{\rm vac} + nl), \qquad L - D = cT_{\rm vac} + \frac{c}{n}T_{\rm gl} = cT_{\rm vac} + l, \tag{17}$$

which lead to,

$$D = \frac{U[L + (n-1)l]}{Mc^2 + U}\,. \tag{18}$$

The final position of the center of mass of the system is related by,

$$\begin{aligned} M' x_{\rm cm,f} &= M\left(\frac{L}{2} - D\right) + m\left(\frac{L}{2} + D_{\rm gl}\right) + \frac{U}{c^2}(L - D) \\ &= \frac{(M+m)L}{2} - \frac{MU[L+(n-1)l]}{Mc^2+U} + \frac{Ul(n-1)}{c^2} + \frac{U}{c^2}\left(L - \frac{U[L+(n-1)l]}{Mc^2+U}\right) \end{aligned}$$

[16]Thus, Nature keeps up from having to confront the issue that $c/n$ is the speed of an electromagnetic wave inside a medium of index $n$ only if that medium is at rest.

$$
\begin{aligned}
&= \frac{(M+m)L}{2} - \left(M + \frac{U}{c^2}\right)\frac{U[L+(n-1)l]}{Mc^2+U} + \frac{U[L+(n-1)l]}{c^2} \\
&= \frac{(M+m)L}{2}\,, \qquad (19)
\end{aligned}
$$

which is the same as its initial position (7).

Although this analysis is doubtful, its result is satisfactory in supporting the center-of-energy theorem, and it is amusing that this success required use of a nonzero displacement, eq. (16), of the glass block by the passing laser pulse.

## 2.4 Analysis with the Glass Block Attached to the Box

### 2.4.1 The Glass Block has Length $l = L$

For the case where the glass block is "rigidly" attached to the box, as in [1], the simplest version is that the length $l$ of the glass block is the same as the length $L$ of the box. Then, we don't need to consider the laser pulse in vacuum, but only inside the glass block where it has energy $U$ and momentum $U\,\hat{\mathbf{x}}/c\,n$, as in sec. 2.3.2.[17] We now take $M$ to be the combined mass of the glass block plus the box when the laser pulse is traveling inside the glass, and $M' = M + U/c^2$ is the total mass of the system. Then, the initial position of the center of mass of the system, $x_{\mathrm{cm,i}}$, is related by,

$$
M' x_{\mathrm{cm,i}} = \frac{U}{c^2}0 + M\frac{L}{2} = \frac{ML}{2}\,. \qquad (20)
$$

In the (doubtful) approximation of a "rigid" box, it recoils with speed,

$$
V = \frac{U}{Mcn}\,, \qquad (21)
$$

in the $-x$ direction when the laser pulse is emitted at time $t = 0$. The laser pulse is reabsorbed at the right end of the box at time $T$, before which the box moved distance $D = VT$ to the left, while the laser pulse moved with speed $c/n$ to the right, such that,

$$
T = \frac{L-D}{c/n}\,, \qquad D = VT = \frac{UL}{Mc^2} - \frac{UD}{Mc^2}\,, \qquad D = \frac{UL}{Mc^2+U} = \frac{UL}{M'c^2}\,. \qquad (22)
$$

The final center of mass of the system, $x_{\mathrm{cm,f}}$, is related by,

$$
M' x_{\mathrm{cm,f}} = \frac{U}{c^2}(L-D) + M\left(\frac{L}{2} - D\right) = \frac{UL}{c^2}\left(1 - \frac{U}{Mc^2+U}\right) + \frac{ML}{2} - \frac{MUL}{Mc^2+U} = \frac{ML}{2}\,, \qquad (23)
$$

which is the same as the initial position (20).

Note the formal similarity of the analysis of this case to that in sec. 2.2.

[17] We again note that in [1, 22], the wave has energy $n^2U$ and momentum $nU\,\hat{\mathbf{x}}/c$ when inside the glass.

### 2.4.2 The Glass Block has Length $l < L$

If the glass block has length $l < L$, then the laser pulse travels in vacuum before and after its time inside the glass block. The initial position of the center of mass of the system is again given by eq. (20). The recoil velocity of the box + glass block when the laser pulse is in vacuum is again given by eq. (4), which we now write as,

$$V_{\rm vac} = \frac{U}{Mc}\,, \qquad (24)$$

where $M$ is the mass of the box + glass while the laser pulse is in flight (as in sec. 2.4.1). However, when the laser pulse is inside the glass, and its momentum is $U\,\hat{\mathbf{x}}/c\,n$, as noted in eq. (10), the speed (to the left) of the box + glass (if "rigid") is only,

$$V_{\rm gl} = \frac{U}{Mc\,n}\,. \qquad (25)$$

The time $T_{\rm gl}$ that the laser pulse spends inside the glass is related to the distance $D_{\rm gl}$ that the box + glass moves (to the left) while the pulse is inside the glass by,

$$T_{\rm gl} = \frac{l - D_{\rm gl}}{c/n}\,, \qquad D_{\rm gl} = V_{\rm gl}T_{\rm gl} = \frac{Ul}{Mc^2} - \frac{UD_{\rm gl}}{Mc^2}\,, \qquad D_{\rm gl} = \frac{Ul}{Mc^2 + U} = \frac{Ul}{M'c^2}\,, \qquad (26)$$

as in eq. (22), replacing $L$ by $l$. Similarly, The time $T_{\rm vac}$ that the laser pulse spends in vacuum is related to the distance $D_{\rm vac}$ that the box + glass moves (to the left) while the pulse is in vacuum by,

$$T_{\rm vac} = \frac{L - l - D_{\rm vac}}{c}\,, \quad D_{\rm vac} = V_{\rm vac}T_{\rm vac} = \frac{U(L-l)}{Mc^2} - \frac{UD_{\rm vac}}{Mc^2}\,, \quad D_{\rm vac} = \frac{U(L-l)}{Mc^2 + U} = \frac{U(L-l)}{M'c^2}\,. \qquad (27)$$

The total distance $D$ that the box + glass travels (to the left) while the laser pulse is in flight is,

$$D = D_{\rm vac} + D_{\rm gl} = \frac{UL}{M'c^2}\,, \qquad (28)$$

The final center of mass of the system, $x_{\rm cm,f}$, is (again) related by,

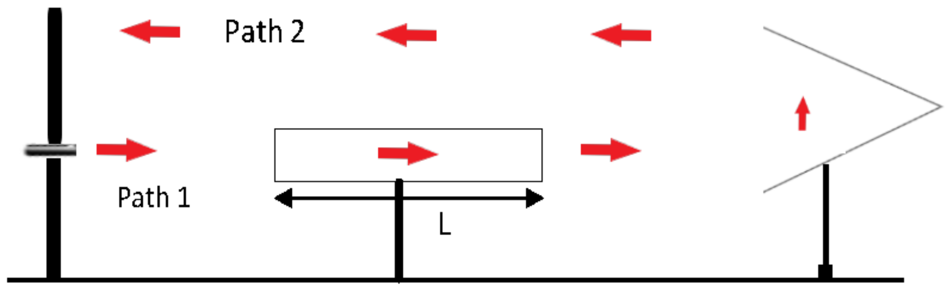
$$M'x_{\rm cm,f} = \frac{U}{c^2}(L - D) + M\left(\frac{L}{2} - D\right) = \frac{UL}{c^2}\left(1 - \frac{U}{Mc^2 + U}\right) + \frac{ML}{2} - \frac{MUL}{Mc^2 + U} = \frac{ML}{2}\,, \qquad (29)$$

which is the same as the initial position (20).

Thus, the center of mass remains at rest in McClymer's example whether or not the glass block is attached to the box. *No "bootstrap spaceships"!*

### 2.4.3 Scenario with a Retroreflector

The actual configuration discussed in [1] includes a retroreflector, as shown below.




This adds the complications that the box moves downwards during the short time that the laser pulse moves upwards inside the retroreflector, and the box also rotates during this time. But, the center of mass of the system still remains at rest (if initially at rest).

## 3 McClymer's Spaceship and the Planck Length

Bekenstein noted [33, 34] that in examples like McClymer's, if the laser pulse consisted of only a single optical photon, then the displacement $D$ of the glass block would be less than the Planck length [35],

$$L_P = \sqrt{\hbar G/c^3} \approx 1.6 \times 10^{-35} \text{ m}. \tag{30}$$

The argument/speculation of Bekenstein was that if space were grainy on the Planck scale (as suggested by Clifford (1876) [36], and Wheeler (1962) [37]), such a tiny displacement would be impossible, and the single photon would not be transmitted, but would be reflected.

If this were so, then in McClymer's example, the photon would return to the laser and presumably be reabsorbed. Indeed, since the box would recoil by less than a Planck length, the speculation might be that the laser could not even emit just a single photon.

The present author is on record [38] as being skeptical about Bekenstein's speculation.

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