

2 Newton's First and Second Laws for Linear Motion

Introduction

The central insights on which Ph101 are based are Newton's three laws of motion. In this lab you can explore the first two law's in a simple situation: linear motion = motion in only one direction.

The lab has two parts:

1. Study of motion when the total applied force vanishes: the velocity v should be constant (First Law).
2. Study of motion when the applied force is constant: the acceleration a should then be constant as related by $F = ma$ (Second Law).

Newton's laws were discovered relatively late in the history of mankind in large part due to the difficulty of arranging that the total force on an object that is free to move be zero or constant. In this lab you can use an 'air track' to provide near ideal conditions for the study of motion in one dimension. Measurements with 1% relative accuracy are possible with this apparatus.

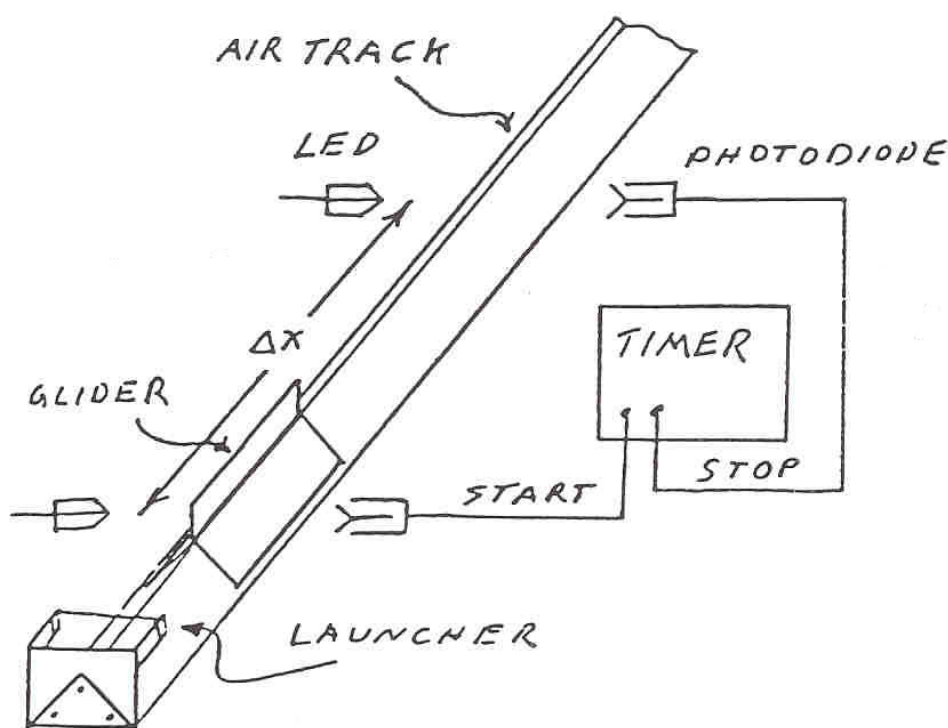


Figure 1: An airtrack with two photogates.

It is strongly recommended that you explore the procedures of secs. 2.1.1 and 2.2.1 to familiarize yourself with the apparatus before recording any measurements in your lab book.

2.1 The First Law

After you launch a ‘glider’ on an air track the air flowing out the small holes along the track should balance the force of gravity. If so, the velocity of the glider should remain constant – until acted on by some other force such as the bumper at the end of the air track.

Measurements of the time when the glider is at various positions can be made with ‘photogates’ = pairs of light-emitting diodes (LED) and phototransistors that are connected to your lab PC via the game port. The phototransistor detects when the light beam from the LED is either blocked or unblocked and the computer records the time of these events. The light from the LED is in the infrared, that is, of wavelengths too long to be seen by your eye (like that used with television remote controls). Figure 1 sketches the setup for an air track with two photogates. Four photogates are available in the present lab.

The manufacturer claims that the accuracy of time measurements with the photogates is 0.0001 sec.

The photogate timing is controlled via the dos program pt in directory c:\timer. Appended to this writeup is a summary of the menus of this program, and a timing diagram for the various modes of use. Select Photogate Status Check to verify that all four of your photogates respond to blocking and unblocking of the light path.

Position the four photogates to cover the range over which the glider can move when the tape and hanging weight are attached. The tape will slide over the curved arm at the opposite end of the air track from the launcher. This tape will not be used until part 2.2 but you can save time later by choosing appropriate locations now. The glider must pass through the photogates in 1-2-3-4 order as determined by the labels on the small interface box. The glider must be started before any part of it intercepts the first photogate, and must completely pass through the 4th photogate before the hanging weight hits the floor.

2.1.1 Leveling the Air Track Via Comparison of Time Intervals

If the total force on an object vanishes the velocity of the object should be constant (possibly zero). Use the **Four Gates** option of the **Gate Timing Modes** of the timer program to record the time intervals during which a moving glider interrupts the photogate beams as it travels along the air track. If the velocity is constant these intervals should, of course, be equal.

The Normal Display mode permits you to view the results of several successive timing runs.

Use a rubber band with the fork at one end of the air track to launch the glider. The four time intervals displayed on the computer should be nearly equal. If they increase or decrease from beginning to end the air track may not be level; if so, adjust the level and repeat until uniform times are observed. Be sure to tighten the lock nut (if present) on the jack screw after each adjustment.

The goal is that all four time intervals be equal to 1/2%. As you near this goal you may notice small variations between different photogate times that suggest the air track is bent slightly.

You can take multiple readings more conveniently if you do not slide the glider back through the first photogate, since this triggers the measurement sequence. Lift the glider off the track after a measurement and place it by the launcher for the next data sample. If

you collect garbled measurements you can, of course, start over, but also you can delete the garbled lines later with the **Delete Data** command of the **Data Analysis Options** menu.

Once the track is level, practice launching the glider until you achieve roughly 2-3% reproducibility from launch to launch. Then record a data set containing at least 5 good launches. Print this data set for your lab book.

In this Lab each group can share data sets.

2.1.2 Distance *vs.* Time

After you have obtained satisfactory results with the **Four Gates** timing mode, perform a study of distance *vs.* time using the **Gate and Pulse (4 Gates)** mode under the **Miscellaneous Timing Modes** menu. Analyze your data for a possible acceleration a according to the relation

$$x = x_0 + v_0t + \frac{1}{2}at^2, \quad (1)$$

and determine if a is 'small' and what $\Delta v/v_0$ it corresponds to.

First determine the positions at which the glider intercepts the photogates. With the air off, slide the glider away from the launcher until it just blocks the first photogate. Record the position (and error estimate!) of the leading edge of the glider using the scale on the air track. Then slide the glider away from the launcher until its trailing edge just unblocks the photogate and record the position. Similarly for gates 2-4.

As a check, also record the length of the aluminum plate on the glider that passes through the photogate. This should equal the difference between the positions at which the leading and trailing edge of the glider intercepts each gate.

If it happens that you launch the glider from the end of the scale with large numbers your velocities will be negative.

Launch the glider with the rubber band as before and collect at least 5 data samples in which 7 times are recorded for each sample. Use the option **Display Table of Data** to make a first evaluation of the quality of your data. For each column the standard deviation should be small compared to the mean. Also, the means of columns 1, 3, 5 and 7 should be nearly equal if the acceleration is small. When you have a good data set save it to a disk file with a **.dat** extension and print it for your lab book.

Use the program **StatMost** for detailed analysis of your data. Exit the timer program to dos and type **win** to start Windows. Click on the **StatMost** icon in the **Phys101** window. Start a new sheet document and click on **Import Data** using your **.dat** file. Choose **Delimited with Comma** and import only the lines of data, probably beginning with line 11 and not including the last ("EOF") line of the file.

The following procedure can be used to average your several data samples. First interchange rows and columns of your spreadsheet by clicking on **Data** and then on **Transpose**. Bring up the **Row Statistics** window by clicking on **Statistics** and then dragging on **Summaries** and then on **Row Statistics**. Choose only the **Mean** option and click on **OK** to create a new column that will contain the mean of the selected columns (the default is all columns selected).

Convert the 7 time intervals into cumulative times as follows. Note that the first row does not contain times but rather just column numbers. Replace the value of the first row of the

Mean column with a 0 by clicking on that box, typing 0 in the data-entry box at the top of the spreadsheet, and then hit Enter/Return. This adds an initial data point corresponding to $t = 0$ to your data set. Bring up the Accumulate window by clicking on Data, then dragging on Transform and then on Accumulate. Select only the Mean column and click on OK to create a new column labelled Accum that contains the cumulative sums of the times. It may be helpful to relabel the new Accum column as Time.

Fill the first empty column with the 8 distances (in cm) you recorded earlier corresponding to the position of the glider when it blocks and unblocks the 4 photogates. Click on the top box of the empty column and type the first distance in the data entry box. Hit Enter/Return to enter the value into the spreadsheet; type in the second distance, etc. Label this column as Distance.

Save your spreadsheet by clicking on File and Save As... and typing a filename with a .dmd extension.

Finally, fit the distances to a second-order polynomial in time (i.e., to eq. (1)) and graph your results. Click on Analyses and drag on Polynomial Regression to fit your data. In the Polynomial Regression window choose Order as 2, and Show Option as Graph. To avoid cluttering your spreadsheet with extra columns, turns off all the Save Options. Select the column with the accumulated times as X: Independent and the column with distances as Y: Dependent. Click on OK to produce the graph and a summary of the fit results. You can relabel the axes of your graph by double clicking on the existing label and typing a new one in the window that pops up. Save the graph with the same name as your spreadsheet but with a .dmg extension. Print the graph and also the summary window (labelled \$RES000n.TXT).

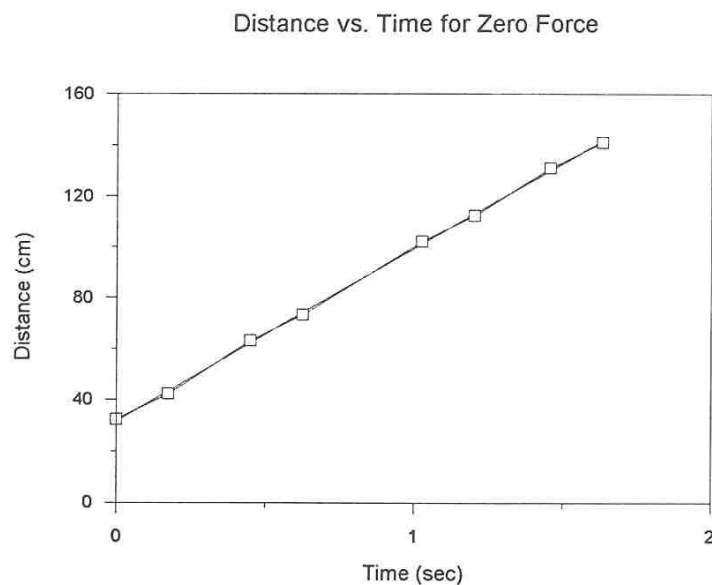


Figure 2: Sample graph of data on distance *vs.* time for zero force.

Figure 2 shows a sample graph of distance *vs.* time which appears to have nearly constant slope, corresponding to nearly uniform velocity.

Scroll down the \$RES000n.TXT window to find the **Table of Estimates** which includes the fit Estimate and Standard Error for coefficients c_i of Order i according to

$$x = c_0 + c_1t + c_2t^2. \quad (2)$$

Comparing with eq. (1) we see that $c_1 = v_0$ and $c_2 = a/2$. How large is your observed acceleration compared to that of gravity? Is your observed acceleration significantly different from zero, using the **Standard Error** reported by the computer as the measure of significance? The **Table of Estimates** reports the **t-Value** for each fit coefficient; this is just the number of standard deviations the coefficient is away from zero. The **P-Value** is the computer's estimate of the probability that the fit coefficient is actually zero (based on eq. (6) of Lab 1 for the bell curve).

Another measure of the significance of the observed acceleration is the resulting change in velocity, $\Delta v = at$, over the time of your measurements. What is $\Delta v/v_0$ for your data?

2.2 The Second Law

2.2.1 Air Track with Constant Force

According to Newton, if a constant force is applied to a system it should undergo a constant acceleration a and its motion should obey eq. (1). *To study this, attach a tape to your air-track glider of mass M and hang a weight $m = 15\text{-}20$ gm from the other end of the tape. The tape should pass over the curve section appended to the air track and air should be flowing out of the holes in this appendage. If necessary, adjust the air-flow rate with the small thumb screw. Use the technique of sec. 2.1.2 to measure the resulting acceleration when the glider is released from rest.*

Take at least 5 data samples, all with the glider released from rest at the same position. Remeasure the positions of the 4 photogates after collecting your data – in case the photogates have been bumped. Weigh your and the hanging mass to determine M and m . Figure 3 shows a sample data analysis.

The total mass of your system is $M + m$ while the total force is just that of gravity on the hanging mass, *i.e.*, mg . Hence Newton's second law tells us that

$$a = \frac{m}{M + m}g. \quad (3)$$

Invert this to make an estimate of g based on the results of the second-order fit to your (averaged) data on distance *vs.* time. Include the corresponding error estimate. If your value of g differs from the standard value of 980 cm/s² by more than twice the error estimate something is likely wrong; if so, recheck your analysis procedure and if necessary take new data.

You might wish to see what happens if you fit your data to a third-order polynomial. If the acceleration is really constant the third-order fit coefficient should be zero within its error estimate.

2.2.2 Simplified Error Estimate

Since the above procedures are a bit complicated it is useful to think about a simplified version. From this we can make a quick estimate of the accuracy of the measurement of g .

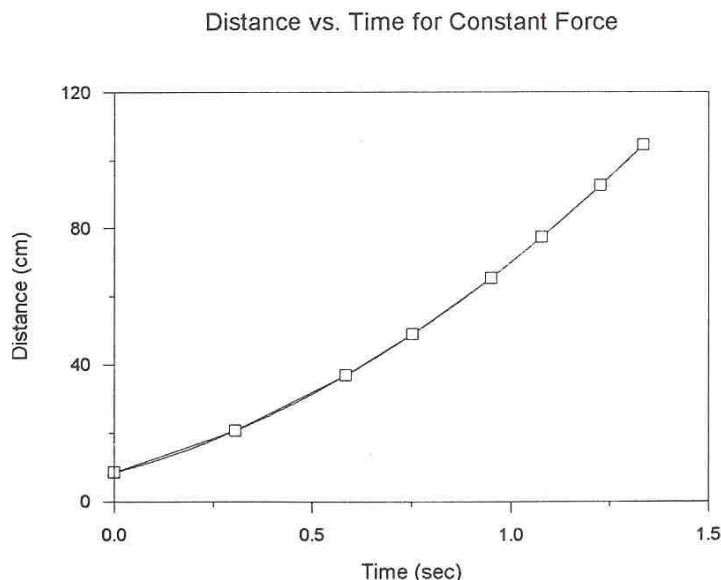


Figure 3: Sample graph of data on distance *vs.* time for constant force.

The simplification is to suppose the glider was released from rest at the position of the first photogate which is at $x_0 = 0$. Then we should have

$$x = \frac{1}{2}at^2, \quad \text{so that} \quad a = \frac{2x}{t^2}, \quad \text{and hence} \quad g = \frac{M+m}{m} \frac{2x}{t^2} \approx \frac{M}{m} \frac{2x}{t^2}, \quad (4)$$

using eq. (3). Then according to eq. (16) of Lab 1

$$\frac{\sigma_g}{g} \approx \sqrt{\left(\frac{\sigma_M}{M}\right)^2 + \left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_x}{x}\right)^2 + 4\left(\frac{\sigma_t}{t}\right)^2} \approx \sqrt{\left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_x}{x}\right)^2}, \quad (5)$$

since $M \gg m$ but $\sigma_M \approx \sigma_m$, and σ_t is claimed to be small.

Taking x to be the distance between the first and last photogates, make estimates of σ_x and σ_m to evaluate σ_g/g according to eq. (5). How does the simplified estimate compare that your result obtained from the computer analysis, and with $\Delta g/g = g_{\text{measured}}/980 - 1$?

2.3 Falling Object

After completing the above analyses, make a measurement of g using a falling object: a ‘picket fence’ consisting of a piece of plastic with 12 pieces of tape on it at equal intervals (Fig. 4).

Remove the first photogate from the air track and position it so the picket fence can be dropped vertically through the gate. Exit StatMost and Windows and restart program pt. Choose the Motion Timer option. Hold the picket fence with its lower end just above the photogate and drop it. Press Enter/Return to end the data collection. If the data sample does not include 11 time intervals, try again.

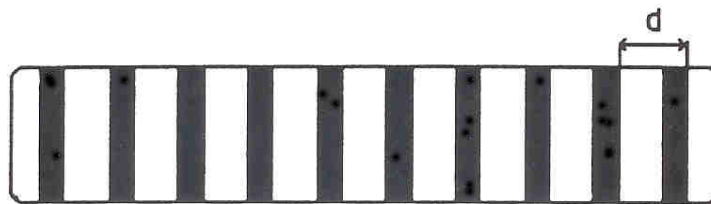


Figure 4: The 'picket fence'. Measure the distance d .

The program `pt` can be used for data analysis consisting of fitting a straight line to a graph, but it cannot perform a polynomial fit. So instead of studying eq. (1), you can study the dependence of velocity on time for a falling object:

$$v = v_0 + gt. \quad (6)$$

To analyse your data choose **Data Analysis Options** from the main menu. Then choose **Graph Data and Velocity vs. Time**. This option assumes you have used a picket fence and asks you to enter the spacing d (see Fig. 4) in meters. The program then calculates a velocity $v = d/\Delta t$ for each time interval Δt in your data sample. For **Graph Style** choose **Point Protectors, Regression Line and Statistics**. Accept the default scaling options and hit **Enter/Return** to produce the graph. Figure 5 is an example.

Figure 5: Example of the analysis of data from a falling picket fence.

The parameter M from the analysis is the slope of the velocity *vs.* time curve, which should just be g . Record this value and the corresponding error estimate. Repeat the whole procedure 10 times. Print one graph for your lab notebook.

Determine the mean and standard deviation of your 10 measurements of g , and plot a histogram of the 10 values. For this you could exit `pt` and run `StatMost` under `Windows` and use the procedure outlined in sec. 1.4 of Lab 1. How does the standard deviation of your 10 measurements of g compare with the error estimate provided by program `pt`?

Recalling the simplified error analysis of sec. 2.2.2, do you expect a better measurement of g from the method of this section or from that of sec. 2.2.1?

2.4 Appendix: Polynomial Regression

In this technical Appendix we sketch the formalism used in the polynomial regression method for fitting data. This is a generalization of the method of linear regression.

We start with a set of data (x_j, y_j) , $j = 1, \dots, m$, and we wish to fit these data to the polynomial

$$y(x) = \sum_{i=0}^n c_i x^i. \quad (7)$$

In general each measurement y_j has a corresponding uncertainty σ_j . That is, if the measurements were repeated many times at coordinate x_j the values of y_j would follow a gaussian distribution of standard deviation σ_j . We indicate below how the programs `pt` and `StatMost` proceed in the absence of input data as to the σ_j .

Because of the uncertainties in the measurements y_j we cannot expect to find the ideal values of the coefficients c_i , but only a set of best estimates we will call \bar{c}_i . However, we will also obtain estimates of the uncertainties in these best-fit parameters which we will label as $\sigma_{\bar{c}_i}$. The best-fit polynomial is then

$$\bar{y}(x) = \sum_{i=0}^n \bar{c}_i x^i. \quad (8)$$

The method to find the \bar{c}_i is called least-squares fitting as well as polynomial regression because we minimize the square of the deviations. We introduce the famous *chi square*:

$$\chi^2 = \sum_{j=1}^m \frac{(y_j - \bar{y}(x_j))^2}{\sigma_j^2} = \sum_j \frac{(y_j - \sum_i^n \bar{c}_i x_j^i)^2}{\sigma_j^2}. \quad (9)$$

Note that $\exp(-\chi^2/2)$ is the (un-normalized) probability distribution for observing a set of variables y_j supposing the true values of the coefficients are \bar{c}_i .

A great insight is that $\exp(-\chi^2/2)$ can be thought of another way. It is also the (un-normalized) probability distribution that the polynomial coefficients have values c_i given that the measurements have values y_j . Expressing this in symbols,

$$\exp(-\chi^2/2) = \text{const} \times \exp\left(-\sum_k^n \sum_l^n \frac{(c_k - \bar{c}_k)(c_l - \bar{c}_l)}{2\sigma_{kl}^2}\right), \quad (10)$$

or equivalently

$$\chi^2/2 = \text{const} + \sum_k^n \sum_l^n \frac{(c_k - \bar{c}_k)(c_l - \bar{c}_l)}{2\sigma_{kl}^2}. \quad (11)$$

The uncertainty on \bar{c}_k is σ_{kk} in this notation. In eqs. (10) and (11) we have introduced the important concept that the uncertainties in the \bar{c}_k are correlated. That is, the quantity σ_{kl}^2 is a measure of the probability that the values of c_k and c_l both have positive fluctuations at the same time. In fact, σ_{kl}^2 can be negative indicating that when c_k has a positive fluctuation then c_l has a correlated negative one.

One way to see the merit of minimizing the χ^2 is as follows. According to eq. (11) the derivative of χ^2 with respect to c_k is

$$\frac{\partial\chi^2/2}{\partial c_k} = \sum_l^n \frac{c_l - \bar{c}_l}{\sigma_{kl}^2}, \quad (12)$$

so that all first derivatives of χ^2 vanish when all $c_l = \bar{c}_l$. That is, χ^2 is a minimum when the coefficients take on their ‘best-fit’ values \bar{c}_i . A further benefit is obtained from the second derivatives:

$$\frac{\partial^2\chi^2/2}{\partial c_k\partial c_l} = \frac{1}{\sigma_{kl}^2}. \quad (13)$$

In practice we evaluate the χ^2 according to eq. (9) based on the measured data. Taking derivatives we find

$$\frac{\partial\chi^2/2}{\partial\bar{c}_k} = \sum_j^m \frac{(y_j - \sum_i^n \bar{c}_i x_j^i)(-x_j^k)}{\sigma_j^2} = \sum_i^n \sum_j^m \frac{\bar{c}_i x_j^i x_j^k}{\sigma_j^2} - \sum_j^m \frac{y_j x_j^k}{\sigma_j^2}, \quad (14)$$

and

$$\frac{\partial^2\chi^2/2}{\partial\bar{c}_k\partial\bar{c}_l} = \sum_j^m \frac{x_j^k x_j^l}{\sigma_j^2} \equiv M_{kl}. \quad (15)$$

To find the minimum χ^2 we set all derivatives (14) to zero, leading to

$$\sum_i^n \sum_j^m \frac{x_j^i x_j^k}{\sigma_j^2} \bar{c}_i = \sum_j^m \frac{y_j x_j^k}{\sigma_j^2} \equiv V_k. \quad (16)$$

Using the matrix M_{kl} introduced in eq. (15) this can be written as

$$\sum_i^n M_{ik} \bar{c}_i = V_k. \quad (17)$$

We then calculate the inverse matrix M^{-1} and apply it to find the desired coefficients:

$$\bar{c}_k = \sum_l^n M_{kl}^{-1} V_l. \quad (18)$$

Comparing eqs. (13) and (15) we have

$$\frac{1}{\sigma_{kl}^2} = M_{kl}. \quad (19)$$

The uncertainty in best-fit coefficient \bar{c}_i is then reported as

$$\sigma_{\bar{c}_i} = \sigma_{ii} = \frac{1}{\sqrt{M_{ii}}}. \quad (20)$$

Programs `pt` and `StatMost` do not provide you directly with the values of σ_{ij} when $i \neq j$, but these values are used in the calculation of the confidence bands described below.

2.4.1 Procedure When the σ_j Are Not Known

This method can still be used even if the uncertainties σ_j on the measurements y_j are not known. When the functional form (7) correctly describes the data we claim that on average the minimum χ^2 has value $m - n - 1$. (The whole fitting procedure does not make sense unless there are more data points than parameters being fitted.) To take advantage of this remarkable result we suppose that all uncertainties σ_j have a common value, σ . Then

$$\chi^2 = \sum_j^m \frac{(y_j - \bar{y}(x_j))^2}{\sigma^2} \approx m - n - 1, \quad \text{so that} \quad \frac{1}{\sigma_j^2} = \frac{1}{\sigma^2} = \frac{m - n - 1}{\sum_j^m (y_j - \sum_i^n \bar{c}_i x_j^i)^2}. \quad (21)$$

which can be used in the numerical evaluation of V_k and M_{kl} above.

In practice it appears that the error estimates from this procedure are more realistic if a fit is made using a polynomial with one order higher than needed for a ‘good’ fit to the data.

2.4.2 Confidence Bands

The Linear and Nonlinear Regression options of program StatMost can also draw ‘confidence bands’ around the best-fit solution. The ‘true’ solution is supposed to lie somewhere inside these bands with 95% probability (confidence). These bands define a relatively narrow region near the center of the data sample, but become farther apart at the edges of the data where our knowledge is poorer.

The confidence bands are the curves defined by $\bar{y}(x) \pm 2\sigma_{\bar{y}(x)}$, where $\bar{y}(x)$ is the best-fit solution, eq. (8) for polynomial fits, and $\sigma_{\bar{y}(x)}$ is the uncertainty in the best-fit curve at coordinate x . Recalling eq. (9) of Lab 1 and writing the general polynomial as in eq. (7) we have

$$\begin{aligned} \sigma_{\bar{y}(x)}^2 &= \langle (y_{\text{general}}(x) - \bar{y}_{\text{best-fit}}(x))^2 \rangle = \left\langle \left(\sum_i^n (c_i - \bar{c}_i) x^i \right)^2 \right\rangle = \sum_i^n \sum_j^n \langle (c_i - \bar{c}_i)(c_j - \bar{c}_j) \rangle x^i x^j \\ &= \sum_i^n \sum_j^n \sigma_{ij}^2 x^i x^j, \end{aligned} \quad (22)$$

where σ_{ij}^2 is obtained from eq. (19).