Princeton University
Ph304 Problem Set 9
Electrodynamics

(Due in class, Wednesday Apr. 16, 2003)

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Text: *Introduction to Electrodynamics, 3rd ed.*
by D.J. Griffiths (Prentice Hall, ISBN 0-13-805326-X, now in 6th printing)
Errata at http://academic.reed.edu/physics/faculty/griffiths.html

Reading: Griffiths secs. 9.1-9.3.

1. Griffiths' prob. 9.6. In part a), solve for the complex reflected and transmitted amplitudes $\tilde{A}_R$ and $\tilde{A}_T$ for strings with dispersion relations $k_i = \omega\sqrt{\mu_i/T}$, $i = 1, 2$, where μ_i is the mass per unit length and T is the tension (assumed to be constant, which ignores the fact that the strings could not stretch unless they were elastic). Part b) then consists of setting $k_2 = 0$ in part a). Note that energy conservation requires waves on massive strings to obey $k_1 \left|\tilde{A}_R\right|^2 + k_2 \left|\tilde{A}_T\right|^2 = k_1 \left|\tilde{A}_I\right|^2$ (energy density $\propto \mu\omega^2 \left|\tilde{A}\right|^2 = k^2 T \left|\tilde{A}\right|^2$, and energy flow is $v_{\text{group}} = d\omega/dk = \omega/k = \sqrt{T/\mu}$ times energy density). A "massless" string cannot have any curvature ($k = 0$), and it carries no energy, so the above condition reduces to $\left|\tilde{A}_R\right| = \left|\tilde{A}_I\right|$ in part b). Indeed, the "massless string" could be replaced by a frictionless rod perpendicular to the massive string, on which the "knot" of mass m slides. In this case, you would not even consider a transmitted wave in part b)...

   This problem illustrates the dilemma of the Maxwellians: Experience from mechanics implies that a medium must have mass to transmit energy via waves. Hence, the search for the æther.

   From 140 years distance it is easy to dismiss this prejudice as naïve, but perhaps the current enthusiasm for "string" theory in higher dimensional space contains its own elements of naïvety that will take a generation of effort to resolve.

2. Griffiths' prob. 9.7.

3. Griffiths' prob. 9.12.

4. Griffiths' prob. 9.16.

5. Griffiths' prob. 9.34. Griffiths seems to suggest working this via matching 5 waves at 2 boundaries – which is fine. But another approach works also. Namely, consider the transmitted wave to be the result of interference of multiple reflections at the 1-2 and 2-3 interfaces. The simplest transmitted wavelet has (relative) amplitude $t_{12}e^{i\Delta}t_{23}$, where t_{12} is the amplitude transmission coefficient at the 1-2 boundary, and Δ is the phase shift of the wave while crossing medium 2. The 2nd piece consists of transmission at 1-2, reflection at 2-3, reflection at 2-1, and finally transmission at 2-3. The amplitude for this is $t_{12}e^{i\Delta}r_{23}e^{i\Delta}r_{21}e^{i\Delta}t_{23}$. And so on. The series is easy to sum, yielding the amplitude t_{13}. Square to find T....

   The real point of this problem is to choose n_2 and d so as to make $T = 1$. Good lenses are coated with thin films to satisfy this desirable condition. The choice for d is "obvious". Verify that the choice $n_2^2 = n_1 n_3$ provides the desired "index matching". It suffices to verfify that $|t_{13}| = 1$ in this case.

6. Griffiths' prob. 9.37