Ph 406: Elementary Particle Physics
Problem Set 5

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1. **Positronium**

   The concept of positronium, an atom made of an electron and a positron, was invented by Princetonian John Wheeler – who called it a polyelectron.[1]

   What symmetry principles would be violated if the following 1-photon transitions between excited states $n^{2s+1}L_j$ of positronium were observed (via microwave pumping)?

   (a) $1^3S_1 \to 2^3S_1$

   (b) $1^3S_1 \to 2^1S_0$

   (c) $1^3S_1 \to 2^1P_1$

   (d) $1^3S_1 \to 2^3P_1$

   Polarized 1^3S_1 positronium can be formed when positrons from ^{22}Na β-decay combine with atomic electrons. This state decays to 3 photons (do you recall why 2-photon decay is forbidden?). Let $\hat{\mathbf{k}}_1$ and $\hat{\mathbf{k}}_2$ be the directions of the two higher-energy decay photons, and $\hat{\mathbf{S}}$ be the direction of the positronium spin. What symmetries would be violated by angular correlations of the following forms?

   (e) $\hat{\mathbf{S}} \cdot \hat{\mathbf{k}}_1 \times \hat{\mathbf{k}}_2$

   (f) $\hat{\mathbf{S}} \cdot \hat{\mathbf{k}}_1$

   (g) $(\hat{\mathbf{S}} \cdot \hat{\mathbf{k}}_1)(\hat{\mathbf{S}} \cdot \hat{\mathbf{k}}_1 \times \hat{\mathbf{k}}_2)$

   (h) $\hat{\mathbf{S}} \cdot \hat{\boldsymbol{\epsilon}}_1 \times \hat{\mathbf{k}}_2$

   where $\hat{\boldsymbol{\epsilon}}_1$ is along the direction of polarization of final-state photon 1.

2. Λ^0 hyperons are produced by a pion beam in the reaction $\pi^- p \to K^0 \Lambda^0$, and observed by the decay $\Lambda^0 \to p\pi^-$ (which is a weak interaction that does not conserve parity). Let J denote the spin of the Λ (considered to be unknown in this problem, while the spins of the π^-, p and K^0 are known), and θ be the angle of a decay product in the Λ rest frame, relative to the direction of the Λ in the lab frame. In the case where the Λ is produced exactly along the beam direction, what are the possible values of J_z?

[1] J.A. Wheeler, *Polyelectrons*, Ann. N.Y. Acad. **48**, 219 (1946), http://kirkmcd.princeton.edu/examples/QM/wheeler_anyas_48_219_46.pdf.

Show that for unpolarized beam protons, and for Λ's produced along the beam direction, the Λ-decay angular distribution depends on J according to

$$\begin{aligned} J &= 1/2, \quad \text{isotropic}, \\ J &= 3/2, \quad 3\cos^2\theta + 1, \\ J &= 5/2, \quad 5\cos^4\theta - 2\cos^2\theta + 1. \end{aligned} \tag{1}$$

Hints in Sakurai, Invariance Principles and Elementary Particles *(1964), p. 17.*

3. The lowest-mass strange baryons Λ, Σ, Ξ decay weakly, and isospin is not conserved in these decays. For example, the isospin-0 particle Λ^0 decays to $p\pi^-$ which can only be in isospin 1/2 and 3/2 states. However, strange-baryon "resonances" such as the $\Lambda^0(1520)$ decay (quickly) via the strong interaction. Use isospin conservation to predict the relative decay rates of this state to pK^- and $n\bar{K}^0$, and the relative rates to $\Sigma^+\pi^-$, $\Sigma^0\pi^0$ and $\Sigma^-\pi^+$. Predict also the relative decay rates for $\Sigma(1660) \to p\bar{K}^0,\ n\bar{K}^0,\ pK^-,\ n\bar{K}^-$, and for $\Sigma(1660) \to \Sigma^+\pi^0,\ \Sigma^0\pi^+,\ \Sigma^+\pi^-,\ \Sigma^0\pi^0,\ \Sigma^-\pi^+,\ \Sigma^-\pi^0,\ \Sigma^0\pi^+$.

   Ignore the effect of small mass differences among particles in an isospin multiplet.

   Table of Clebsch-Gordan Coefficients: http://pdg.lbl.gov/2002/clebrpp.pdf

4. **Meson Theory of Hyperdeuterons**

   Estimate the relative binding energies of the 64 possible pairs of baryons in the spin-1/2 octet: $n,\ p,\ \Lambda,\ \Sigma^-,\ \Sigma^0,\ \Sigma^+,\ \Xi^-,\ \Xi^0$.

   For this, use a simplified one-pion-exchange model that the nuclear force is entirely due to exchanges of a single π meson, and that the operator $g^2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ characterizes the charge independence of this interaction.[2] Here g is a coupling constant, and $\boldsymbol{\tau}$ is the isospin-1 operator (because pions form an $I = 1$ multiplet). That is, ignore electromagnetic effects and spin-dependent effects.

   (A harder version of the problem would be to deduce that $g^2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ is the appropriate operator.)

   A hint is that the Hamiltonian relevant to binding of the dibaryons is $H \propto g^2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$. Hence, we should consider the matrix elements $\langle B_1B_2|\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2|B_1B_2\rangle$, where B is any member of the baryon octet. As for electricity, we infer that a negative matrix element implies an attractive force, and bound states, while a positive matrix element implies repulsion.

   Note that

$$\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 = \frac{1}{2}\left(\boldsymbol{\tau}^2 - \boldsymbol{\tau}_1^2 - \boldsymbol{\tau}_2^2\right).$$

   Also, charge independence means you don't have to look at each of the 64 pairs separately, but you can more simply consider pairs of isospin multiplets, each of which leads to one or more multiplets of total isospin exactly as for combinations of ordinary

[2] If this interaction is represented by a Feynman diagram with single pion exchange, then each of the $BB\pi$ vertices has strength $g\boldsymbol{\tau}$.

spin. For this, note that the nucleons, N, and the cascade particles, Ξ, each form an isodoublet, the Λ is an isosinglet, and the Σ's form an isotriplet.

Give the isospin wavefunctions of the candidate bound states.

I found that 11 of the 64 pairs should have bounds states, and that none of these would be more weakly bound than the deuteron.

Considerations somewhat similar to those of this problem are given in D.B. Lichtenberg and M.H. Ross, Pion Contribution to Hyperon-Nucleon Forces, Phys. Rev. **107***, 1714 (1957),* http://kirkmcd.princeton.edu/examples/EP/lichtenberg_pr_107_1714_57.pdf*.*
No dibaryon bound state other than the deuteron has ever been observed, although searches continue.[3] *The lightest known hypernucleus is* ${}^3_\Lambda H$,[4] *and even its antiparticle has been observed.*[5] *A* Σ*-hypernucleus is* ${}^4_\Sigma He$.[6] *A handful of examples of hyper-He nuclei containing two* Λ*'s have been reported.*[7]

5. The J/ψ meson is an (electrically neutral) $c\bar{c}$ state with mass = 3.1 GeV, $J^{PC} = 1^{--}$ and $I^G = 0^-$. Which of the following possible decay modes are allowed, and if forbidden, what symmetry would be violated if such decay were observed: $N\bar{N}$, $\pi^+\pi^-$, $\pi^0\pi^0$, $\gamma\gamma$, $\pi^0\gamma$, $\pi^0\gamma\gamma$, $\pi^+\pi^-\gamma$, $\pi^+\pi^-\pi^0$, $3\pi^0$, $4\pi^0$, $\pi^+\pi^-\pi^+\pi^-$?

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[3]See, for example, B.H. Kim *et al.*, *Search for an H-Dibaryon with a Mass near* $2m_\Lambda$ *in* $\Upsilon(1S)$ *and* $\Upsilon(2S)$ *Decays*, Phys. Rev. Lett. **110**, 222002 (2013),
http://kirkmcd.princeton.edu/examples/EP/kim_prl_110_222002_13.pdf.
For a review of ongoing modeling of such possible states, see S.R. Beane *et al.*, *Light nuclei and hypernuclei from quantum chromodynamics in the limit of SU(3) flavor symmetry*, Phys. Rev. D **87**, 034506 (2013),
http://kirkmcd.princeton.edu/examples/EP/beane_prd_87_034506_13.pdf.

[4]R.J. Prem and P.H. Steinberg, *Lifetimes of Hypernuclei,* ${}_\Lambda H^3$, ${}_\Lambda H^4$, ${}_\Lambda He^4$, Phys. Rev. **136**, B1803 (1964), http://kirkmcd.princeton.edu/examples/EP/prem_pr_136_B1803_64.pdf.

[5]STAR Collaboration, *Observation of an Antimatter Hypernucleus*, Science **328**, 58 (2010),
http://kirkmcd.princeton.edu/examples/EP/star_science_328_58_10.pdf.

[6]T. Nagae *et al.*, *Observation of a* ${}^4_\Sigma He$ *Bound State in the* ${}^4He(K^-,\pi^-)$ *Reaction at 600 MeV/c*, Phys. Rev. Lett. **80**, 1605 (1998),
http://kirkmcd.princeton.edu/examples/EP/nagae_prl_80_1605_98.pdf.

[7]See, for example, K. Nakazawa *et al.*, *Double-*Λ *Hypernuclei via the* Ξ^- *Hyperon Capture at Rest Reaction in a Hybrid Emulsion*, Nucl. Phys. A **835**, 207 (2010),
http://kirkmcd.princeton.edu/examples/EP/nakazawa_npa_835_207_10.pdf.