Ph 406 PROBLEM SET 6

DUE MARCH 29, 1993.

(1) The energies (in MeV), spins and parities of the ground and first four excited states of $^{207}_{82}Pb$ are as follows: $0.0\,\frac{1}{2}^-$; $0.57\,\frac{5}{2}^-$; $0.90\,\frac{3}{2}^-$; $1.63\,\frac{13}{2}^+$; $2.33\,\frac{7}{2}^-$. Discuss the shell model description of these states. The 0.57 MeV state has a lifetime of 10^{-10} s, while the lifetime of the 1.63 MeV state is 0.8 s. Account for this difference and predict the multipolarity of the γ-decay of the 1.63 MeV state. Estimate the lifetime of the 2.33 MeV state.

(2) Calculate the fraction of electrons emitted within 100 eV of the end-point in tritium β-decay, assuming the mass of the neutrino is zero. ($^3H \rightarrow {}^3He + e^- + \bar{\nu}_e + 18.60$ keV.)

(3) The possibility exists that neutrinos have mass and that there is an interaction between the neutrinos ν_e and ν_μ produced in weak-interaction processes, so that the neutrinos which are mass eigenstates ν_1 and ν_2 are in fact linear combinations of ν_e and ν_μ:

$$\nu_1 = \nu_e \cos\theta + \nu_\mu \sin\theta$$

$$\nu_2 = \nu_\mu \cos\theta - \nu_e \sin\theta$$

If at time $t=0$ electron neutrinos are produced in a weak interaction show that at time t the probability P of detecting a muon neutrino is given by:

$$P = \sin^2 2\theta \, \sin^2(1.27\Delta m^2 L/pc)$$

where $\Delta m^2 = (m_1^2 - m_2^2)c^4$ is in $(\text{eV})^2$, L is the distance from the source in metres, and p is the neutrino beam momentum in MeV/c. (The masses m_1 and m_2 may be taken to be so small that $E_i = pc + m_i^2 c^3/2p$, $i = 1, 2$.)

(4) Calculate a value for G_β from the observed ft value for ^{14}O β-decay of 3127 s.

(5) In an alternate universe there are three charge states of the spin-$\frac{1}{2}$ nucleons, +, 0, and −. These all have masses roughly that of the proton, and experience strong, electromagnetic, and weak interactions of strengths similar to those in our universe.

(a) Indicate the form of a semi-empirical mass formula for nuclei made from these nucleons, in terms of the nucleon numbers $A = N_+ + N_0 + N_-$, $Z = N_+ - N_-$, and N_0. You may ignore any pairing term.

(b) Deduce the trend of Z and N_0 *vs.* A for beta-stable nuclei. Are heavier nuclei more likely to decay via fission or 'α'-decay (where an 'α' particle in this universe has $N_+ = N_0 = N_- = 2$)?

Would you like to live in this universe?

(6) Excited nuclei:

(a) The spin-parity J^P and excitation energy E of the first few states of $^{170}_{72}$Hf are

J^P	0^+	2^+	4^+	6^+	8^+
E(keV)	0	100	321	641	1041

Explain how these states are indicative of rotational excitations, and deduce the moment of inertia of the Hf nucleus. Compare this with the moment of inertia supposing the Hf nucleus is a sphere ($I = \frac{2}{5}MR^2$).

(b) Give the shell-model configurations for the ground state and first five excited states of $^{15}_{7}$N shown below, supposing all states can be accounted for by single-particle excitations.