# Ph 406: Elementary Particle Physics
# Problem Set 6

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1. **Two Body Decay**

   Consider the decay of the neutral $\pi$ meson of (total) energy $E_\pi$ to two photons, $\pi^0 \to \gamma\gamma$.

   (a) If the two photons are observed in the laboratory with energies $E_1$ and $E_2$ and angle $\alpha$ between them, what is their invariant mass?

   (b) If the decay of the $\pi^0$ is isotropic in its rest frame, what is the laboratory distribution $dN/dE_\gamma$ of the energies of the decay photons?

   (c) What is the minimum opening angle, $\alpha_{\rm min}$, between the two photons in the lab frame?

   (d) What is the distribution $dN/d\alpha$ of the opening angle between the two photons in the lab frame?

   (e) If the two photons are detected at positions $x_1$ and $x_2$ in a plane perpendicular to the direction of the $\pi^0$ at a distance $D$, what is the projected impact point $x$ of the $\pi^0$ had it not decayed? You may assume that $|x_1 - x_2| \ll D$, which is true for most, but not quite all, decays if $E_\pi/m_\pi \gg 1$.

   (f) What is the maximum laboratory angle $\theta_{\rm max}$ between the direction of a photon from $\pi^0$ decay and the direction of the $\pi^0$, supposing the photon is observed to have energy $E_\gamma \gg m_\pi$?

   (g) Suppose $\pi^0$'s are produced in some scattering process with distribution $N_\pi(E_\pi, \theta_\pi)$, where angle $\theta_\pi$ is measured with respect to the beam direction. That is, $N_\pi(E_\pi, \theta_\pi)\ dE_\pi\ d\Omega_\pi$ is the number of $\pi^0$'s in energy interval $dE_\pi$ centered about energy $E_\pi$ that point towards solid angle $d\Omega_\pi$ centered about angles $(\theta_\pi, \phi_\pi)$.

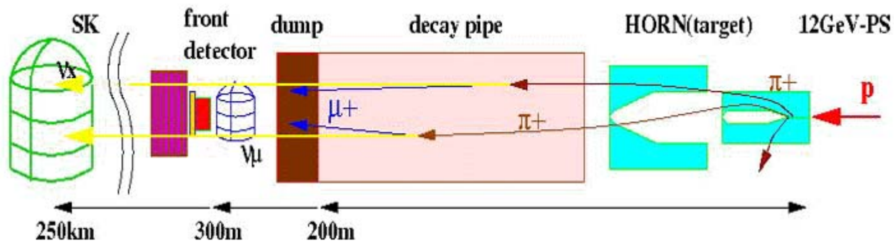
   A detector is placed at angle $\theta$ to the beam and records the energy spectrum $N_\gamma(E_\gamma, \theta)$ of the photons that strike it. Show that the $\pi^0$ spectrum can be related to the photon spectrum by

$$N_\pi(E_\pi, \theta) = -\frac{E_\pi}{2} \frac{dN_\gamma(E_\gamma = E_\pi, \theta)}{dE_\gamma}\,, \qquad (1)$$

   if $E_\pi \gg m_\pi$.

2. **Neutrino Beam from Pion Decay**

A typical high-energy neutrino beam is made from the decay of $\pi$ mesons that have been produced in proton interactions on a target, as sketched in the figure below.




Suppose that only positively charged particles are collected by the "horn." The main source of neutrinos is then the decay $\pi^+ \to \mu^+\nu_\mu$.

(a) Give a simple estimate of the relative number of other types of neutrinos than $\nu_\mu$ in the beam (due to decays in the decay pipe).

(b) If the decay pions have energy $E_\pi \gg m_\pi$, what is the characteristic angle $\theta_C$ of the decay neutrinos with respect to the direction of the $\pi^+$?

(c) If a neutrino is produced with energy $E_\nu \gg m_\pi$, what is the maximum angle $\theta_{\max}(E_\nu)$ between it and the direction of its parent pion (which can have any energy)? What is the maximum energy $E_\nu$ at which a neutrino can be produced in the decay of a pion if it appears at a given angle $\theta$ with respect to the pion's direction?

Parts (d) and (f) explore consequences of the existence of these maxima.

(d) Deduce an analytic expression for the energy-angle spectrum $d^2N/dE_\nu d\Omega$ for neutrinos produced at angle $\theta \le \theta_C$ to the proton beam. You may suppose that $E_\nu \gg m_\pi$, that the pions are produced with an energy spectrum $dN/dE_\pi \propto (E_p - E_\pi)^5$, where $E_p$ is the energy of the proton beam, and that the "horn" makes all pion momenta parallel to that of the proton beam.

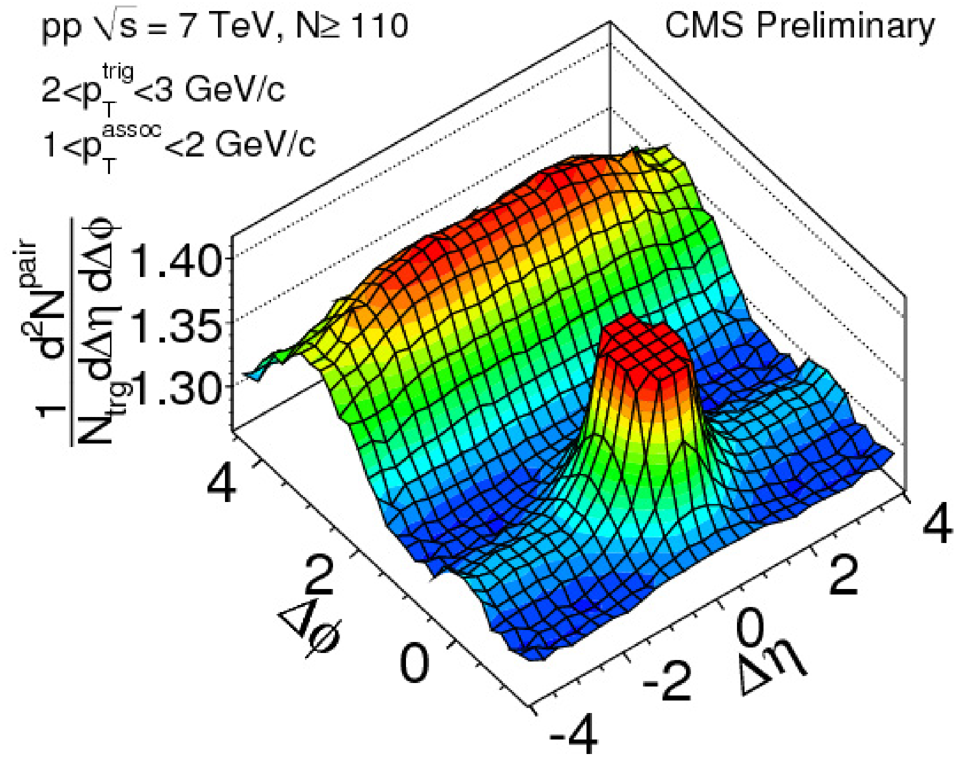
(e) At what energy $E_{\nu,\text{peak}}$ does the neutrino spectrum peak for $\theta = 0$?

(f) Compare the characteristics of a neutrino beam at $\theta = 0$ with an off-axis beam at angle $\theta$ such that $E_{\nu,\max}(\theta)$ is less than $E_{\nu,\text{peak}}(\theta = 0)$.

Facts: $m_\pi = 139.6$ MeV/$c^2$, $\tau_\pi = 26$ ns, $m_\mu = 105.7$ MeV/$c^2$, $\tau_\mu = 2.2$ $\mu$s. In this problem, neutrinos can be taken as massless.

3. **Pseudorapidity Ridge**

An unexpected feature in recent data from high energy *pp* collisions is the appearance of a "ridge" along $\Delta\phi = 0$ in $\Delta\eta$-$\Delta\phi$ space in 2-particle correlations in events that contain at least 2 particles at moderately high transverse momentum, where $\eta = -\ln\tan(\theta/2)$ and $\phi$ are the pseudorapidity and azimuthal angle of a particle relative to the *pp* axis. See, for example, CMS Collaboration, *Observation of long-range, near-side angular correlations in proton-proton collisions at the LHC*, JHEP09, 091 (2010), http://kirkmcd.princeton.edu/examples/EP/cms_jhep09_091_10.pdf.




The peak at $\Delta\eta = 0 = \Delta\phi$ is due to $\rho \to \pi\pi$ decay (although this is also attributed to Bose-Einstein correlations among pions), and the "ridge" at $\Delta\phi \approx \pi$ is attributed to pairs of particles with transverse momentum opposite to that of the $\rho \to \pi\pi$.

The (open-ended) problem is to explain the "same-side ridge."

The answer to this is not considered to be clear yet. You may, of course, consult recent literature on this topic.