Ph 406: Elementary Particle Physics
Problem Set 8

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1. The spin-1 vector mesons can be taken to have quark content: $\rho^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$, $\omega^0 = (u\bar{u} + d\bar{d})/\sqrt{2}$, $\phi = s\bar{s}$, $J/\psi = c\bar{c}$, $\Upsilon = b\bar{b}$ ($V_{\rm top} = t\bar{t}$ will not exist).

   The decays $V \to e^+e^-$ proceed via a single intermediate photon, where V is a vector meson. In the quark model, this corresponds to the reaction $q\bar{q} \to \gamma \to e^+e^-$, whose cross section was discussed on p. 108, Lecture 7 of the Notes. Deduce the decay rate Γ for this by recalling (p. 13, Lecture 1 of the Notes) that

   $$\text{Rate} = \Gamma_{a+b\to c+d} = N v_{\rm rel}\, \sigma_{a+b\to c+d}, \qquad (1)$$

   where N is the number of candidate scatters per second per unit volume, and $v_{\rm rel}$ is the relative velocity of the initial-state particles a and b. In case of a two-particle bound state, $N = |\psi(0)|^2$ is the probability that both particles are at the origin.

   Predict the decay rates to e^+e^- for the five vector mesons in the model that the strong interaction between (colored) quarks at short distances can be described by the Coulomb-like potential $V(r) \approx -4\alpha_S/3r$.

   Compare with data summarized at http://pdg.lbl.gov/2013/tables/contents_tables_mesons.html.

   What do we learn from this about possible energy dependence of α_S?

2. In the vector-meson decays $V \to \pi^0\gamma$, $\eta\gamma$, the meson spin changes from 1 to 0. Hence, this must be an M1 (magnetic dipole) transition. In the quark model the M1 electromagnetic transition flips a single quark spin, but does not change quark flavor, with matrix element proportional to the relevant quark magnetic moment(s). Suppose the quarks have Dirac moments $Q_q/2m_q$ where $m_u \approx m_d \approx \frac{2}{3}m_s$. Predict the relative decay rates (not just matrix elements) to $\pi^0\gamma$ and $\eta\gamma$ for the ρ^0, ω^0, ϕ and J/ψ vector mesons.

   Recall that in the quark model the spin-0-octet neutral mesons have quark wavefunction $\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$ and $\eta(548) = (2s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{6}$. Compare with data summarized at http://pdg.lbl.gov/2013/tables/contents_tables_mesons.html.

3. The $\psi'(3685)$ vector meson can decay to $\chi(3415) + \gamma$. The χ particle is believed to be a ${}^3P_0$ $c\bar{c}$ state. If so, predict the angular distribution of the γ relative to the direction of the electron supposing the ψ' is produced in a colliding-beam experiment $e^+e^- \to \psi' \to \chi\gamma$. Recall that at high energies the one-photon annihilation of e^+e^- proceeds entirely via transversely polarized photons ($S_z = \pm 1$).

Crossing Symmetry

We have previously noted that the inverse processes $a+b \leftrightarrow c+d$ have common matrix elements, and that these process may proceed via single-particle exchange in any of the s-, t- or u-channels with related matrix elements. Such relations among matrix elements for related processes are sometimes called crossing symmetry.

In the next 3 problems you will use crossing symmetry to convert the matrix element for muon decay, $\mu \to e\overline{\nu}_e\nu_\mu$ to results for 3 related processes.

The square of the matrix element for the 4-particle vertex $\mu\nu_\mu e\nu_e$ of unpolarized particles in the Fermi theory of the weak interaction is, in terms of the particle 4-vectors p,

$$|M|^2 = 32G_F^2(p_\mu \cdot p_{\nu_e})(p_e \cdot p_{\nu_\mu}), \tag{2}$$

where G_F is Fermi's constant, and the average over initial spins and sum over final spins is the same for all variants of the vertex.

4. Deduce the cross section for the neutrino-scattering reaction $\nu_\mu + e^- \to \mu^- + \nu_e$.

Recall that the differential cross section for 2-particle scattering $a+b \to c+d$ can be written in the center of mass frame as (p. 80, Lecture 5 of the Notes)

$$\frac{d\sigma}{d\Omega^\star} = \frac{|M|^2\ P_f}{64\pi^2 s P_i}\,, \tag{3}$$

where P_f is the momentum of the final-state particles c and d, P_i is the momentum of the initial-state particles a and b, and $s = (p_a + p_b)^2 = (p_c + p_d)^2$ is the square of the total energy in the center of mass frame. Express the cross section in terms of s, and then evaluate this in the lab frame where the electron is at rest and the muon neutrino has energy E.

5. The process $\mu^+e^- \to \mu^-e^+$ was considered by Pontecorvo in 1957 as a possible example of quantum oscillations of a two-particle system,[1] as this reaction could proceed via a two-neutrino intermediate state.

While the reaction $\mu^+e^- \to \overline{\nu}_\mu\nu_e$ is unlikely ever to be observed, it is now understood that the related reaction $e^+e^- \to \overline{\nu}_e\nu_e$ is the main source of neutrino production in supernovae, and a key process in their history.

Use a suitable variant of the matrix element (2) to deduce the cross section for $e^+e^- \to \overline{\nu}_e\nu_e$. *Work in the center-of-mass frame, and express the result in terms of the invariant* s.

[1] B. Pontecorvo, *Mesonium and Antimesonium*, Sov. Phys. JETP **6**, 429 (1957), http://kirkmcd.princeton.edu/examples/EP/pontecorvo_spjetp_6_429_57.pdf.
This landmark paper introduced the term mesonium, raised the possibility that ν_e and ν_μ are different particles, made the first speculations about neutrino oscillations, and led to the notion of conservation of lepton number, as developed by G. Feinberg and S. Weinberg, *Law of Conservation of Muons*, Phys. Rev. Lett. **6**, 381 (1961), http://kirkmcd.princeton.edu/examples/EP/feinberg_prl_6_381_61.pdf.

The divergence of this cross section at low energy is avoided by Nature in that the electron and positron would not scatter but rather would bind into a positronium atom.

6. Some positronium atoms (which of the ortho- and para- states?) can decay to two neutrinos. Deduce the decay rate Γ for this by recalling (p. 13, Lecture 1 of the Notes) that

$$\text{Rate} = \Gamma_{a+b\to c+d} = N v_{\text{rel}} \sigma_{a+b\to c+d}, \tag{4}$$

where N is the number of candidate scatters per second per unit volume, and v_{rel} is the relative velocity of the initial-state particles a and b. In case of a two-particle bound state, $N = |\psi(0)|^2$ is the probability that both particles are at the origin.