Ph 406 Problem Set 9

Due April 26, 1993

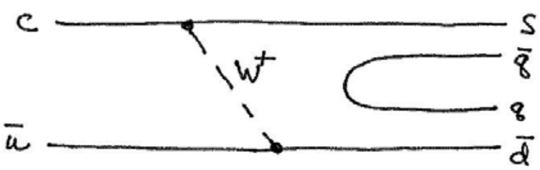
The spin-1 **vector mesons** can be taken to have quark content: $\rho^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$, $\omega^0 = (u\bar{u} + d\bar{d})/\sqrt{2}$, $\phi = s\bar{s}$, $J/\psi = c\bar{c}$, and $\Upsilon = b\bar{b}$.

1. The decays $V \to e^+e^-$ proceed via a single intermediate photon, where V is a vector meson. Suppose the decay rates $\Gamma(V \to e^+e^-)$ are independent of quark mass and the meson mass, but do depend on quark charge. Predict the relative decay rates to e^+e^- for the five vector mesons.

2. In the decay $V \to \pi^0\gamma$ the meson spin changes from 1 to 0. Hence this must be an $M1$ magnetic dipole transition. In the quark model the decay rate depends on the size of the relevant quark magnetic moments. Suppose the quarks have Dirac moments $Q_q/2m_q$ where $m_u \approx m_d \approx \frac{2}{3}m_s$. Predict the relative decay rates to $\pi^0\gamma$ for the ρ^0, ω^0, and ϕ mesons.

3. (a) The $\psi'(3685)$ vector meson can decay to $\chi(3415) + \gamma$. The χ particle is believed to be a ${}^3P_0$ $c\bar{c}$ state. If so, predict the angular distribution of the γ relative to the direction of the electron supposing the ψ' is produced in a colliding-beam experiment $e^+e^- \to \psi' \to \chi\gamma$. Recall that at high energies the one-photon annihilation of e^+e^- proceeds entirely via transversely polarized photons ($S_z = \pm 1$).

   (b) The charmed meson D^0 can decay to $K\pi$ via the Cabbibo-favored W-exchange diagram (with gluons not shown)

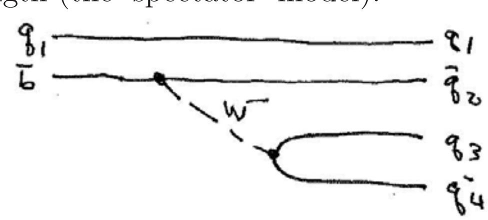


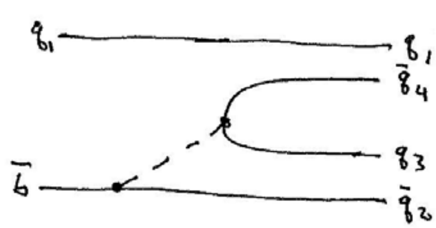
   If this were the only possible diagram, predict the ratio of branching ratios:

$$\frac{\Gamma(D^0 \to K^-\pi^+)}{\Gamma(D^0 \to \bar{K}^0\pi^0)}.$$

   You may assume the $K\pi$ system is in an isospin-$\frac{1}{2}$ state.
   Draw any other Cabbibo-favored diagrams for these decays.

4. There are four spin-0 mesons that contain one bottom quark: $B_u^+ = u\bar{b}$, $B_d^0 = d\bar{b}$, $B_s^0 = s\bar{b}$, and $B_c^+ = c\bar{b}$. These decay via the weak interaction by two graphs with roughly equal strength (the 'spectator' model):





Here we consider only nonleptonic final states. Suppose the four final-state quarks form exactly two mesons (as happens a few percent of the time). List the two dominant two-body decays for each of the four bottom mesons.

A complication arises for the B_c meson. The charm quark has a slightly shorter lifetime than the bottom quark. Hence there are two more prominent two-body decays of the B_c involving $c \to Wq$ rather than $b \to Wq$ transitions. List these.

According to the measured values of the C-K-M matrix elements

$$\frac{V_{ub}}{V_{cb}} \approx \frac{V_{us}}{V_{ud}} \approx \frac{V_{cd}}{V_{cs}} \approx \lambda = \text{Cabbibo angle.}$$

List the two-body nonleptonic decays of the four bottom mesons that are suppressed by one power of λ in the matrix element (and hence by $\lambda^2 \approx 1/25$ in rate).

Note that $D^+ = c\bar{d}$, $D^0 = c\bar{u}$, and $D_s^0 = c\bar{s}$. If a meson is produced from, say, a $d\bar{d}$ state it could be a π^0, η, ρ^0, or ω^0. Here it is sufficient to list only the π^0...

5. Both the $B_d^0 = d\bar{b}$ and $\bar{B}_d^0 = \bar{d}b$ can decay to common final states, such as $J/\psi K_S^0$ as you found in prob. 4. Hence there are transitions between B^0 and $\bar{B}^0$ and so the states of definite mass and lifetime are not these but

$$B_1 = \frac{B^0 + \bar{B}^0}{\sqrt{2}}, \qquad \text{and} \qquad B_2 = \frac{B^0 - \bar{B}^0}{\sqrt{2}}.$$

So far this is much like the K^0-$\bar{K}^0$ system, ignoring the possibility of CP violation. (Can you readily show that B_1 and B_2 are the eigenstates of the 2×2 Hamiltonian, assuming the off-diagonal elements are equal, as is the case for time-reversal invariance (CP conservation)?)

In practice the lifetimes of B_1 and B_2 are essentially identical (unlike the case for K_1 and K_2), so

$$|B_1(t)\rangle = e^{-\Gamma t/2} e^{i m_1 t} |B_1(0)\rangle,$$
$$|B_2(t)\rangle = e^{-\Gamma t/2} e^{i m_2 t} |B_2(0)\rangle.$$

Deduce the probabilities $P(t)$ and $\bar{P}(t)$ of having a B^0 and $\bar{B}^0$ at time t in terms of the initial amplitudes $|B^0(0)\rangle$ and $|\bar{B}^0(0)\rangle$ and the mass difference $\Delta m \equiv m_1 - m_2$.

Suppose at $t = 0$ we have a pure B^0. What is the probability that it decays as a $\bar{B}^0$ rather than as a B^0, in terms of the 'mixing' parameter $x \equiv \Delta m/\Gamma$?

For the B_d^0, x_d has been measured to be 0.7, and it is expected that for the B_s^0, $x_s \approx 10$.