

ELECTRIC MATERIALS

EXTENDING OUR VIEWPOINT FROM COLLECTIONS OF POINT CHARGES TO THAT OF BULK MATTER, WE IDENTIFY TWO SIMPLE CASES OF INTEREST: CONDUCTORS AND DIELECTRICS.

CONDUCTORS (BECKER SEC. 20)

A CONDUCTOR IS A SUBSTANCE IN WHICH CHARGES CAN MOVE FREELY. (IT IS SUFFICIENT IF ONLY ONE SIGN OF CHARGE IS FREE TO MOVE). MOST PRACTICAL 'CONDUCTORS' ACTUALLY EXHIBIT A FINITE 'RESISTANCE' TO THE MOTION OF CHARGE - AS WILL BE CONSIDERED LATER. ASTONISHINGLY, THERE EXISTS A STATE OF MATTER WHICH OBEYS OUR IDEALIZED VIEW OF CONDUCTORS - THE SO-CALLED SUPERCONDUCTORS SUCH AS Ni_3Sn FOR WHICH ELECTRICAL RESISTANCE VANISHES BELOW A CERTAIN TEMPERATURE.

IN ELECTROSTATICS, THE CHARGES MUST NOT BE IN MOTION. SINCE $\vec{F} = q\vec{E}$, WE CONCLUDE THAT

$$\vec{E} = 0 \text{ INSIDE A CONDUCTOR.}$$

IF A FIELD \vec{E} IS SUDDENLY APPLIED, THE CHARGES RAPIDLY REARRANGE THEMSELVES SO AS TO CANCEL OUT THE APPLIED FIELD.

(IGNORING QUANTUM PROCESSES)

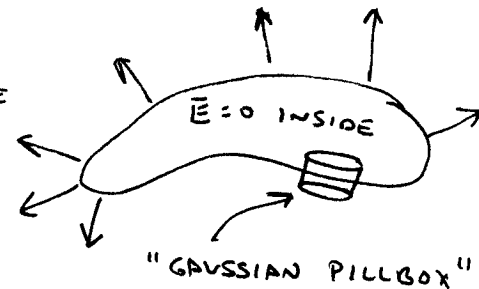
HOWEVER, CHARGES CANNOT LEAVE THE SURFACE OF A CONDUCTOR. ^ATHUS, IT IS POSSIBLE THAT \vec{E} IS PERPENDICULAR TO THE SURFACE OUTSIDE THE CONDUCTOR.

BUT AT THE BOUNDARY, \vec{E}_{\parallel} MUST VANISH! OTHERWISE THE SURFACE CHARGES WOULD MOVE.

APPLYING GAUSS' LAW TO A 'PILLBOX' CONTAINING A PIECE OF THE SURFACE,

$$\int \vec{E} \cdot d\vec{S} = 4\pi Q_{\text{INSIDE}},$$

$$\text{SO } E_{\perp} = 4\pi \sigma \text{ WHERE } \sigma = \frac{Q}{\text{AREA}} = \text{SURFACE CHARGE DENSITY.}$$



"GAUSSIAN PILLBOX"

SINCE $\vec{E} = 0$ INSIDE THE CONDUCTOR, AND $\vec{E} = -\nabla\phi$, THE ENTIRE CONDUCTOR IS AT THE SAME POTENTIAL. IN PARTICULAR, THE SURFACE IS AN EQUIPOTENTIAL.

(SINCE $\vec{E}_{\parallel} = 0$ AT SURFACE, $\vec{E} \cdot d\vec{l} = 0$ FOR $d\vec{l}$ ALONG THE SURFACE. EQUIPOTENTIAL $\Rightarrow \nabla^2\phi = 0$ INSIDE CONDUCTOR.

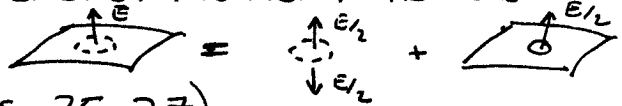
BUT $\nabla^2\phi = -4\pi\rho \Rightarrow \rho = 0$ INSIDE CONDUCTOR, AS WELL AS $\vec{E} = 0$.

A FINAL COMMENT ABOUT CONDUCTORS CONCERNS THE FORCE ON THE SURFACE CHARGES. FROM THE BASIC RELATION

$$\vec{F} = q\vec{E}, \text{ WE MIGHT EXPECT } \vec{F} = \sigma\vec{E} \text{ AT THE SURFACE.}$$

BUT IN FACT $\vec{F} = \frac{\sigma}{2}\vec{E}$. THIS MAY BE JUSTIFIED BY SUPPOSING THAT THE SURFACE CHARGE IS DISTRIBUTED OVER A LAYER OF FINITE THICKNESS. THEN $\vec{E} = \vec{E}_{\text{OUTSIDE}}$ AT THE TOP OF THE LAYER, AND $\vec{E} = 0$ AT THE BOTTOM, SO $\vec{E}_{\text{AVE}} = \frac{1}{2}\vec{E}_{\text{OUTSIDE}}$.

WE MAY ALSO CONFIRM THIS BY AN ENERGY ARGUMENT AS TO BE CONSIDERED IN LECTURE 3. ALSO



DIELECTRICS (BECKER SECS. 25-27)

IN A DIELECTRIC WE SUPPOSE THE CHARGES ARE NOT COMPLETELY FREE, BUT ARE BOUND IN MOLECULES. THE MOLECULES MAY, HOWEVER, DEFORM SO THAT THE + AND - CHARGES ARE DISPLACED RELATIVELY TO ONE ANOTHER UNDER THE INFLUENCE OF AN ELECTRIC FIELD.

THE CHARGES CANNOT MOVE FAR ENOUGH TO COMPLETELY CANCEL AN APPLIED FIELD, SO \vec{E} CAN BE NON-ZERO INSIDE A DIELECTRIC.

THE SIMPLEST DEFORMATION OF A MOLECULE IS THAT IT TAKES ON A NON-ZERO DIPOLE MOMENT \vec{p} .

WE DEFINE THE DIELECTRIC POLARIZATION $\vec{P} = \frac{\sum \vec{p}}{\text{VOL}} = \frac{\text{AVE DIPOLE MOMEN}}{\text{VOLUME}}$
($N = \#$ OF MOLECULES)

TO A GOOD APPROXIMATION WE CAN NEGLECT QUADRUPOLE MOMENTS IN A DIELECTRIC.

WHAT ARE THE RULES GOVERNING THE BEHAVIOR OF \vec{E} INSIDE A DIELECTRIC? HOW IS \vec{P} RELATED TO \vec{E} ?

FROM A MICROSCOPIC POINT OF VIEW, $\vec{\nabla} \cdot \vec{E} = 0$, $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$. AND ρ VARIES WILDLY DUE TO THE POSSIBLE POLARIZATION OF MOLECULES.

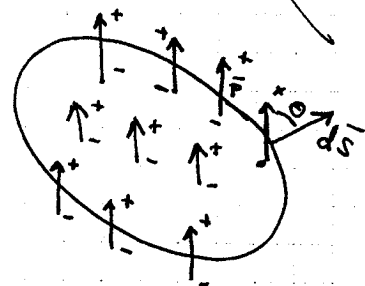
WE WISH TO IGNORE THIS MICROSCOPIC EFFECT AND CONSIDER ONLY FIELDS AVERAGED OVER VOLUMES WHICH CONTAIN MANY MOLECULES (BUT ARE STILL SMALL COMPARED TO 'ROOM SIZE' OBJECTS).

WE HOPE THAT THE 'MACROSCOPIC' MAXWELL EQUATION KEEPS THE FORM OF THE MICROSCOPIC EQUATION, AND LOOK FOR

$$\vec{\nabla} \cdot \vec{E}_{\text{AVE}} = 4\pi \rho_{\text{AVE}}$$

TO DETERMINE P_{AVE} , CONSIDER A SMALL VOLUME, CONTAINING MANY MOLECULES.

THEN $P_{AVE} = \frac{Q_{INSIDE}}{VOLUME}$



MOLECULES WHICH ARE ENTIRELY CONTAINED WITHIN THE VOLUME CONTRIBUTE NOTHING TO Q_{IN} (SUPPOSING $Q_{MOLECULE} = 0$)

BUT MOLECULES WHICH STRADDLE THE SURFACE GIVE A CONTRIBUTION, DUE TO POLARIZATION.

THE AMOUNT OF CHARGE LEFT INSIDE NEXT TO A SURFACE ELEMENT dS IS

$dQ_{INSIDE} = -\vec{P} \cdot d\vec{S}$ (- SINCE THE -CHARGES ARE INSIDE)

TO SEE THIS SUPPOSE EACH MOLECULE HAS DIPOLE MOMENT $p = qd$

THEN THE CHARGE LEFT INSIDE A VOLUME $d \cdot dS \cos \theta$ JUST INSIDE THE VOLUME IS

$dQ_{INSIDE} = -nq = -\frac{P dvol}{p} q = -\frac{P d \cdot dS \cos \theta}{qd} q = -\vec{P} \cdot d\vec{S}$

INTEGRATING OVER THE SURFACE

$Q_{INSIDE} = -\int_{SURFACE} \vec{P} \cdot d\vec{S} = -\int (\vec{\nabla} \cdot \vec{P}) dvol = \int P_{AVE} dvol$ BY DEF. OF P_{AVE}

HENCE $P_{AVE} = -\vec{\nabla} \cdot \vec{P}$

HENCE OUR MAXWELL EQUATIONS FOR ^(NEUTRAL) DIELECTRICS ARE

$\vec{\nabla} \times \vec{E} = 0$, $\vec{\nabla} \cdot \vec{E} = -4\pi \vec{\nabla} \cdot \vec{P}$ INSIDE

NOTE THAT AT THE PHYSICAL SURFACE OF THE DIELECTRIC, THERE WILL BE A SURFACE CHARGE DISTRIBUTION

$\sigma = +\vec{P} \cdot \hat{n} = \vec{P} \cdot \hat{n}$ (\hat{n} = UNIT NORMAL TO SURFACE)

FOR WHAT IT'S WORTH, WE CAN WRITE DOWN A FORMAL SOLUTION FOR THE POTENTIAL DUE TO THE DIELECTRIC CHARGE DISTRIBUTION!

$\phi = \int \frac{\rho dvol}{\gamma} = -\int_{VOLUME} \frac{\vec{\nabla} \cdot \vec{P}}{\gamma} dvol + \int_{SURFACE} \frac{\vec{P} \cdot d\vec{S}}{\gamma}$

WE WILL FIND LATER THAT $\vec{\nabla} \cdot \vec{P} = 0$ INSIDE MOST SIMPLE DIELECTRICS.

IF THE MOLECULES WERE CHARGED, OR OTHER CHARGES WERE SOMEHOW EMBEDDED IN THE DIELECTRIC, WE DESCRIBE THE RESULTING CHARGE DISTRIBUTION AS ρ_{EXT}

$$\text{THEN } \nabla \cdot \vec{E} = 4\pi \rho_{\text{EXT}} - 4\pi \nabla \cdot \vec{P}$$

TYPICALLY WE WOULD HAVE SOME OPINION ABOUT ρ_{EXT} , BUT THE RELATION BETWEEN \vec{E} AND \vec{P} IS NOT YET SPECIFIED. THIS LED PEOPLE TO DEFINE

$$\underline{\vec{D}} = \underline{\vec{E}} + 4\pi \underline{\vec{P}} \equiv \underline{\text{DISPLACEMENT}}$$

$$\text{SO THAT } \underline{\nabla \cdot \vec{D}} = 4\pi \rho_{\text{EXT}}$$

\vec{D} IS OFTEN READILY DETERMINED.

MAXWELL FELT THAT EVEN EMPTY SPACE WAS A KIND OF DIELECTRIC MATERIAL - CALLED THE ETHER. HE CONSIDERED THE EXISTENCE OF AN ELECTRIC FIELD IN EMPTY SPACE AS DUE TO DISPLACEMENTS OF CHARGES IN THE ETHER. THE FIELD \vec{D} IS THEN THE MEASURE OF THE "VACUUM POLARIZATION" COMBINED WITH THE TRUE DIELECTRIC POLARIZATION.

IN GENERAL THERE IS NO RESTRICTION THAT $\nabla \times \vec{P} = 0$ SO $\nabla \times \vec{D} \neq 0$ IN GENERAL, AND \vec{D} CANNOT BE DERIVED FROM A POTENTIAL. FOR EXAMPLE, \vec{P} CAN BE PARALLEL TO THE EDGE OF A SLAB OF DIELECTRIC AND ZERO OUTSIDE THE DIELECTRIC. THEN $\oint \vec{P} \cdot d\vec{l} \neq 0$ FOR A LOOP PART IN AND PART OUTSIDE THE DIELECTRIC....

TO MAKE SIGNIFICANT PROGRESS, WE MUST HAVE SOME UNDERSTANDING OF THE RELATION BETWEEN \vec{P} AND \vec{E} .

EXPERIMENTALLY IT HAS BEEN DETERMINED THAT FOR MOST DIELECTRICS A LINEAR RELATION HOLDS: ALSO, THE FIELD SHOULD BE 'WEAK'. A 'STRONG' FIELD ON THE ATOMIC SCALE HAS VOLTAGE DROP ≈ 10 V ACROSS AN ATOM $\Rightarrow \approx 10^8$ V/cm

$$P_i = \chi_{ij} E_j$$

WHERE χ_{ij} IS THE ELECTRIC SUSCEPTIBILITY TENSOR

FOR MATERIALS OF RANDOM, OR OF CUBIC CRYSTALLINE STRUCTURE, THE 3 PRINCIPAL COMPONENTS OF THIS TENSOR ARE EQUAL, AND SO

$$\underline{\vec{P}} = \chi \underline{\vec{E}} \quad \left[\begin{array}{l} \text{MKSA: } \vec{P} = \epsilon_0 \chi_{\text{MKSA}} \vec{E} \\ \Rightarrow \chi_{\text{GAUSSIAN}} = \frac{\chi_{\text{MKSA}}}{4\pi} \end{array} \right] !$$

WE WILL ONLY CONSIDER SUCH SIMPLE SUBSTANCES. MATERIALS WITH NON-TRIVIAL χ_i LEAD TO COMPLICATED BEHAVIOR, SUCH AS OPTICAL BIREFRINGENCE...

IF $\vec{P} = \chi \vec{E}$

THEN $\vec{D} = \vec{E} + 4\pi \vec{P} = (1 + 4\pi \chi) \vec{E} = \epsilon \vec{E}$

WHERE $\epsilon \equiv$ DIELECTRIC CONSTANT

[$\epsilon = 1$ IN THE VACUUM]
 [$\epsilon = 1 + \frac{4\pi}{c} (\frac{E}{E_{crit}})^2$ DUE TO VACUUM POLARIZATION]

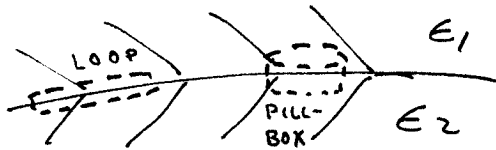
MKSA:
 $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$
 $= \epsilon_0 (1 + \chi) \vec{E}$
 $= \epsilon_r \epsilon_0 \vec{E}$
 $\epsilon_r = \epsilon_{GAUSSIAN}$

IF THE DIELECTRIC CONSTANT IS KNOWN WE HAVE ALL THE INGREDIENTS NEEDED TO SOLVE DIELECTRIC PROBLEMS.
 [IN ELECTROSTATICS, ϵ CAN VARY IN SPACE, BUT NOT IN TIME.]

BOUNDARY CONDITIONS

IT MAY BE USEFUL TO RECORD THE BOUNDARY CONDITIONS ON ϕ , \vec{E} , AND \vec{D} AT A SURFACE BETWEEN TWO DIFFERENT DIELECTRICS.

A. CHARGE FREE BOUNDARY



CONSIDERING THE LOOP AS SHOWN,
 $\nabla \times \vec{E} = 0 \Rightarrow \oint_{\text{LOOP}} \vec{E} \cdot d\vec{l} = 0$

$\Rightarrow E_{||1} = E_{||2}$

\Rightarrow TANGENTIAL COMPONENT OF \vec{E} IS CONTINUOUS AT THE BOUNDARY.

CONSIDERING THE PILL BOX: $\nabla \cdot \vec{D} = \rho_{EXT} = 0 \Rightarrow \int \vec{D} \cdot d\vec{S} = 0$

$\Rightarrow D_{\perp 1} = D_{\perp 2} \Rightarrow$ NORMAL COMPONENT OF \vec{D} IS CONTINUOUS AT THE BOUNDARY.

THUS $\epsilon_1 E_{\perp 1} = \epsilon_2 E_{\perp 2}$ AT THE BOUNDARY.

FOR THE POTENTIAL, $\nabla \times \vec{E} = 0$ ALLOWS US TO DEFINE A POTENTIAL WHICH IS CONTINUOUS AT THE BOUNDARY.

HOWEVER, WRITING $E_{\perp} = -\frac{\partial \phi}{\partial n}$ WHERE $\frac{\partial}{\partial n} \equiv$ NORMAL DERIVATIVE $= \hat{n} \cdot \nabla$

WE SEE THAT $\epsilon_1 \frac{\partial \phi}{\partial n} \Big|_1 = \epsilon_2 \frac{\partial \phi}{\partial n} \Big|_2$

HENCE THE SLOPE OF ϕ MAY TAKE A STEP AT THE BOUNDARY.

B. CHARGED BOUNDARY

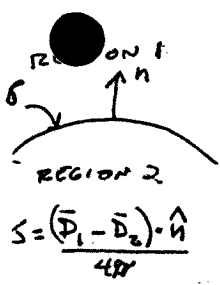
IF THE BOUNDARY SURFACE

HAS AN EXTERNAL CHARGE DENSITY σ , THEN $\nabla \cdot \vec{D} \neq 0$

AT THE BOUNDARY. BUT $\int \vec{D} \cdot d\vec{S} = 4\pi \sigma \cdot \text{AREA}$

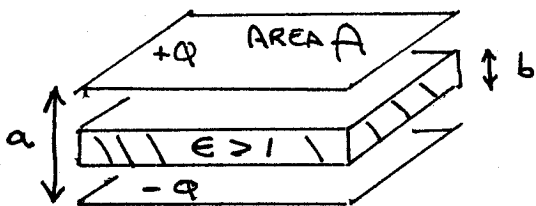
$\Rightarrow D_{\perp 1} = D_{\perp 2} + 4\pi\sigma$; $\epsilon_1 E_{\perp 1} = \epsilon_2 E_{\perp 2} + 4\pi\sigma$

$\nabla \times \vec{E} = 0$ IS STILL TRUE, SO $E_{\parallel 1} = E_{\parallel 2}$ ETC...



EXAMPLE

PARALLEL PLATE CAPACITOR WITH DIELECTRIC SLAB. THE PLATES HAVE FIXED CHARGES $\pm Q$.



a) MACROSCOPIC SOLUTION:

EMPHASIZE \vec{D} AND ρ_{EXTERNAL}

$\nabla \cdot \vec{D} = 4\pi \rho_{\text{EXT}}$

CONSTRUCT A PILL BOX SURROUNDING THE UPPER PLATE

$\int \vec{D} \cdot d\vec{S} = 4\pi Q_{\text{IN}} \Rightarrow D = \frac{4\pi Q}{A}$ BETWEEN THE PLATES; $D = 0$ OUTSIDE

IN THE AIR, $E = D = \frac{4\pi Q}{A}$; IN THE SLAB, $E = \frac{D}{\epsilon} = \frac{4\pi Q}{\epsilon A}$

$\vec{P} = \frac{\epsilon - 1}{4\pi} \vec{E} \Rightarrow P_{\text{AIR}} = 0$; $P_{\text{SLAB}} = \frac{\epsilon - 1}{\epsilon} \frac{Q}{A}$

THE POTENTIAL DIFFERENCE BETWEEN THE PLATES IS

$V = \Delta\phi = \int E \cdot d\ell = \frac{4\pi Q}{A} \left[(a-b) + \frac{b}{\epsilon} \right] < \frac{4\pi Q a}{A}$

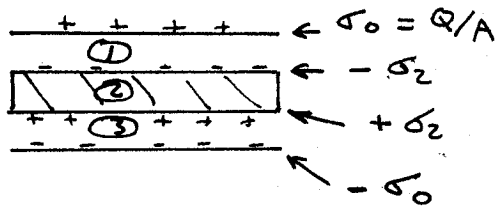
b) 'MICROSCOPIC' SOLUTION:

EMPHASIZE \vec{E} AND $\rho_{\text{TOTAL}} = \rho_{\text{EXT}} + \rho_{\text{POLARIZATION}}$

FOR A LINEAR DIELECTRIC, $\vec{P} = \chi \vec{E}$ WITH $\rho_{\text{EXT}} = 0$ ALSO.

$\nabla \cdot \vec{D} = 0 \Rightarrow \nabla \cdot \vec{E} = 0 \Rightarrow \nabla \cdot \vec{P} = 0 \Rightarrow \rho_{\text{POLARIZATION}} = 0$ AS WELL.

BUT AT THE DIELECTRIC SURFACE, $\sigma_2 = \vec{P} \cdot \hat{n} = \chi E_2 = \frac{\epsilon - 1}{4\pi} E_2$



THEN $E_1 = E_3 = 4\pi \sigma_0$ (VIA GAUSS' LAW)

ALSO $E_2 - E_1 = -4\pi \sigma_2 = -(\epsilon - 1) E_2$

$\Rightarrow \epsilon E_2 = E_1$, OR $E_2 = \frac{4\pi \sigma_0}{\epsilon} = \frac{4\pi Q}{\epsilon A}$

AND $\sigma_2 = \frac{\epsilon - 1}{\epsilon} \frac{Q}{A}$ ETC ...

CAPACITANCE:

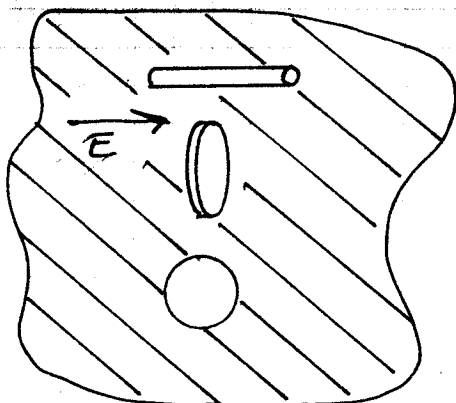
$$C = \frac{Q}{V}$$

$$= \frac{\epsilon A}{4\pi} \frac{1}{b + \frac{\epsilon - 1}{\epsilon} b}$$

$$> \frac{A}{4\pi a}$$

$b \rightarrow a \Rightarrow$
 $C \rightarrow \frac{\epsilon A}{4\pi a}$

EXAMPLE: FIELDS IN CAVITIES



WE CONSIDER CAVITIES IN A DIELECTRIC WHICH ARE SO SMALL THAT THEY DO NOT APPRECIABLY DISTURB THE ELECTRIC FIELD IN THE DIELECTRIC.

A. NEEDLE SHAPED CAVITY WITH AXIS OF CAVITY ALONG THE DIRECTION OF \vec{E} .

THEN THE CONTINUITY OF $E_{||}$ AT A BOUNDARY $\Rightarrow \vec{E}_{CAVITY} = \vec{E}_{||} = \vec{E}_{DIELECTRIC}$

B. 'PILL BOX' WITH FLAT FACE OF CAVITY \perp TO \vec{E} .

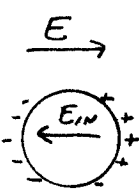
IT IS D_{\perp} RATHER THAN E_{\perp} WHICH IS CONTINUOUS.

SO $\vec{D}_{CAV} = \vec{D}_{DIELECTRIC} = \epsilon \vec{E}_{DIELECTRIC}$

AND SO $\vec{E}_{CAVITY} = \epsilon \vec{E}_{DIELECTRIC}$ (SINCE $\epsilon_{VACUUM} = 1$)

C. SMALL SPHERE OF PARTICULAR INTEREST IS THE CASE WHERE THE SPHERE IS EXACTLY THE SIZE OF ONE MOLECULE. THEN WE WILL DETERMINE THE 'EXTERNAL' FIELD FELT BY A MOLECULE.

WE PROCEED VIA A 'TRICK': THE SUPERPOSITION PRINCIPLE. IF THERE WERE NO CAVITY THE FIELD WOULD BE JUST \vec{E} , AND THE DIELECTRIC POLARIZATION $\vec{P} = \chi \vec{E}$. NOW IMAGINE WE REMOVE THE MATERIAL INSIDE THE CAVITY, BUT HOLD THE POLARIZATION \vec{P} OF THIS MATERIAL FIXED. WE ALSO SUPPOSE THE FIELD IN THE LARGE BLOCK OF DIELECTRIC REMAINS CONSTANT WHEN THE CAVITY IS CREATED.

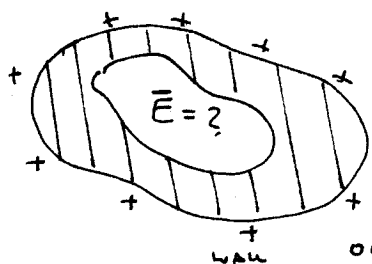


THEN $\vec{E} = \vec{E}_{CAVITY} + \vec{E}_{INSIDE SPHERE OF DIELECTRIC WHICH WAS REMOVED}$.

ON THE PROBLEM SET YOU WILL SHOW THAT $\vec{E}_{POLARIZED SPHERE} = -\frac{4\pi}{3} \vec{P}$

HENCE $\vec{E}_{CAVITY} = \vec{E} + \frac{4\pi}{3} \vec{P}$

D. ANY CAVITY INSIDE A CONDUCTOR. EVEN THO THE CONDUCTOR MAY BE CHARGED, $\vec{E} = 0$ INSIDE



THE CONDUCTOR. CAN $\vec{E} \neq 0$ IN THE CAVITY?

SUPPOSE IT IS. THEN CONTINUITY OF $E_{||}$ AT A BOUNDARY $\Rightarrow E_{||} = 0$ JUST INSIDE CAVITY.

SO IF $E \neq 0$, $E_{\perp} \neq 0$. BUT $\int \vec{E} \cdot d\vec{s} = Q_{INSIDE} = 0$

\therefore COULD ONLY HAVE FIELD LINES WHICH START ON CAVITY WALL AND END ELSEWHERE ON THE WALL.

BUT THEN $\int_{WALL} \vec{E} \cdot d\vec{l} \neq 0$. BUT THE SURFACE OF A CAVITY MUST BE AN

EQUIPOTENTIAL, A CONTRADICTION, HENCE THE FIELD IN THE CAVITY MUST VANISH NO MATTER WHAT THE CHARGES ON THE CONDUCTOR MAY BE (IF CAVITY IS CHARGE FREE).

THIS IS THE BASIS OF ELECTRICAL SHIELDING - IF YOU ARE IN A METAL BOX YOU ARE FREE OF ANY INFLUENCE OF EXTERNAL CHARGES AND FIELDS.

IS THE CONVERSE TRUE? I.O. IF THERE ARE CHARGES IN THE CAVITY CAN AN OBSERVER OUTSIDE THE CONDUCTOR TELL THIS? NOT IF THE CONDUCTOR IS GROUNDED = PUT IN ELECTRICAL CONTACT WITH A GIANT CHARGE RESERVOIR (THE EARTH)

MOLECULAR MODEL OF THE DIELECTRIC CONSTANT

WE SUPPOSE THAT AN EXTERNAL FIELD APPLIED TO A MOLECULE DEFORMS IT, INDUCING A DIPOLE MOMENT

$$\bar{P}_{\text{MOLECULE}} = \alpha \bar{E}_{\text{ON MOLECULE}} = \alpha \left(\bar{E} + \frac{4\pi\bar{P}}{3} \right)$$

$\alpha \equiv$ MOLECULAR POLARIZABILITY, $\bar{E} =$ AVE FIELD IN DIELECTRIC

ON THE PROBLEM SET YOU WILL CONSIDER TWO CLASSICAL MODELS OF A MOLECULE TO CALCULATE α .

WHAT IS THE RELATION BETWEEN α (A MOLECULAR PROPERTY) AND ϵ (A BULK PROPERTY)?

NOW $\bar{P} = N \bar{P}_{\text{MOLECULE}}$ WHERE $N = \#$ OF MOLECULES / VOLUME

$$\text{SO } \bar{P} = N \alpha \left(\bar{E} + \frac{4\pi\bar{P}}{3} \right)$$

BUT WE ALSO HAVE $\bar{P} = \chi \bar{E} = \frac{\epsilon-1}{4\pi} \bar{E}$ IN TERMS OF THE DIELECTRIC CONSTANT.

$$\text{HENCE } \frac{\epsilon-1}{4\pi} \bar{E} = N \alpha \bar{E} \left(1 + \frac{\epsilon-1}{3} \right) = N \alpha \bar{E} \frac{\epsilon+2}{3}$$

$$\text{OR } \underline{\underline{\frac{\epsilon-1}{\epsilon+2} = \frac{4\pi}{3} N \alpha}} \quad (\text{CLAUSIUS - MOSOTTI})$$

IF $\epsilon \approx 1$, AS IN A GAS, WE ESTIMATE

$$\underline{\underline{\epsilon \sim 1 + 4\pi N \alpha}} \quad (\text{LORENZ - LORENTZ})$$

TRY THIS OUT ON THE PROBLEM SET...

THE SUSCEPTIBILITY $\chi = \frac{\epsilon-1}{4\pi} = \frac{\epsilon+2}{3} N \alpha > N \alpha$ IF $\epsilon > 1$

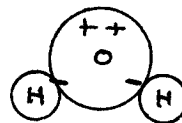
THIS IS BECAUSE THE DIPOLE MOMENTS OF THE MOLECULES

LINE UP SO AS TO INCREASE THE FIELD ON OTHER MOLECULES, WHICH IN TURN CAUSES MORE MOLECULAR POLARIZATION....

PERMANENT DIPOLE MOMENTS

IT IS POSSIBLE THAT EVEN IN THE ABSENCE OF EXTERNAL FIELDS A MOLECULE MIGHT NOT BE SPHERICAL. FOR EXAMPLE, A POLAR MOLECULE SUCH AS H_2O .

LET \vec{p}_0 = THE PERMANENT MOMENT OF A MOLECULE

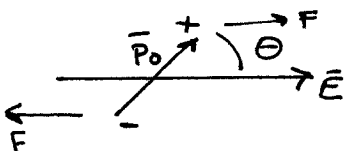


THEN $\vec{p}_{MOL} = \vec{p}_0 + \alpha \vec{E}_0$ IN MOLECULE

IF THE \vec{p}_0 'S ARE RANDOMLY ORIENTED FROM MOLECULE TO MOLECULE THEN THE PERMANENT MOMENTS GIVE NO NET CONTRIBUTION TO THE DIELECTRIC POLARIZATION \vec{P} .

(THERE ARE CERTAIN SOLIDS, CALLED ELECTRETS, IN WHICH THE \vec{p}_0 'S ARE ALL ALIGNED A CERTAIN WAY IN THE CRYSTAL LATTICE FOR EXAMPLE $BaTiO_3$)

HOWEVER, IF A FIELD \vec{E} IS APPLIED TO A POLAR MOLECULE, THE MOLECULE WILL TEND TO LINE UP WITH THE FIELD.



IF $p_0 = qd$ THE FIELD CAUSES A TORQUE

$$\tau = 2 q E \frac{d}{2} \sin \theta = p_0 E \sin \theta$$

IF A MOLECULE EXISTED IN ISOLATION FROM OTHER MOLECULES, IT WOULD CLEARLY LINE UP WITH \vec{E} . BUT IN A MATERIAL AT TEMPERATURE T , THE MOLECULES INTERACT AND EXCHANGE ENERGIES - OBEYING THE LAWS OF THERMODYNAMICS.

WE CAN THINK OF THE DIPOLE \vec{p}_0 HAS HAVING A CERTAIN ENERGY DEPENDING ON ITS ORIENTATION, θ , WITH RESPECT TO \vec{E} . WORK IS DONE AS THE TORQUE CHANGES THE ANGLE

WORK DONE ON MOLECULE \equiv ENERGY STORED $\equiv U = \int \tau d\theta = -p_0 E \cos \theta = -\vec{p}_0 \cdot \vec{E}$

BOLTZMANN TELLS US THAT THE CONDITIONS OF THERMAL EQUILIBRIUM AT TEMPERATURE T REQUIRE STATES OF ENERGY U TO BE POPULATED ACCORDING TO A PROBABILITY DISTRIBUTION

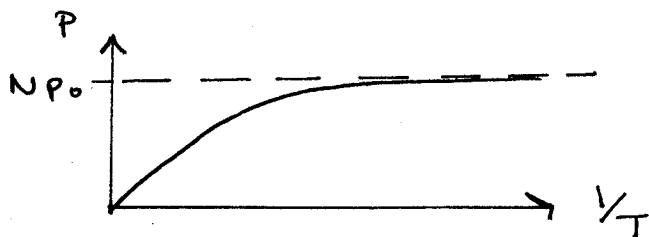
$$P(U) dU \sim e^{-\frac{U}{kT}} dU \quad \text{WHERE } k = \text{BOLTZMANN'S CONSTANT}$$

AS $T \rightarrow \infty$, ALL STATES ARE EQUALLY LIKELY, AND THERMAL EFFECTS DESTROY THE TENDENCY OF THE DIPOLES TO LINE UP.

WE MAY CALCULATE THE AVERAGE POLARIZATION DUE TO THE ALIGNMENT OF THE POLAR MOLECULES AT A FINITE TEMPERATURE

$$P_{AVE} = \frac{N \int p_0 \cos \theta e^{-\frac{(-p_0 E \cos \theta)}{KT}} d\omega \theta}{\int e^{\frac{+p_0 E \cos \theta}{KT}} d\omega \theta}$$

$$= N p_0 \left[\coth \left(\frac{p_0 E}{KT} \right) - \frac{KT}{p_0 E} \right]$$



FOR LARGE T $P \sim \frac{N p_0^2 E}{3KT} = N \alpha' E$ WITH $\alpha' = \frac{p_0^2}{3KT}$

RETURNING TO THE CLAUSIUS-MOSOTTI EQUATION, WE SEE THAT THE ALIGNMENT OF THE PERMANENT MOMENTS MAKES THE APPARENT MOLECULAR POLARIZABILITY BECOME

$$\alpha + \alpha' = \alpha + \frac{p_0^2}{3KT}$$

AND SO $\frac{\epsilon - 1}{\epsilon + 2} = \frac{4\pi N}{3} \left(\alpha + \frac{p_0^2}{3KT} \right)$ (T LARGE)

$$\sim a + \frac{b}{T}$$

↑ PERMANENT MOMENT
↑ INDUCED MOMENT

THUS BY MEASUREMENT OF THE TEMPERATURE DEPENDENCE OF THE DIELECTRIC CONSTANT THE PERMANENT MOMENT p_0 , AND THE MOLECULAR POLARIZABILITY α CAN BOTH BE DETERMINED.

[NOTE: THE MOLECULAR NUMBER DENSITY N HAS A TEMPERATURE DEPENDENCE IF THE MEDIUM (OR CONTAINER) IS SUBJECT TO THERMAL EXPANSION...]

HISTORICAL NOTE ON THE CONCEPT OF CHARGE.

WHEN COULOMB WROTE $\vec{F} = q_1 q_2 \frac{\hat{r}}{r^2}$ HE DID NOT IMPLY THE

NECESSARY EXISTENCE OF POINT CHARGES. RATHER, THIS EXPRESSION SERVED AS A USEFUL LIMIT OF SOME PHYSICAL PROPERTY - CALLED CHARGE - THAT APPEARED TO BE UNIFORMLY DISTRIBUTED OVER THE SURFACE OF METAL SPHERES IN SOME CIRCUMSTANCES.

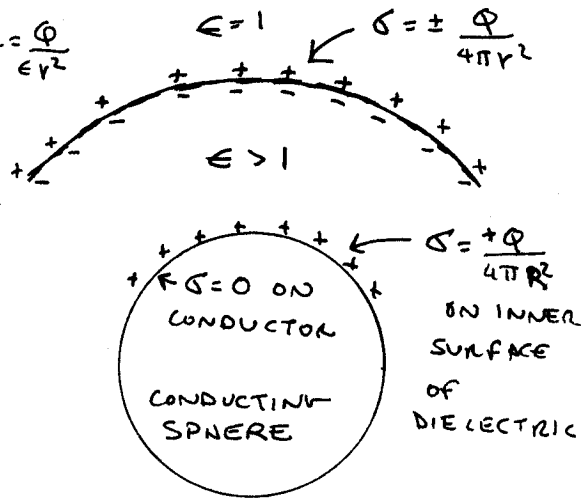
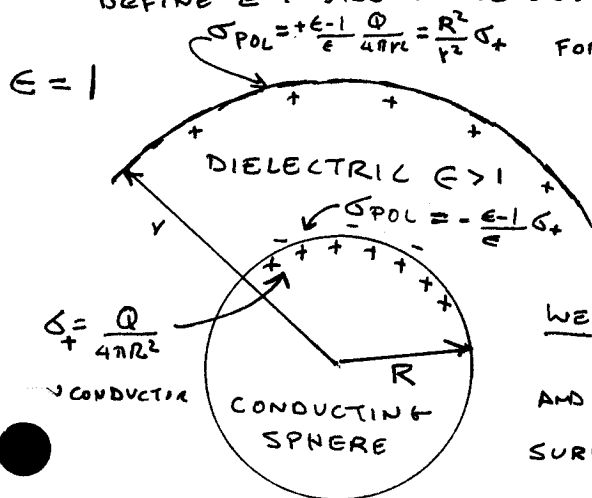
SINCE J.J. THOMSON'S DISCOVERY OF THE ELECTRON IN 1895 AND H.A. LORENTZ' RE-INTERPRETATION OF MAXWELL'S THEORY IN TERMS OF CHARGE AS A PROPERTY OF POINT (OR VERY TINY) MASS, WE ARE COMFORTABLE IN SAYING THE CHARGE ON METAL SPHERES CONSISTS OF AN EXCESS (OR DEFICIT) OF ELECTRONS THERE.

MAXWELL HAD A VERY DIFFERENT VIEW!

FOR HIM, THE EQUATION $\vec{\nabla} \cdot \vec{D} = 4\pi \rho$ WAS THE KEY TO ELECTROSTATICS. FURTHER, THE FIELD \vec{D} WAS THE CENTRAL CONCEPT. IT DESCRIBED A STRAIN OR DISPLACEMENT OF THE ETHER. ELECTRIC FORCES ON SOME OBJECT COULD BE CALCULATED VIA THE AUXILIARY CONCEPT $\rho = \frac{\vec{\nabla} \cdot \vec{D}}{4\pi}$, WHICH IS A MEASURE OF LOCALIZED ACCUMULATION OF STRAIN.

A (PERFECT) CONDUCTOR IS A REGION OF SPACE THAT CAN SUPPORT NO DISPLACEMENT. SO THEN $\vec{\nabla} \cdot \vec{D} = 0$ IN A CONDUCTOR, AND CONDUCTORS HAVE NO ρ , i.e., NO CHARGE. MAXWELL FURTHER INTERPRETED THE SURFACE CHARGE AS RESIDING NOT ON THE CONDUCTOR, BUT ON THE 'SURFACE' OF THE MEDIUM (DIELECTRIC, OR ETHER) NEXT TO THE CONDUCTOR.

MAXWELL DID NOT HAVE A MODEL OF DIELECTRIC POLARIZABILITY IN THE MODERN SENSE. HE WROTE $\vec{D} = \epsilon \vec{E}$ BUT NOT $\vec{P} = \chi \vec{E}$. HE DID NOT SPEAK OF POLARIZATION SURFACE CHARGE $\sigma = \vec{P} \cdot \hat{n}$ (P.16), NOR DID HE DEFINE \vec{E} INSIDE A DIELECTRIC.



WE SAY, $\vec{P} = \frac{\epsilon-1}{4\pi} \vec{E} = \frac{\epsilon-1}{\epsilon} \frac{Q}{4\pi r^2} \hat{r}$
 AND σ_{POL} ON DIELECTRIC SURFACE: $\sigma_{POL} = \vec{P} \cdot \hat{n} = \frac{\epsilon-1}{\epsilon} \frac{Q}{4\pi r^2}$

PRESENT VIEW
 [NO ETHER, NO CHARGES AT $r > R$ IF $\epsilon = 1$]

MAXWELL'S VIEW
 (TREATISE, VOL 1, SEC 111)
 [MUST HAVE ETHER, SINCE ALL CHARGE IS AT $r > R$]

111.] I have not been able to make the next step, namely, to account by mechanical considerations for these stresses in the dielectric. I therefore leave the theory at this point, merely stating what are the other parts of the phenomenon of induction in dielectrics.

I. *Electric Displacement.* When induction is transmitted through a dielectric, there is in the first place a displacement of electricity in the direction of the induction. For instance, in a Leyden jar, of which the inner coating is charged positively and the outer coating negatively, the direction of the displacement of positive electricity in the substance of the glass is from within outwards.

Any increase of this displacement is equivalent, during the time of increase, to a current of positive electricity from within outwards, and any diminution of the displacement is equivalent to a current in the opposite direction.

The whole quantity of electricity displaced through any area of a surface fixed in the dielectric is measured by the quantity which we have already investigated (Art. 75) as the surface-integral of induction through that area, multiplied by $K/4\pi$, where K is the specific inductive capacity of the dielectric.

II. *Surface charge of the particles of the dielectric.* Conceive any portion of the dielectric, large or small, to be separated (in imagination) from the rest by a closed surface, then we must suppose that on every elementary portion of this surface there is a charge measured by the total displacement of electricity through that element of surface reckoned inwards.

In the case of the Leyden jar of which the inner coating is charged positively, any portion of the glass will have its inner side charged positively and its outer side negatively. If this portion be entirely in the interior of the glass, its surface charge will be neutralized by the opposite charge of the parts in contact with it, but if it be in contact with a conducting body, which is incapable of maintaining in itself the inductive state, the surface charge will not be neutralized, but will constitute that apparent charge which is commonly called the Charge of the Conductor.

The charge therefore at the bounding surface of a conductor and the surrounding dielectric, which on the old theory was called the charge of the conductor, must be called in the theory of induction the surface charge of the surrounding dielectric.

According to this theory, all charge is the residual effect of the polarization of the dielectric. The polarization exists throughout the interior of the substance, but it is there neutralized by the juxtaposition of oppositely charged parts, so that it is only at the surface of the dielectric that the effects of the charge become

The theory completely accounts for the theorem of Art. 71, that the total induction through a closed surface is equal to the total quantity of electricity within the surface multiplied by 4π . For what we have called the induction through the surface is simply the electric displacement multiplied by 4π , and the total displacement outwards is necessarily equal to the total charge within the surface.

The theory also accounts for the impossibility of communicating an 'absolute charge' to matter. For every particle of the dielectric has equal and opposite charges on its opposite sides, if it would not be more correct to say that these charges are only the manifestations of a single phenomenon, which we may call Electric Polarization.

A dielectric medium, when thus polarized, is the seat of electric energy, and the energy in unit of volume of the medium is numerically equal to the electric tension on unit of area, both quantities being equal to half the product of the displacement and the resultant electromotive intensity, or

$$p = \frac{1}{2} \mathcal{D} \mathcal{E} = \frac{1}{8\pi} K \mathcal{E}^2 = \frac{2\pi}{K} \mathcal{D}^2,$$

where p is the electric tension, \mathcal{D} the displacement, \mathcal{E} the electromotive intensity, and K the specific inductive capacity.

If the medium is not a perfect insulator, the state of constraint, which we call electric polarization, is continually giving way. The medium yields to the electromotive force, the electric stress is relaxed, and the potential energy of the state of constraint is converted into heat. The rate at which this decay of the state of polarization takes place depends on the nature of the medium. In some kinds of glass, days or years may elapse before the polarization sinks to half its original value. In copper, a similar change is effected in less than the billionth of a second.

We have supposed the medium after being polarized to be simply left to itself. In the phenomenon called the electric current the constant passage of electricity through the medium tends to restore the state of polarization as fast as the conductivity of the medium allows it to decay. Thus the external agency which maintains the current is always doing work in restoring the polarization of the medium, which is continually becoming relaxed, and the potential energy of this polarization is continually becoming transformed into heat, so that the final result of the energy expended in maintaining the current is to gradually raise the temperature of the conductor, till as much heat is lost by conduction and radiation from its surface as is generated in the same time by the electric current.