

Ph 206 SET 12

DUE: TUESDAY, MAY 1, 1984 ; MAXIMUM RECORDED SCORE = 70 POINTS

① A PLANE WAVE IS INCIDENT ON A FREE ELECTRON. CALCULATE THE AVERAGE FORCE ON THE ELECTRON, SUPPOSING THE VELOCITY REMAINS SMALL, SO WE MAY NEGLECT TERMS IN v/c .

a) TO THE FIRST APPROXIMATION THE MOTION OF THE ELECTRON IS TRANSVERSE TO THE DIRECTION OF WAVE MOTION. THUS, REFERRING TO P 237 OF THE NOTES, THE MOMENTUM RADIATED BY THE CHARGE AVERAGES TO ZERO. SO ALL MOMENTUM LOST FROM THE INCIDENT WAVE MUST BE ABSORBED BY THE ELECTRON. USE THIS TO SHOW THAT

$$\bar{F} = \frac{\sigma_{\text{THOMSON}} \langle E^2 \rangle \hat{n}}{4\pi}$$

b) WE SHOULD ALSO BE ABLE TO CALCULATE THE FORCE VIA THE RADIATION REACTION. SHOW THAT THIS LEADS TO THE SAME RESULT AS a). HINT: CALCULATE IN THE INSTANTANEOUS REST FRAME OF THE ELECTRON.

② A CIRCULARLY POLARIZED PLANE WAVE OF FREQUENCY ω IS INCIDENT ON A FREE ELECTRON. FIND THE RESULTANT MOTION OF THE ELECTRON EVEN FOR STRONG FIELDS FOR WHICH $v_{\text{EL}} \rightarrow c$. IGNORE THE LONGITUDINAL FORCE FOUND IN PROBLEM ①

a) SHOW THAT THE TOTAL RADIATED INTENSITY IS

$$\frac{dU}{dt} = \frac{2}{3} \frac{e^4}{m_0^2 c^3} \gamma^2 E^2 = \frac{2}{3} \frac{e^4 E^2}{m_0^2 c^3} \left[1 + \left(\frac{eE}{m_0 \omega c} \right)^2 \right] \quad \left(\begin{array}{l} \text{MEASURED AT} \\ \text{THE ELECTRON} \end{array} \right)$$

USING OUR RELATIVISTIC TRANSFORMATION METHODS.

b) SUPPOSE INSTEAD WE USED A MULTIPOLE EXPANSION TO CALCULATE THE RADIATED INTENSITY. REFERRING TO PREVIOUS WORK (WITHOUT REPEATING IT) SHOW

$$\frac{dU}{dt} \sim \frac{2}{3} \frac{e^4 E^2}{m_0^2 c^3} \left(1 + \frac{7}{5} \left(\frac{eE}{m_0 \omega c} \right)^2 + \dots \right) \quad \left(\begin{array}{l} \text{MEASURED BY AN} \\ \text{EXTERNAL} \\ \text{OBSERVER.} \end{array} \right)$$

PART OF
NOTE THAT THE 2ND TERM IN THIS EXPANSION CORRESPONDS TO RADIATION AT FREQUENCY 2ω . THUS WE HAVE A FREQUENCY DOUBLING MECHANISM!

[FROM THE QUANTUM VIEW POINT, THIS CORRESPONDS TO THE ABSORPTION OF 2 PHOTONS FROM THE INCIDENT WAVE, AND THE RE-EMISSION OF ONE PHOTON OF TWICE THE ENERGY OF THE INCIDENT PHOTONS.]

- ③ AN ELLIPTICALLY POLARIZED PLANE WAVE IS INCIDENT ON A FREE ELECTRON. CALCULATE THE CROSS SECTION FOR SCATTERING OF RADIATION. (IGNORE THE STRONG FIELD EFFECTS OF PROBLEM ②)

NOTE THAT THE ELECTRIC FIELD HAS THE FORM

$$\vec{E} = \vec{A} \cos(\omega t - kz) + \vec{B} \sin(\omega t - kz)$$

WHERE \vec{A} IS \perp TO \vec{B} (\neq BOTH \perp TO \hat{z}), BUT $|\vec{A}| \neq |\vec{B}|$ IN GENERAL.

$$\text{SHOW } \frac{d\sigma}{d\Omega} = r_0^2 \frac{(\vec{A} \times \hat{n})^2 + (\vec{B} \times \hat{n})^2}{A^2 + B^2}$$

NOTE THAT FOR CIRCULARLY POLARISED LIGHT, $\sigma_{\text{LEFT}} = \sigma_{\text{RIGHT}} = \sigma_{\text{THOMSON}}$, CONSISTENT WITH PROBLEM ②.

- ④ CALCULATE THE SCATTERING OF LINEARLY POLARIZED RADIATION OFF POLAR MOLECULES OF FIXED DIPOLE MOMENT \vec{d} . YOU MAY SUPPOSE THE WAVELENGTH OF THE INCIDENT RADIATION SATISFIES $\lambda \gg d/e$. ← SIZE OF THE MOLECULE. IGNORE ANY MOTION OF THE MOLECULES OTHER THAN THAT CAUSED BY THE INCIDENT WAVE.

$$\text{SHOW } \sigma = \frac{16\pi d^4}{9I^2 c^4}$$

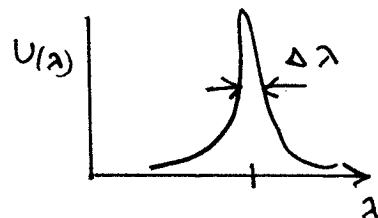
WHERE I = MOMENT OF INERTIA OF MOLECULE ABOUT AN AXIS \perp TO \vec{d}
ASSUME ALL ANGLES OF ORIENTATION OF THE DIPOLES \vec{d} TO THE FIELD \vec{E} ARE EQUALLY LIKELY - AS IN A GAS.

⑤ SPECTRAL LINE BROADENING

- a) A SPECTRAL 'LINE' OF CENTRAL WAVELENGTH λ IS ALWAYS OBSERVED TO HAVE A FINITE SPREAD OF WAVELENGTHS $\Delta\lambda$.

CONSIDER THE YELLOW SODIUM LINE WITH $\lambda = 5893 \text{ \AA}$

WHAT IS THE CONTRIBUTION TO $\frac{\Delta\lambda}{\lambda}$ FROM THE DAMPING DUE TO THE RADIATION REACTION?



- b) DOPPLER BROADENING SUPPOSE WE OBSERVE GLOWING SODIUM VAPOR AT 600°K . THEN SOME OF THE GLOWING ATOMS ARE MOVING TOWARDS US, AND SOME AWAY. THE OBSERVED WAVELENGTH IS THEN DOPPLER SHIFTED. HOW BIG IS THE SPREAD $\Delta\lambda/\lambda$ DUE TO THIS EFFECT? (THERE IS NO NEED TO GO INTO DETAILS OF THE MAXWELL DISTRIBUTION...).

c) COLLISION BROADENING AN ATOM STARTS GLOWING AT $t=0$, DUE TO SOME PROCESS SUCH AS A COLLISION WHICH SET THE ATOM'S "SPRING" OSCILLATING. AT SOME LATER TIME $T > 0$ ANOTHER COLLISION OCCURS WHICH IN EFFECT STOPS THE RADIATION (OR AT LEAST DESTROYS THE COHERENCE OF THE RADIATION). THUS THE WIDTH IN TIME OF THE PULSE OF RADIATION IS REDUCED, SO OUR UNCERTAINTY RELATION $\Delta \omega \Delta t \sim 1$ TELLS US THAT THE SPECTRUM IS BROADENED.

SUPPOSE ν = MEAN COLLISION FREQUENCY (#/SEC). THEN

$$\nu = [\text{COLLISION CROSS-SECTION}] \times \left[\frac{\# \text{ OF ATOMS}}{\text{VOLUME}} \right] \times [\text{AVE RELATIVE VELOCITY}].$$

(CHECK THE DIMENSIONS!) A PLAUSIBLE COLLISION CROSS SECTION IS $\sim 10^{-14} \text{ cm}^2$. THE # OF ATOMS/VOLUME DEPENDS ON THE PRESSURE. VELOCITY DEPENDS ON TEMPERATURE. CALCULATE $\nu(P)$ AT $T = 600^\circ \text{K}$, FOR P IN ATMOSPHERES.

CONVINCE YOURSELF THAT THE PROBABILITY OF A COLLISION DURING TIME dt IS νdt , SO THAT THE PROBABILITY OF NO COLLISION HAVING OCCURED BETWEEN $t=0$ AND t IS $e^{-\nu t}$.

AVERAGING OVER MANY COLLISIONS, THIS MEANS THAT THE AVERAGE INTENSITY OF THE RADIATION AT TIME t AFTER THE BEGINNING OF EMISSION IS $I_0 e^{-\nu t}$. THIS IS VERY SIMILAR TO THE EFFECT OF THE OTHER DAMPING MECHANISMS, FOR WHICH $I = I_0 e^{-2\gamma t}$ (SINCE $I \sim E^2 \sim (E_0 e^{-\gamma t})^2$). HENCE THE EFFECTIVE DAMPING CONSTANT IN THE PRESENCE OF COLLISIONS IS $\gamma = \frac{\nu}{2} + \gamma_{\text{OTHER}}$.

AT WHAT PRESSURE IS $\frac{\Delta \lambda}{\lambda} |_{\text{COLLISIONS}} \sim \frac{\Delta \lambda}{\lambda} |_{\text{DOPPLER}}?$

⑥ OPTICAL THEOREM IN LECTURE 17 ON DIFFRACTION, WE NOTED THAT IF WE WRITE $\frac{d\sigma_{\text{SCAT}}}{d\Omega} = |f(\theta)|^2$

THEN $\sigma_{\text{TOT}} = \frac{4\pi}{k} |\text{Im } f(0^\circ)|$ - THE OPTICAL THEOREM.

GIVE A SUITABLE FORM FOR $f(\theta)$ FOR THE SCATTERING OF RADIATION OFF ELECTRONS BOUND IN ATOMS, TO SHOW THAT THE OPTICAL THEOREM INDEED HOLDS FOR $\sigma_{\text{TOT}} = \sigma_{\text{THOMSON}} \frac{(\frac{\gamma}{\gamma_0}) \omega^2 \omega_0^2}{(\omega^2 - \omega_0^2) + \gamma^2 \omega^2}$

TRY THIS OUT FOR A FREE ELECTRON, WHERE $\sigma_{TOT} = \sigma_{THOMSON}$.
WHAT PHYSICAL EFFECT SAVES THE OPTICAL THEOREM HERE?

- ⑦ LEVINGER-BETHE SUM RULE IN A SERIES OF EXPERIMENTS, THE SCATTERING OF RADIATION OFF COPPER NUCLEI WAS MEASURED, AND THE INTEGRATED CROSS SECTION WAS DETERMINED TO BE

$$\int \sigma dE = \int \sigma d(k\omega) \sim 1.5 \times 10^{-24} \text{ MeV cm}^2$$

$$(1 \text{ MeV} = 10^6 \text{ ELECTRON VOLTS})$$

CALCULATE THIS INTEGRAL ASSUMING ALL THE RADIATION IS DUE TO DIPOLE MOMENT OSCILLATIONS. (START FROM P. 282 OF THE NOTES).

- SUPPOSE THE ENTIRE NUCLEUS MOVES AS A WHOLE.
- SUPPOSE THE NEUTRONS IN THE NUCLEUS REMAINS FIXED WHILE ONLY THE PROTONS MOVE ABOUT.
- SUPPOSE THE PROTONS MOVE AS A GROUP, AND THE NEUTRONS MOVE AS A GROUP, BUT THE C.M. OF THE NUCLEUS REMAINS FIXED. ANS: $\int \sigma dE = 2\pi \frac{N^2}{A} \alpha \left(\frac{\hbar}{Mc}\right)^2 Mc^2$

WHERE $Z = \#$ OF PROTONS = 29 FOR COPPER

$N = \#$ OF NEUTRONS

$A = N + Z = 64$ FOR COPPER

$$\alpha = e^2/\hbar c = 1/137 \quad ; \quad M = M_p = M_n$$

HOW WELL DOES THE SUM RULE WORK NUMERICALLY?

REFERENCES: GOLDHABER & TELLER PHYS. REV. 74, 1046 (1948)
LEVINGER & BETHE PHYS. REV. 78, 115 (1950)

- ⑧ WHAT IS THE MINIMUM FREQUENCY WHICH CAN BE PROPAGATED IN FREE SPACE WITHOUT ATTENUATION, SUPPOSING THERE IS ONE ELECTRON PER CUBIC CENTIMETER?

FOR FREQUENCIES VERY MUCH HIGHER THAN THE CRITICAL FREQUENCY IS THERE REALLY 'NO ATTENUATION'? FOR EXAMPLE, AT OPTICAL FREQUENCIES $\lambda \ll 1 \text{ cm} =$ AVERAGE DISTANCE BETWEEN SCATTERING CENTERS. THEN WE MAY DOUBT THAT THE SCATTERED RADIATION ADDS UP COHERENTLY FROM ELECTRON TO ELECTRON TO PRODUCE 'NO ATTENUATION'! RATHER, THE SCATTERED RADIATION IS EFFECTIVELY LOST TO AN OBSERVER LOOKING AT, SAY, A DISTANT STAR. HOW MUCH IS LOST?

RECALL THE MEANING OF THE SCATTERING CROSS SECTION TO FIND THE DIFFERENTIAL EQUATION FOR THE LOSS OF INTENSITY OF THE INCIDENT WAVE:

$$\frac{dI}{I} = -f(\sigma \dots) dL$$

WHERE dL = THICKNESS OF THE MEDIUM TRAVERSED

FIND $I = I(L)$. WHAT IS L SUCH THAT $I_{\text{REMAINING}} = 1\%$ OF I_0 ? (FOR 1 ELECTRON / cm^3). EXPRESS YOUR ANSWER IN LIGHT YEARS.

THIS EFFECT LEADS TO THE OBSCURATION OF STARLIGHT BY GALACTIC 'DUST CLOUDS'.

9) GRAVITATIONAL RED SHIFT

IN 1911 EINSTEIN GAVE A SIMPLE NON-QUANTUM DERIVATION OF THE GRAVITATIONAL RED SHIFT. AS IN THE NOTES THERE ARE 2 KEY INGREDIENTS

- THE PRINCIPLE OF EQUIVALENCE, TO RELATE PHYSICS IN A UNIFORM GRAVITATIONAL FIELD TO THAT IN A UNIFORMLY ACCELERATED FRAME.
- THE USE OF INSTANTANEOUS LORENTZ FRAMES TO RELATE PHYSICS IN THE ACCELERATED FRAME TO THAT IN AN APPROPRIATE INERTIAL FRAME FOR A SHORT TIME.

AS IN THE NOTES, ACCELERATED FRAME A COINCIDES WITH INERTIAL FRAME I' AT $t = t' = 0$, AND HAS ACCELERATION g RELATIVE TO I' THEREAFTER, ALONG THE z' AXIS.

AT $t = 0$ LIGHT OF FREQUENCY ν_0 IS EMITTED BY A SOURCE AT THE ORIGIN OF A.

THIS LIGHT IS DETECTED BY AN OBSERVER AT $x=0, y=0, z=z_1$ IN A WHERE IT IS FOUND TO HAVE FREQUENCY ν_1 .

INTRODUCE THE ADDITIONAL INERTIAL FRAME I'' WHICH IS THE INSTANTANEOUS LORENTZ FRAME CORRESPONDING TO FRAME A AT THE MOMENT THE LIGHT IS OBSERVED AT z_1 .

USE SPECIAL RELATIVITY DOPPLER SHIFTS TO SHOW

THAT
$$\nu_1 = \frac{\nu_0}{1 + \frac{gz_1}{c^2}} + O\left(\frac{g^2}{c^2}\right) \quad \left(\text{SEE BECKER SEC 84 IF NEEDED}\right)$$

$$\Rightarrow \nu = \frac{\nu_0}{1 + \frac{\Phi}{c^2}} \quad \text{IN THE UNIFORM GRAVITATIONAL FIELD.}$$