TESTS OF THE GLASHOW - SALAM - WEINBERG MODEL

1. COUPLING OF THE $W^\pm$ AND Z^0 BOSONS TO LEPTONS

WE FIRST ILLUSTRATE HOW THE WEAK INTERACTIONS OF THE $W^\pm$ AND Z^0 BOSONS CAN BE READ OFF OUR EXPRESSION FOR THE GAUGE COVARIANT DERIVATIVE (P.381)

$$D_\mu = \partial_\mu + i\frac{g}{\sqrt{2}}\left(\sigma^+ W_\mu^- + \sigma^- W_\mu^+\right) + ieA_\mu + i\frac{g}{\cos\theta_W} Z_\mu \left(\cos^2\theta_W I_3 - \sin^2\theta_W \frac{Y}{2}\right)$$

WE HAVE USED THE IDENTIFICATION $e = g\sin\theta_W$ IN WRITING ieA_μ.

WE INTERPRET THE TERMS INVOLVING THE POTENTIALS AS INDICATING THE COUPLING OF THE VARIOUS FERMIONS OF THE $SU(2)_L \times U(1)_Y$ CLASSIFICATION TO THE QUANTA OF THE POTENTIALS W_μ^+, W_μ^-, A_μ AND Z_μ. FOR EACH COUPLING THERE IS A CORRESPONDING FEYNMAN DIAGRAM WHICH SERVES AS A CONVENIENT WAY OF SUMMARIZING THE RESULT.

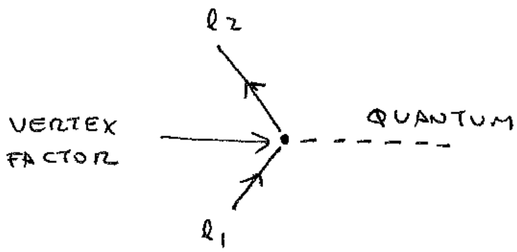


Table 11.1 Weak isospin and hypercharge assignments for fermions.

	I	I_3	y	Q
ν_e, ν_μ	1/2	1/2	−1	0
e_L, μ_L	1/2	−1/2	−1	−1
e_R, μ_R	0	0	−2	−1
u_L, c_L	1/2	1/2	1/3	2/3
$(d_C)_L, (s_C)_L$	1/2	−1/2	1/3	−1/3
u_R, c_R	0	0	4/3	2/3
$(d_C)_R, (s_C)_R$	0	0	−2/3	−1/3

WE CONSIDER THE COUPLINGS TO THE e AND ν_e AS AN EXAMPLE. THE CASE OF THE ELECTROMAGNETIC COUPLING IS OF COURSE THE SIMPLEST. THE PHOTON CANNOT COUPLE AT ALL TO THE ν AS THIS HAS NO CHARGE. (CAN THE ν HAVE A MAGNETIC MOMENT?) THE PHOTON CAN COUPLE TO BOTH LEFT HANDED AND RIGHT HANDED ELECTRONS WITH THE SAME STRENGTH - THE ELECTRIC CHARGE e.

THE 4-VECTOR NATURE OF THE POTENTIAL A_μ REMINDS US THAT THE PHOTON HAS POLARIZATION. WE KNOW THAT GAUGE INVARIANCE LIMITS THE PHOTON TO ONLY 2 INDEPENDENT POLARIZATION STATES. THE VERTEX FACTOR MUST INCLUDE A 4-VECTOR WHICH CAN COUPLE TO THE POLARIZATION 4-VECTOR TO YIELD A SCALAR. IN ALL CASES WE TAKE THIS TO BE THE DIRAC MATRIX ELEMENT

$$(\bar{u}_{\ell_2} | \gamma_\mu | u_{\ell_1})$$

THE POSSIBILITY OF PARITY VIOLATION IS ENTIRELY CONTAINED WITHIN THE DIFFERENT COUPLINGS TO LEFT-HANDED AND RIGHT-HANDED PARTICLES. THUS WE WILL SOMETIMES WRITE

$$\text{VERTEX FACTOR} = e\,C_L\,(\bar{u}_L | \gamma_\mu | u_L) + e\,C_R\,(\bar{u}_R | \gamma_\mu | u_R)$$

WHERE C_L AND C_R ARE THE LEFT- AND RIGHT- HANDED COUPLING CONSTANTS. FOR THE PHOTON - ELECTRON VERTEX WE HAVE $C_L = C_R = 1$, WHILE FOR THE PHOTON - NEUTRINO CASE, $C_L = C_R = 0$

WE SEE THAT THE IDEA OF HELICITY CONSERVATION (p. 113) IS BUILT INTO THE WEINBERG-SALAM MODEL. THERE IS NO COUPLING AT ALL OF THE TYPE $(\bar{u}_L | \gamma_\mu | u_R)$ BETWEEN LEFT-HANDED AND RIGHT HANDED PARTICLES, AS THESE BELONG TO DIFFERENT MULTIPLETS OF WEAK ISOSPIN.

[RECALL THAT WE CAN ANNIHILATE A LEFT-HANDED ELECTRON WITH A RIGHT-HANDED POSITRON : $(\bar{v}_R | \gamma_\mu | u_L) \neq 0$, WHICH IS CONSISTENT BOTH WITH HELICITY CONSERVATION AND THE WEINBERG-SALAM MODEL.]

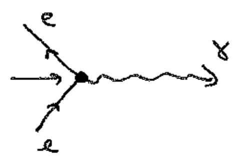
AS REMARKED ON p 382 WE TAKE $|u_L)$ TO MEAN $\frac{1-\gamma_5}{2} |u)$

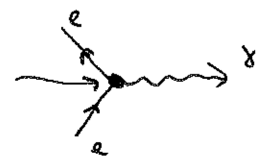
i.e. $|u_L)$ = RESULT OF APPLYING THE HELICITY PROJECTION OPERATOR (p. 116) TO THE STATE $|u)$. THEN $|u_R) = \frac{1+\gamma_5}{2} |u)$, AND $|u_L) + |u_R) = |u)$.

WITH THIS INTERPRETATION $(\bar{u}_L | \gamma_\mu | u_R) = (\bar{u}_R | \gamma_\mu | u_L) = 0$ FOR ALL ELECTRON ENERGIES, AS A CONSEQUENCE OF DIRACOLOGY. THUS

$$(\bar{u}_L | \gamma_\mu | u_L) + (\bar{u}_R | \gamma_\mu | u_R) = (\bar{u} | \gamma_\mu | u)$$

AND THE PHOTON ELECTRON VERTEX FACTOR BECOMES $e(\bar{u} | \gamma_\mu | u)$ EXACTLY AS IN FEYNMANS VIEW OF QED. WE SYMBOLIZE THIS AS

$e\gamma_\mu$ 

or $\frac{e}{2}\gamma_\mu\left[(1-\gamma_5) + (1+\gamma_5)\right]$ 

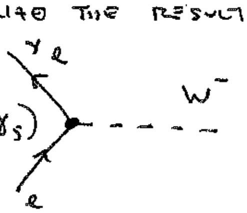
NEXT WE CONSIDER THE COUPLING OF THE W^- BOSON. IN THE GAUGE COVARIANT DERIVATIVE THE W^- IS ASSOCIATED WITH THE RAISING OPERATOR OF WEAK ISOSPIN, σ^+. THE BEHAVIOR OF σ^+ IS

$$\sigma^+ | I, I_3 \rangle = \sqrt{I(I+1) - I_3(I_3+1)}\; | I, I_3 + 1 \rangle$$

THUS $\sigma^+ | \nu_e \rangle = 0$, $\sigma^+ | e_R \rangle = 0$ AND $\sigma^+ | e_L \rangle = | \nu_e \rangle$

THUS THE W^- BOSON CAN ONLY CAUSE TRANSITIONS $e_L^- \to W^- + \nu_e$ (OR $\bar{\nu}_e \to W^- + e_R^+$)
THE STRENGTH OF THIS TRANSITION IS JUST $g/\sqrt{2}$, AS WE READ OFF THE EXPRESSION FOR D_μ (p. 386).
AGAIN THE VERTEX FACTOR INCLUDES THE DIRAC 4-VECTOR $(\bar{u}_{\nu_L} | \gamma_\mu | u_{e_L})$
THIS CAN BE REWRITTEN $\left(\bar{u}_\nu \middle| \left(\frac{1+\gamma_5}{2}\right) \gamma_\mu \left(\frac{1-\gamma_5}{2}\right) \middle| u_e\right) = \left(\bar{u}_\nu \middle| \gamma_\mu \left(\frac{1-\gamma_5}{2}\right) u_e\right)$

WE SUMMARIZE THE RESULT AS

$\frac{g}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)$ 

SO FOR THIS CASE $C_L = \frac{g}{2\sqrt{2}} = \sqrt{\frac{G M_W^2}{\sqrt{2}}}$

RECALLING THAT $\frac{G}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$ AS ON p 382.

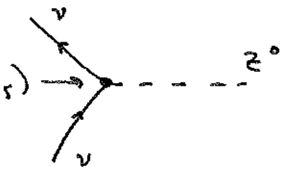
WE ALSO HAVE $C_R = 0$. THUS THE WEINBERG-SALAM MODEL EXACTLY REPRODUCES THE V-A THEORY OF THE CHARGED CURRENT.

WE LEARN SOMETHING NEW WHEN WE CONSIDER THE Z^0. LIKE THE PHOTON THE Z^0 CAN COUPLE BOTH TO LEFT-HANDED AND RIGHT-HANDED LEPTONS, ALTHOUGH THE COUPLINGS ARE NOT EQUAL. BUT IT IS SIMPLEST TO CONSIDER FIRST THE NEUTRINO, WHICH CAN ONLY BE LEFT-HANDED (IF $M_\nu = 0$). THE NEUTRINO HAS $I_3 = 1/2$ AND $Y = -1$ ACCORDING TO GLASHOW (SEE THE TABLE ON P 386). THUS THE FACTOR

$$\cos^2\theta_W\, I_3 - \sin^2\theta_W \frac{Y}{2} = \frac{1}{2} \qquad \text{FOR THE NEUTRINO}$$

IMMEDIATELY WE CAN WRITE THE VERTEX FACTOR AS

$$\frac{g}{2\cos\theta_W}\left(\bar{u}_{\nu_L}|\gamma_\mu|u_L\right) = \frac{g}{4\cos\theta_W}\left(\bar{u}_\nu|\gamma_\mu(1-\gamma_5)|u_\nu\right)$$

$\frac{g}{4\cos\theta_W}\gamma_\mu(1-\gamma_5)$ 

NOTE THAT $\frac{g}{4\cos\theta_W} = \frac{1}{\sqrt{2}}\sqrt{\frac{G M_Z^2}{\sqrt{2}}} = \frac{e}{2\sin 2\theta_W}$

WE COULD ALSO WRITE $\frac{g}{4\cos\theta_W} = \frac{g}{4\cos\theta_W}C_L = \frac{e\, C_L}{2\sin 2\theta_W}$ WITH $C_L = 1$, $C_R = 0$

ACCORDING TO A CERTAIN CONVENTION.

FOR THE ELECTRON WE HAVE $I_3 = -1/2$, $Y = -1$ FOR e_L
$I_3 = 0$, $Y = -2$ FOR e_R

$$\text{SO} \quad \cos^2\theta_W\, I_3 - \sin^2\theta_W\frac{Y}{2} = \begin{cases} \dfrac{2\sin^2\theta_W - 1}{2} & \text{FOR } e_L \\ \sin^2\theta_W & \text{FOR } e_R \end{cases}$$

THE VERTEX FACTOR IS THEN

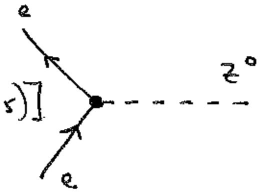
$$\frac{g}{2\cos\theta_W}\left[\left(\bar{u}_{eL}|\gamma_\mu(2\sin^2\theta_W - 1)u_{eL}\right) + \left(u_{eR}|\gamma_\mu\, 2\sin^2\theta_W|u_{eR}\right)\right]$$

$$= \frac{g}{4\cos\theta_W}\left(\bar{u}_e|\gamma_\mu\left[(2\sin^2\theta_W - 1)(1-\gamma_5) + 2\sin^2\theta_W(1+\gamma_5)\right]|u_e\right)$$

$$= \frac{g}{4\cos\theta_W}\left(\bar{u}_e|\gamma_\mu(4\sin^2\theta_W - 1 + \gamma_5)|u_e\right)$$

WE NOTE THAT THE EXPERIMENTAL RESULT: $\sin^2\theta_W = .23 \pm .01$ INDICATES THAT $4\sin^2\theta_W - 1 \approx 0$. THUS THE Z^0 COUPLING TO <u>ELECTRONS</u> IS ALMOST PURE AXIAL VECTOR, COMPLEMENTING THE PURE VECTOR COUPLING OF THE PHOTON. (THIS NICE RELATION DOES NOT HOLD FOR THE Z^0 COUPLING TO QUARKS). IT IS AMUSING TO SPECULATE AS TO WHETHER $\sin^2\theta_W = 1/4$ IN A DEEPER VERSION OF THE WEINBERG-SALAM MODEL...

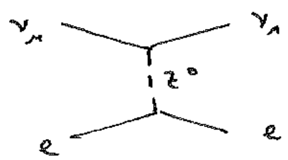
WE SUMMARIZE:

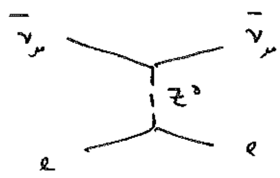
$\frac{g}{4\cos\theta_W}\gamma_\mu\left[C_L(1-\gamma_5) + C_R(1+\gamma_5)\right]$ 

WITH $C_L = 2\sin^2\theta_W - 1$
$C_R = 2\sin^2\theta_W$
FOR ELECTRONS.

2. NEUTRINO ELECTRON SCATTERING

WE SEE THAT EXPERIMENTAL DETERMINATION OF THE Z^0-ELECTRON COUPLINGS C_L AND C_R WILL LEAD TO A MEASUREMENT OF $\sin^2\theta_W$. FOR THIS WE CONSIDER THE REACTIONS

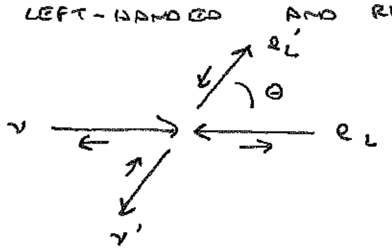




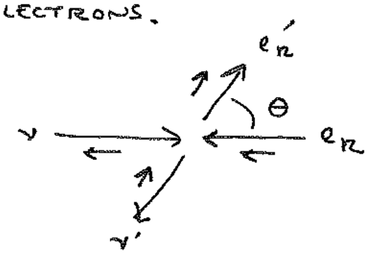
$\nu_\mu e \to \nu_\mu e$ $\qquad\qquad$ $\bar\nu_\mu e \to \bar\nu_\mu e$

EVEN IF $E_{CM} \ll M_{Z^0}$ WE CAN LEARN ABOUT C_L AND C_R, AS THESE REACTIONS CAN ONLY TAKE PLACE VIA Z^0 EXCHANGE. WE ANALYZE THEM IN A MANNER SIMILAR TO OUR CONSIDERATIONS OF $\nu q \to \mu q'$ (p 348, LECTURE 19).

IT IS SIMPLEST TO VIEW EACH REACTION AS THE SUM OF SCATTERING OFF LEFT-HANDED AND RIGHT-HANDED ELECTRONS.



$\frac{d\sigma}{d\Omega} \sim$ CONSTANT



$\frac{d\sigma}{d\Omega} \sim \left(\frac{1+\cos\theta}{2}\right)^2$

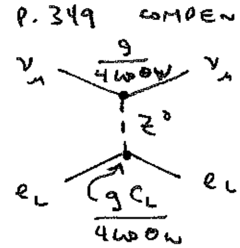
THE ANGLE θ IS MEASURED BETWEEN THE DIRECTION OF THE SCATTERED ELECTRON AND THE INCIDENT NEUTRINO. THIS CONVENTION FOLLOWS BECAUSE IT IS HARD TO OBSERVE THE SCATTERED ν. THE SCATTERING OFF RIGHT-HANDED ELECTRONS LEADS TO A $\left(\frac{1+\cos\theta}{2}\right)^2$ ANGULAR DISTRIBUTION VIA OUR USUAL ARGUMENTS INVOLVING THE SPIN 1 ROTATION MATRIX.

IN THIS REACTION THERE IS NO NEED TO INTRODUCE THE VARIABLE X (AS X=1 ALWAYS HERE). BUT WE DEFINE $y = \frac{E_\nu - E_e'}{E_\nu}$, SO THAT

$\frac{1+\cos\theta}{2} = 1-y$ AS ON p. 349.

[THE CHANGE IN THE DEFINITIONS OF θ AND y COMPARED TO p. 349 COMPENSATE...]

THEN $\frac{d\sigma}{dy}\,\nu_\mu e_L \sim |M|^2 \sim \left[\frac{g}{4\cos\theta_W}\,\frac{1}{q^2-M_Z^2}\,\frac{g}{4\cos\theta_W}\,C_L\right]^2$



IF $q^2 \ll M_Z^2 \implies$ $\sim\left(\frac{g^2}{16\,M_Z^2\cos^2\theta_W}\right)^2 C_L^2 \sim \left(\frac{g^2}{M_W^2}\right)^2 C_L^2 \sim G^2 C_L^2$

ON DIMENSIONAL GROUNDS WE EXPECT A FACTOR $\sim E_{CM}^2 = 2M_e E_\nu$ IN THE CROSS SECTION. THE FULL RESULT OF DIRACOLOGY IS

$$\frac{d\sigma}{dy}\,\nu_\mu e_L \to \nu_\mu e_L = \frac{G^2 m_e E_\nu}{\pi}\,C_L^2$$

(COMPARE p. 349)

FOR SCATTERING OFF RIGHT HANDED ELECTRONS THE ANGULAR FACTOR LEADS TO $(1-y)^2$ IN THE CROSS SECTION:

$$\frac{d\sigma}{dy}\,\nu_\mu e_R \to \nu_\mu e_R = \frac{G^2 m_e E_\nu}{\pi} C_R^2 (1-y)^2$$

FOR A TARGET OF UNPOLARIZED ELECTRONS WE THEN HAVE

$$\frac{d\sigma}{dy}\,\nu_\mu e \to \nu_\mu e = \frac{G^2 m_e E_\nu}{2\pi}\left(C_L^2 + C_R^2 (1-y)^2\right)$$

IF WE USE A $\bar{\nu}_\mu$ BEAM, THEN THE $\bar{\nu}_\mu e_R$ SCATTERING WILL BE ISOTROPIC (IN THE C.M. FRAME), WHILE THE $\bar{\nu}_\mu e_L$ VARIES LIKE $\left(\frac{1 \pm \cos\theta}{2}\right)^2 \to (1-y)^2$

HENCE WE CAN WRITE AT ONCE (NOTING THAT THE COUPLING OF Z^0 TO ν AND $\bar{\nu}$ HAS THE SAME STRENGTH)

$$\frac{d\sigma}{dy}\,\bar{\nu}_\mu e \to \bar{\nu}_\mu e = \frac{G^2 m_e E_\nu}{\pi}\left(C_L^2 (1-y)^2 + C_R^2\right)$$

WITH $\int_0^1 (1-y)^2 dy = 1/3$ WE ALSO HAVE

$$\sigma_{\nu_\mu e \to \nu_\mu e} = \frac{G^2 m E}{2\pi}\left(C_L^2 + \frac{C_R^2}{3}\right) \qquad \sigma_{\bar{\nu}_\mu e \to \bar{\nu}_\mu e} = \frac{G^2 m E}{2\pi}\left(\frac{C_L^2}{3} + C_R^2\right)$$

IN THE LITERATURE PEOPLE OFTEN USE ANOTHER NOTATION FOR THE COUPLING CONSTANTS (IN AN ATTEMPT TO DESCRIBE THE CROSS SECTIONS IN A 'MODEL INDEPENDENT' WAY)

$$g_V \equiv \frac{C_L + C_R}{2} \qquad g_A \equiv \frac{C_L - C_R}{2}$$

THIS OF COURSE COMES FROM REWRITING $\langle \bar{u} | \gamma_\mu \left[C_L \left(\frac{1-\gamma_5}{2}\right) + C_R \left(\frac{1+\gamma_5}{2}\right)\right] | u \rangle$

AS $\langle \bar{u} | \gamma_\mu \left(\frac{C_L + C_R}{2}\right) - \gamma_\mu \gamma_5 \left(\frac{C_L - C_R}{2}\right) | u \rangle$

THEN $C_L = g_V + g_A \qquad C_R = g_V - g_A$

AND
$$\sigma_{\nu_\mu e \to \nu_\mu e} = \frac{2 G^2 m E}{\pi}\left(\frac{g_V^2 + g_V g_A + g_A^2}{3}\right)$$

$$\sigma_{\bar{\nu}_\mu e \to \bar{\nu}_\mu e} = \frac{2 G^2 m E}{\pi}\left(\frac{g_V^2 - g_V g_A + g_A^2}{3}\right)$$

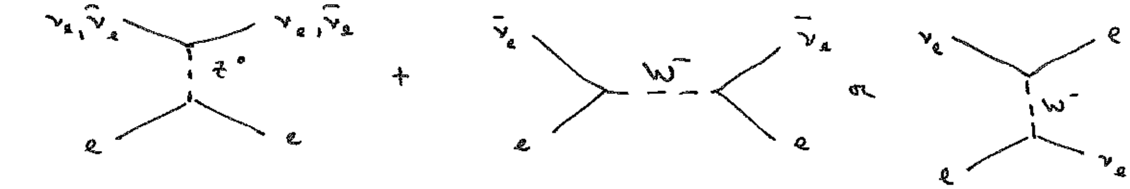
IN THE WEINBERG-SALAM MODEL WE PREDICT $g_V = 2\sin^2\theta_W - \frac{1}{2}$

$g_A = -\frac{1}{2}$

In a similar way we can analyze the scattering reactions

$\nu_e\, e \to \nu_e\, e$ and $\bar\nu_e\, e \to \bar\nu_e\, e$

These are a bit more complicated because there are 2 diagrams which can interfere



The interference occurs only for left handed electrons, for which the matrix element has the form

$$\mathcal{M}_L = \frac{g^2}{4\cos\theta_W} c_L (\bar u_{\nu'_L}|\gamma_\lambda|u_{\nu_L}) \frac{1}{q^2 - M_Z^2} (\bar u_{e'_L}|\gamma_\mu|u_{e_L})$$

$$+ \frac{g^2}{2} (\bar u_{\nu_L}|\gamma_\lambda|u_{e_L}) \frac{1}{E_{cm}^2 - M_W^2} (\bar u_{\nu'_L}|\gamma_\mu|u_{e'_L})$$

In the intermediate energy range where $E_{cm} \gg m_e$, but $E_{cm} \ll M_W, M_Z$ the propagators become $\frac{1}{M_Z^2}$ and $\frac{1}{M_W^2}$. There is a fancy result called the 'Fierz reshuffle theorem' which claims the plausible result that the products of the Dirac matrix elements are the same in both terms above.

Then
$$\mathcal{M}_L \sim \frac{g^2}{4M_W^2}(c_L + 2)$$

As before
$$\mathcal{M}_R \sim \frac{g^2}{4M_W^2} c_R$$

Then comparing to the $\nu_\mu e$ scattering case, we find

$$\sigma_{\nu_e e \to \nu_e e} = \frac{G^2 mE}{2\pi}\left[(c_L+2)^2 + \frac{c_R^2}{3}\right]$$

$$\sigma_{\bar\nu_e e \to \bar\nu_e e} = \frac{G^2 mE}{2\pi}\left[\frac{(c_L+2)^2}{3} + c_R^2\right]$$

Rewriting this in terms of g_V and g_A

$$\sigma_{\nu_e e \to \nu_e e} = \frac{2G^2 mE}{\pi}\left(\frac{g_V^2 + g_V g_V + g_A^2}{3} + g_V + g_A + 1\right)$$

$$\sigma_{\bar\nu_e e \to \bar\nu_e e} = \frac{2G^2 mE}{\pi}\left(\frac{g_V^2 - g_V g_A + g_A^2 + g_V + g_A + 1}{3}\right)$$

Turning to experiment such results are available as

$$\sigma_{\nu_\mu e \to \nu_\mu e} = 1.7 \pm 0.5 \times 10^{-42}\ \text{cm}^2 \cdot (E_\nu \text{ in GeV})$$

$$\sigma_{\bar\nu_\mu e \to \bar\nu_\mu e} = 1.8 \pm 0.9 \times 10^{-42}\ \text{cm}^2 \cdot (E \text{ in GeV})$$

Such equality is certainly consistent with the Weinberg-Salam model if $\sin^2\theta_W \sim 1/4$.

COMBINING THE DATA AVAILABLE FROM ALL DIFFERENT TYPES OF ν-e SCATTERING, ONE FINDS

$$\sin^2\theta_W \sim 0.27 \pm .07$$

THIS MEASUREMENT IS NOT TOO ACCURATE AS THERE ARE ONLY ABOUT 150 EVENTS AVAILBE FOR ALL REACTIONS COMBINED.

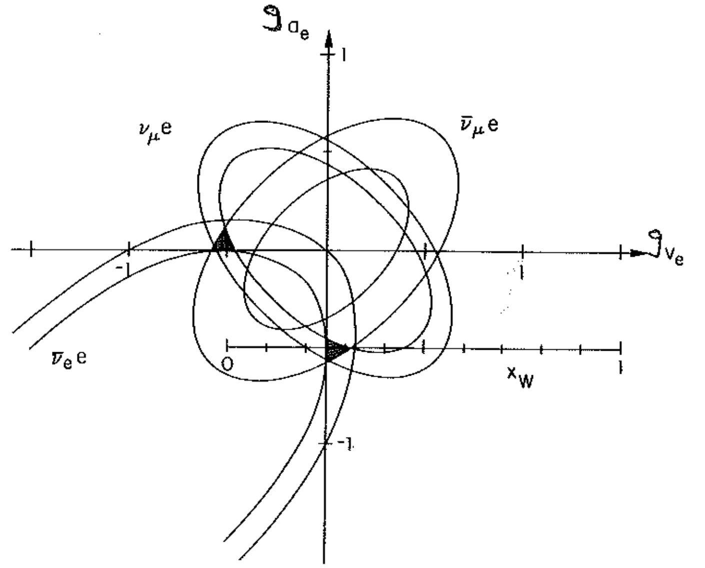
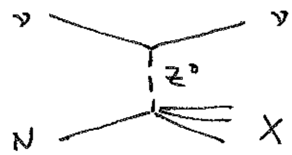


FIG. 6-18. Constraints on the neutral current parameters a_e and v_e from leptonic interactions (after F. Büsser, *Neutrino 81*, Proceedings of the 1981 International Conference on Neutrino Physics and Astrophysics, Maui, Hawaii, edited by R. J. Cence, E. Ma, and A. Roberts, High Energy Physics Group, University of Hawaii, Honolulu, 1981, Vol. II, p. 351).

3. INELASTIC NEUTRINO-NUCLEON SCATTERING

IN LECTURE 19, p353 WE INTRODUCED THE NEUTRAL CURRENT REACTION $\nu N \to \nu X$



TO ANALYZE THIS WE NEED TO KNOW THE COUPLING OF THE Z^0 TO QUARKS, SUPPOSING AS IN THE PARTON MODEL THAT ONLY A SINGLE QUARK ABSORBS THE EXCHANGED Z^0. AS FOR ELECTRONS, BOTH LEFT-HANDED AND RIGHT-HANDED QUARKS COUPLE TO THE Z_0. AS BEFORE THE COUPLING HAS THE FORM

$$\frac{g}{2\cos\theta_W}\left(\bar{u}_{L,R}\,|\,\gamma_\mu\,(1\mp\gamma_5)\,|\,u_{L,R}\right)\left(\cos^2\theta_W I_3 - \sin^2\theta_W \frac{Y}{2}\right)$$

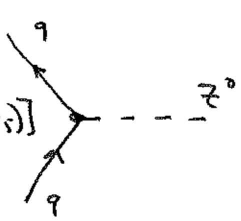
$$\downarrow$$

$$\left(I_3 - \sin^2\theta_W\left(I_3 + \frac{Y}{2}\right)\right)$$

$$\frac{g}{4\cos\theta_W}\left(\qquad\qquad\right)\left(2I_3 - 2Q\sin^2\theta_W\right)$$

USING $Q = I_3 + Y/2$ = QUARK CHARGE IN UNITS OF ELECTRON CHARGE

$$\frac{g}{4\cos\theta_W}\gamma_\mu\left[c_L(1-\gamma_5) + c_R(1+\gamma_5)\right]$$



$$c_L = 2I_3 - 2Q\sin^2\theta_W$$

$$c_R = -2Q\sin^2\theta_W$$

NOTING $I_3 = 0$ FOR ALL RIGHT HANDED QUARKS.

THEN $c_{L_u} = c_{L_c} = c_{L_t} = 1 - \frac{4}{3}\sin^2\theta_W \qquad c_{R_u} = c_{R_c} = c_{R_t} = -\frac{4}{3}\sin^2\theta_W$

$c_{L_d} = c_{L_s} = c_{L_b} = -1 + \frac{2}{3}\sin^2\theta_W \qquad c_{R_d} = c_{R_s} = c_{R_b} = \frac{2}{3}\sin^2\theta_W$

THESE COUPLINGS ARE PERHAPS SURPRISINGLY VARIED, SHOWING THE RICHNESS OF THE GLASHOW-WEINBERG-SALAM MODEL.

WE RECALL THE RESULTS OF p.350 FOR $\nu N \to \mu X$

$$\frac{d\sigma}{dx\,dy}\,\nu N \to \mu^- X = \frac{G^2 M_p E_\nu}{\pi}\, x \left[\left(u(x) + d(x)\right) + \left(\bar{u}(x) + \bar{d}(x)\right)(1-y)^2 \right]$$

$$\frac{d\sigma}{dx\,dy}\,\bar{\nu} N \to \mu^+ X = \frac{G^2 M E}{\pi}\, x \left[\left(u(x) + d(x)\right)(1-y)^2 + \left(\bar{u}(x) + \bar{d}(x)\right) \right]$$

IN THIS $u(x) = u^P(x) =$ DISTRIBUTION OF u QUARKS IN THE PROTON

$d(x) = d^P(x) =$ . . . . d . . . . .

SO THAT $d^n(x) = u^P(x)$ AND $u^n(x) = d^P(x)$

ALSO RECALL THAT THE $(1-y)^2$ FACTOR ARISES FROM THE $\left(\frac{1 \pm \cos\theta}{2}\right)^2$ ANGULAR DISTRIBUTION IN CASE THE ν AND QUARK SPINS ADD UP TO $J_z = 1$.

FOR $\nu N \to \nu X$ THE SCATTERING OFF LEFT-HANDED QUARKS IS VERY SIMILAR TO THE ABOVE. BUT THE Z^0 CAN SCATTER OFF BOTH u AND d QUARKS, WHILE THE W^+ CAN ONLY SCATTER OFF d QUARKS. THEN SINCE $u(x)$ STANDS FOR $\frac{u^P(x) + d^n(x)}{2}$ WE SHOULD MULTIPLY THE $u(x)$ BY $\frac{C_{L_u}^2 + C_{L_d}^2}{2}$. THIS ARGUMENT HOLDS FOR ALL 4 QUARK DISTRIBUTIONS, SO THE FACTOR $\frac{C_{L_u}^2 + C_{L_d}^2}{2}$ ACTUALLY MULTIPLIES THE ENTIRE CROSS-SECTION FOR SCATTERING OFF LEFT-HANDED QUARKS.

FOR THE CASE OF RIGHT-HANDED QUARKS THERE WILL BE A SIMILAR OVERALL FACTOR $\frac{C_{R_u}^2 + C_{R_d}^2}{2}$. FURTHER, THE ANGULAR FACTOR $\left(\frac{1 \pm \cos\theta}{2}\right)^2 = (1-y)^2$ APPLIES TO THE ANTIQUARKS COMPARED TO THE LEFT HANDED CASE.

FINALLY, THERE IS AN EXTRA $\frac{1}{\sqrt{2}}$ IN THE $Z^0 q q$ COUPLING COMPARED TO $W q q$ COUPLING (p 392 & 387). THEN

$$\frac{d\sigma}{dx\,dy}\,\nu N \to \nu X = \frac{G^2 M E}{4\pi}\, x \left\{ \begin{array}{l} \left(C_{L_u}^2 + C_{L_d}^2\right)\left[\left(u(x) + d(x)\right) + \left(\bar{u}(x) + \bar{d}(x)\right)(1-y)^2\right] \\ + \left(C_{R_u}^2 + C_{R_d}^2\right)\left[\left(u(x) + d(x)\right)(1-y)^2 + \bar{u}(x) + \bar{d}(x)\right] \end{array} \right\}$$

$$\frac{d\sigma}{dx\,dy}\,\bar{\nu} N \to \bar{\nu} X = \frac{G^2 M E}{4\pi}\, x \left\{ \begin{array}{l} \left(C_{L_u}^2 + C_{L_d}^2\right)\left[\left(u(x) + d(x)\right)(1-y)^2 + \left(\bar{u}(x) + \bar{d}(x)\right)\right] \\ + \left(C_{R_u}^2 + C_{R_d}^2\right)\left[\left(u(x) + d(x)\right) + \left(\bar{u}(x) + \bar{d}(x)\right)(1-y)^2\right] \end{array} \right\}$$

IN COMPARING THE NEUTRAL CURRENT REACTIONS TO THE CHARGED CURRENT REACTIONS WE NEGLECT THE SEA QUARKS AS A FIRST APPROXIMATION. THEN, INTEGRATING OVER x AND y,

$$\frac{\sigma_{\nu N\to\nu X}}{\sigma_{\nu N\to\mu^- X}} = \frac{(C_{Lu}^2+C_{Ld}^2)+\frac{1}{3}(C_{Ru}^2+C_{Rd}^2)}{4} = \frac{1}{2} - \sin^2\theta_W + \frac{20}{27}\sin^4\theta_W$$

$$\frac{\sigma_{\bar\nu N\to\bar\nu X}}{\sigma_{\bar\nu N\to\mu^+ X}} = \frac{\frac{1}{3}(C_{Lu}^2+C_{Ld}^2)+(C_{Ru}^2+C_{Rd}^2)}{4/3} = \frac{1}{2} - \sin^2\theta_W + \frac{20}{9}\sin^4\theta_W$$

USING

$$C_{Lu}^2+C_{Ld}^2 = 2 - 4\sin^2\theta_W + \frac{20}{9}\sin^4\theta_W$$

$$C_{Ru}^2+C_{Rd}^2 = \frac{20}{9}\sin^4\theta_W$$

ANOTHER RELATION WHICH CAN BE COMPARED TO EXPERIMENT IS

$$\frac{\sigma_{\nu N\to\nu X}-\sigma_{\bar\nu N\to\bar\nu X}}{\sigma_{\nu N\to\mu^- X}-\sigma_{\bar\nu N\to\mu^+ X}}$$

$$= \frac{(C_{Lu}^2+C_{Ld}^2)-(C_{Ru}^2+C_{Rd}^2)}{4} = \frac{1}{2}-\sin^2\theta_W$$

THE EXPERIMENTS THEN INDICATE

$$\sin^2\theta_W = 0.23 \pm .015$$

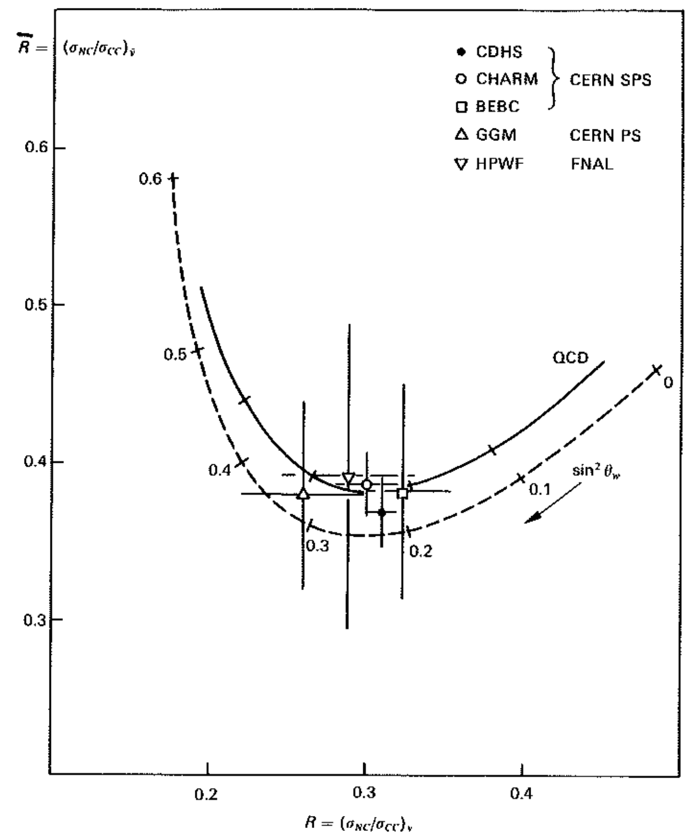
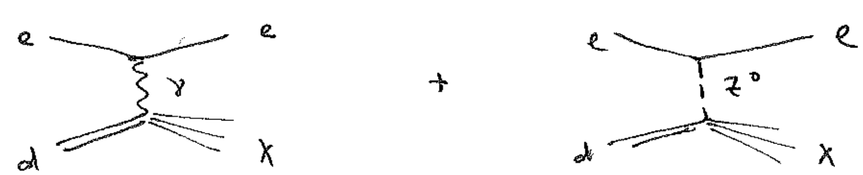


Fig. 8.13 Ratios of neutral-current to charged-current total cross-sections on nucleons, for antineutrinos ($\bar R$), and neutrinos, (R), as found in various experiments (see Table 8.3). The curve shows the prediction of the standard (Weinberg-Salam) model, based on quark distributions in the nucleon, with (full line) and without (dashed line) QCD corrections.

4. PARITY VIOLATION IN INELASTIC e-d SCATTERING

PERHAPS THE BEST MEASUREMENT OF $\sin^2\theta_W$ IN A SINGLE EXPERIMENT (PRIOR TO THE DISCOVERY OF THE W AND Z) CAME IN A STUDY OF THE REACTION $e\,d \to e\,X$. IN THE WEINBERG-SALAM MODEL WE HAVE 2 DIAGRAMS FOR THIS



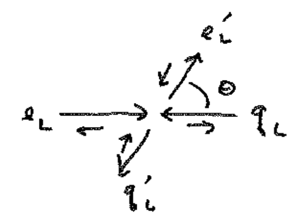
THE Z^0 EXCHANGE CAN LEAD TO PARITY VIOLATION.

Evidence for the Z^0 exchange was sought by comparing the scattering rate with a beam of left-handed and right-handed electrons. An asymmetry parameter was measured for scattering at a fixed angle [Prescott et al, Phys Lett, 77B, 347 (1978); 84B, 524 (1979)]

$$A = \frac{d\sigma_{e_R d} - d\sigma_{e_L d}}{d\sigma_{e_R d} + d\sigma_{e_L d}} \sim 10^{-4}$$

We can give an analysis of this in the Weinberg-Salam model which is somewhat simplified if we neglect the sea quarks. Then the scattering is off either a u quark or a d quark, each of which can be either left- or right-handed. Altogether there are 8 reactions to consider.

$$d\sigma_{e_L q_L \to e_L q_L} \sim \mathcal{M}^2 \sim \left[\frac{e^2 Q_q}{t} + \frac{g^2}{4\cos^2\theta_W} C_{Le} \frac{1}{t - M_Z^2} C_{Lq}\right]^2$$



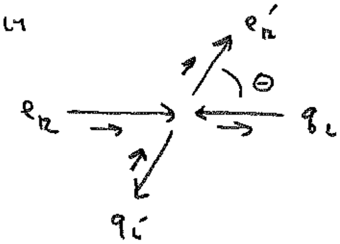
$t = (\text{momentum transfer})^2 << M_Z^2$

We also write $\dfrac{g^2}{4\cos^2\theta_W M_Z^2} = \dfrac{g^2}{4M_W^2} = \dfrac{2G}{\sqrt{2}} = \dfrac{e^2 G}{2\sqrt{2}\pi\alpha}$ with $\alpha = \dfrac{e^2}{4\pi}$

Then $$d\sigma_{e_L q_L} \sim \left(\frac{Q_q}{t} + \frac{G}{2\sqrt{2}\pi\alpha} C_{Le} C_{Lq}\right)^2$$

From the sketch of the spin orientations for this reaction in the c.m. frame we see that the angular factor is isotropic. Similarly

$$d\sigma_{e_R q_L} \sim \left(\frac{Q_q}{t} + \frac{G}{2\sqrt{2}\pi\alpha} C_{Re} C_{Lq}\right)^2 (1-y)^2$$



The $(1-y)^2$ factor arises as now the spins are aligned into a $J_z = 1$ state.

Likewise

$$d\sigma_{e_R q_L} \sim \left(\frac{Q_q}{t} + \frac{G}{2\sqrt{2}\pi\alpha} C_{Re} C_{Rq}\right)^2$$

$$d\sigma_{e_L q_R} \sim \left(\frac{Q_q}{t} + \frac{G}{2\sqrt{2}\pi\alpha} C_{Re} C_{Rq}\right)^2 (1-y)^2$$

In the above 4 sub-reactions, the q can be either u or d. At low t the square of the Z^0 exchange amplitude is negligible compared to photon exchange. We keep only the γ-Z^0 interference term in the following.

$$A = \frac{\sum_q d\sigma_{e_R q_R} + d\sigma_{e_R q_L} - d\sigma_{e_L q_R} - d\sigma_{e_L q_L}}{\sum_q d\sigma_{e_R q_R} + d\sigma_{e_R q_L} + d\sigma_{e_L q_R} + d\sigma_{e_L q_L}}$$

$$= \frac{Gt}{2\sqrt{2}\pi\alpha} \frac{\sum_q Q_q\left[C_{Re}C_{Rq} + C_{Re}C_{Lq}(1-y)^2 - C_{Le}C_{Rq}(1-y)^2 - C_{Le}C_{Lq}\right]}{\sum_q Q_q^2\left[1 + (1-y)^2\right]}$$

In the denominator we have ignored the interference term altogether.

With the coupling constants C_L and C_R from p 388 & 392 we find after a bit of algebra:

$$A = -\frac{9\,G t}{20\sqrt{2}\,\pi\alpha}\left[1 - \frac{20}{9}\sin^2\theta_W + \underbrace{(1-4\sin^2\theta_W)}_{\approx 0}\frac{1-(1-y)^2}{1+(1-y)^2}\right]$$

The results from the asymmetry experiment alone then indicates

$$\sin^2\theta_W = 0.224 \pm .02$$

Fig. 6.4. The experimental asymmetry shows the expected variation (dashed line) as the beam polarization changes. The polarization is written as $P_e = |P_e|\cos 2\phi$, where ϕ is an angle giving the orientation of the prism that polarizes the light. (From Prescott *et al.*, 1978.)

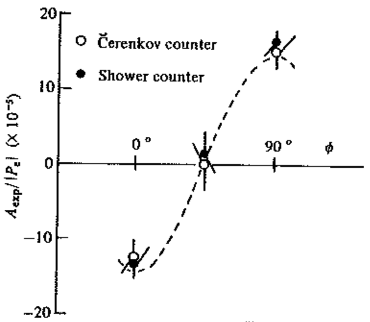
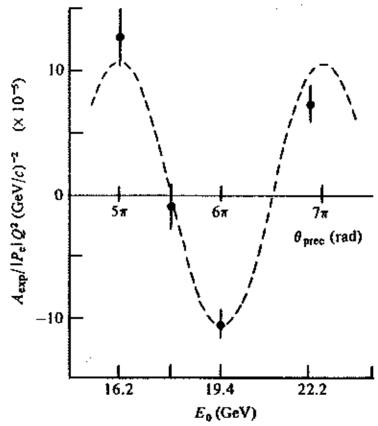


Fig. 6.5. The experimental asymmetry shows the expected variation (dashed line) as the beam polarization changes as a function of beam energy owing to the g-2 precession in the beam transport system. (From Prescott *et al.*, 1978.)



When this result appeared in 1978 it was the most striking evidence then available in favor of the details of the Glashow - Salam - Weinberg model, which may have influenced the Nobel committee in making their awards for 1979. About this time a number of experiments were seeking evidence of Z^0 exchange in atoms, with only varying amounts of success. In recent years the atomic physics experiments have improved and are now considered to be in agreement with the Weinberg - Salam model. See chapter 9 of the book of Commins & Bucksbaum for details.

5. Decay of the W and Z Bosons

The decay rates (= resonance widths) of the $W^\pm$ and Z^0 bosons have not been well measured yet, but will provide interesting tests of the Weinberg - Salam model. The W boson can decay to each of the left handed doublets of weak isospin:

$$W^+ \to e^+\nu_e\,,\ \mu^+\nu_\mu\,,\ \tau^+\nu_\tau\,,\ u\bar{d}_\theta\,,\ c\bar{s}_\theta\,,\ (t\bar{b}_\theta)\ \ldots$$

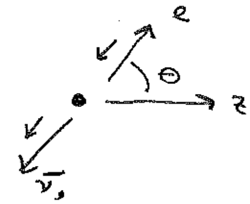
The Z^0 can decay to $l\bar{l}$ or $q\bar{q}$ pairs of all possible types. Especially interesting are the decays $Z^0 \to \nu\bar{\nu}$. If there are n types of neutrinos (with masses $\ll M_Z$) we expect $\Gamma_{Z^0\to\nu\bar{\nu}} = n\,\Gamma_{Z^0\to\nu_e\bar{\nu}_e}$

Thus if $\Gamma_{Z^0\to all}$ and $\Gamma_{Z^0\to \text{charged particles}}$ can be measured,

$$\Gamma_{Z^0\to\nu\bar{\nu}} = \Gamma_{Z^0\to all} - \Gamma_{Z^0\to \text{charged}} \qquad (\text{Can } Z^0 \to \gamma\gamma\,?)$$

It is conjectured that there are as many quark doublets as there are types of neutrinos, so this measurement could be an important indication of the variety of matter in the universe.

WE NOW CONSIDER THE CALCULATION OF DECAY RATES IN MORE DETAIL. AS THE FIRST EXAMPLE WE TAKE $W^- \to e^- \bar{\nu}_e$.



LET THE z AXIS BE THE AXIS ALONG WHICH THE W^- SPIN IS ALIGNED. AND LET THE z' AXIS BE THAT OF THE e AND $\bar{\nu}_e$ IN THE W^- REST FRAME. THEN THE $e\bar{\nu}_e$ STATE HAS $J_{z'} = -1$

HENCE IF $J_z = +1$ FOR THE W^- $\quad \frac{d\Gamma}{d\Omega} \sim \left(\frac{1-\cos\theta}{2}\right)^2$

$J_z = 0 \quad \frac{d\Gamma}{d\Omega} \sim \left(\frac{\sin\theta}{\sqrt{2}}\right)^2$

$J_z = -1 \quad \frac{d\Gamma}{d\Omega} \sim \left(\frac{1+\cos\theta}{2}\right)^2$

REFERRING TO THE SPIN-1 ROTATION MATRIX (p. 118)

FOR ALL 3 POSSIBLE SPIN STATES OF THE W^- THE INTEGRAL OF THE ANGULAR FACTOR IS THE SAME - NAMELY $\frac{4\pi}{3}$ FOR WHAT IT'S WORTH. THUS THE W DECAY RATE WILL BE INDEPENDENT OF J_z, AS IS TO BE EXPECTED.

FOR A PARTICULAR ORIENTATION OF THE W SPIN, WE MAY USE OUR ANALYSIS OF 2 BODY DECAY RATE (p 193) TO WRITE

$$\frac{d\Gamma}{d\Omega} = \frac{P_f}{32\pi^2 M_W^2}|\mathcal{M}|^2 = \frac{|\mathcal{M}|^2}{64\pi^2 M_W} \qquad \text{AS } P_f = \frac{M_W}{2} \text{ WHEN } M_e \ll M_W$$

$$\mathcal{M} = \frac{g}{2\sqrt{2}}\left(\bar{v}_{\bar{\nu}} | \gamma_\mu (1-\gamma_5) | u_e\right)\epsilon^\mu$$

g ← $\sqrt{\frac{G M_W^2}{\sqrt{2}}}$ (p 387)

ϵ^μ ← W POLARISATION, WHICH IS WELL BEHAVED IN THE W REST FRAME

SO $|\mathcal{M}|^2 \approx \frac{G M_W^2}{\sqrt{2}} E_e^2 \cdot$ ANGULAR FACTOR

E_e ← $M_W/2$

NOTING THAT AT HIGH ENERGIES THE SPINOR INCLUDE FACTORS $\sim\sqrt{E}$ IN THEIR NORMALISATION. A DETAILED CALCULATION VIA FEYNMAN'S TRACE METHOD INDICATES $|\mathcal{M}|^2 = 8\frac{G M_W^4}{\sqrt{2}} \cdot$ ANGULAR FACTOR

SO $\frac{d\Gamma}{d\Omega} = \frac{1}{64\pi^2 M_W} \cdot 8\frac{G M_W^4}{\sqrt{2}} \cdot$ ANGULAR FACTOR

AND $\underline{\Gamma_{W\to e\nu}} = \frac{G M_W^3}{8\pi^2\sqrt{2}} \cdot \frac{4\pi}{3} = \underline{\frac{G M_W^3}{6\pi\sqrt{2}}} \approx 225\text{ MeV}$ WITH $M_W \sim 80\text{ GeV}$

THE RATE FOR $W^+ \to u\bar{d}_\theta$ IS EXACTLY THE SAME, FOR A GIVEN COLOR OF QUARKS. HENCE

$$\Gamma_{W^+\to u\bar{d}_\theta} = \frac{G M_W^3}{2\pi\sqrt{2}} \sim 675\text{ MeV} \quad \text{, USING 3 COLORS.}$$

AND $\Gamma_{W^+\to \text{ALL}} = 225\text{ MeV}$ (# OF LEPTON DOUBLETS $+ 3 \cdot$ # OF QUARK DOUBLETS)

IF THE TOP QUARK WAS MASS $M_t < M_W - M_b$ THEN WE HAVE (AT LEAST) 3 KINDS OF LEPTON AND 3 KINDS OF QUARK DOUBLETS.

$$\Gamma_{W^+ \to ALL} = 12\ \Gamma_{W^+ \to \ell^+ \nu} \sim 12 \cdot 225 \approx 2.7\ GeV$$

PUTTING THIS ANOTHER WAY, THE BRANCHING RATIO TO $\ell\nu$ IS $\frac{1}{12}$.

THE PRESENT LIMIT ON Γ_W IS ABOUT 7 GeV BASED ON THE EXPERIMENTS DISCUSSED AT THE END OF LECTURE 21.

TURNING TO THE Z^0, THE SIMPLEST DECAY TO CALCULATE IS $Z^0 \to \nu_e \bar{\nu}_e$.

FOR THIS, $$\mathcal{M} = \frac{g}{4 \cos\theta_W} (\bar{u}_{\bar{\nu}} | \gamma_\mu (1-\gamma_5) | u_\nu) \epsilon^\mu \qquad (p\ 388)$$

$\frac{g}{4\cos\theta_W} = \sqrt{\frac{G M_Z^2}{2\sqrt{2}}}$ $\qquad$ ϵ^μ : Z POLARIZATION

THE REST OF THE CALCULATION IS IDENTICAL TO THAT FOR $W^+ \to \ell^+ \bar{\nu}_\ell$, SUBSTITUTING M_Z FOR M_W. THUS

$$\Gamma_{Z^0 \to \nu_\ell \bar{\nu}_\ell} = \frac{G M_Z^3}{12\pi\sqrt{2}} \sim 160\ MeV \quad \text{IF} \quad M_Z \sim 90\ GeV$$

FOR $Z^0 \to \ell^+ \ell^-$ OR $Z^0 \to q\bar{q}$ THE CALCULATION IS SIMILAR, EXCEPT THAT WE CAN HAVE BOTH LEFT- AND RIGHT-HANDED COUPLINGS, WITH STRENGTHS C_L AND C_R.

$$\Gamma_{Z^0 \to \left\{ \begin{smallmatrix} \ell^+\ell^- \\ q\bar{q} \end{smallmatrix} \right.} = \Gamma_{Z^0 \to \nu\bar{\nu}} \left[C_L^2 + C_R^2 \right]$$

FOR $\ell^+\ell^-$ WE HAVE $\quad C_L = 2\sin^2\theta_W - 1 \qquad C_R = 2\sin^2\theta_W \qquad (p\ 388)$

$$\Gamma_{Z^0 \to \ell^+\ell^-} = \Gamma_{Z^0 \to \nu\bar{\nu}} \left[1 - 4\sin^2\theta_W + 8\sin^4\theta_W \right]$$
$$\approx \tfrac{1}{2} \Gamma_{Z^0 \to \nu\bar{\nu}} \qquad \text{FOR } \sin^2\theta_W \sim 0.23$$
$$\approx 80\ MeV$$

FOR u, c, OR t QUARKS $\qquad C_L = 1 - \tfrac{4}{3}\sin^2\theta_W \qquad C_R = -\tfrac{4}{3}\sin^2\theta_W \qquad (p\ 392)$

SO $$\Gamma_{Z^0 \to u\bar{u},\, c\bar{c},\, t\bar{t}} = 3\,\Gamma_{Z^0 \to \nu\bar{\nu}} \left[1 - \tfrac{8}{3}\sin^2\theta_W + \tfrac{32}{9}\sin^4\theta_W \right]$$
$$\approx 3.6\,\Gamma_{Z^0 \to \nu\bar{\nu}} \sim 575\ MeV$$

FOR d, s, OR b QUARKS, $\qquad C_L = -1 + \tfrac{2}{3}\sin^2\theta_W \qquad C_R = \tfrac{2}{3}\sin^2\theta_W$

$$\Gamma_{Z^0 \to d\bar{d},\, s\bar{s},\, b\bar{b}} = 3\,\Gamma_{Z^0 \to \nu\bar{\nu}} \left[1 - \tfrac{4}{3}\sin^2\theta_W + \tfrac{8}{9}\sin^4\theta_W \right]$$
$$\sim 2.3\,\Gamma_{Z^0 \to \nu\bar{\nu}} \sim 365\ MeV$$

IF $M_t < M_Z/2$ THEN $$\Gamma_{Z^0 \to ALL} = \underset{\text{FAMILIES}}{3}\,\Gamma_{Z^0 \to \nu\bar{\nu}} \Big[\underset{\nu}{1} + \underset{\ell}{\tfrac{1}{2}} + \underset{u\text{-LIKE}}{3.6} + \underset{d\text{-LIKE}}{2.3} \Big]$$
$$\approx 22\,\Gamma_{Z^0 \to \nu\bar{\nu}} \sim \underline{3.5\ GeV}$$

A USEFUL RESULT IS

$$\frac{\Gamma_{Z^0 \to e^+e^-}}{\Gamma_{Z^0 \to ALL}} \sim \frac{1}{44} \sim 2\%$$

$$\frac{\Gamma_{Z^0 \to \nu_e \bar{\nu}_e}}{\Gamma_{Z^0 \to ALL}} \sim \frac{1}{22} \sim 4\%$$

IF WE WISH TO MEASURE $\Gamma_{Z^0 \to ALL}$ WITH SUFFICIENT ACCURACY TO COUNT THE NUMBER OF NEUTRINO TYPES, SAY TO 3 STANDARD DEVIATION ACCURACY, THEN WE NEED AN ERROR OF $\frac{4}{3}\%$. THIS WILL REQUIRE SOME 5000 EVENTS WITH A RELATIVE ENERGY CALIBRATION ACCURACY OF $\frac{.160/3}{90} < .001$. THIS WILL PROBABLY NOT BE POSSIBLE WITH THE $\bar{p}p$ COLLIDER EXPERIMENTS THAT DISCOVERED THE Z^0.

6. PRODUCTION OF THE Z^0 BOSON IN e^+e^- COLLISIONS

A GOOD MEASUREMENT OF $\Gamma_{Z^0 \to ALL} \equiv \Gamma_T$ WILL BE POSSIBLE IN THE REACTION $e^+e^- \to Z^0 \to ALL$, e^+e^-, $\mu^+\mu^-$, ETC. IGNORING FOR THE MOMENT THE ELECTROMAGNETIC INTERACTION OF e^+e^-, WE WOULD HAVE

$$\sigma_{e^+e^- \to Z \to ALL} = 12\pi \frac{s}{M_Z^2} \frac{\Gamma_{Z^0 \to e^+e^-} \Gamma_T}{(s - M_Z^2)^2 + \Gamma_T M_Z^2} \qquad (s = E_{CM}^2) \quad (P\ 210, 216)$$

BY OBSERVING THE CROSS SECTION AS A FUNCTION OF s, THE WIDTH Γ_T IS DIRECTLY MEASURED.

THE RESONANCE PEAK CROSS SECTION IS $\sigma_{PEAK} = \frac{12\pi}{M_Z^2} \frac{\Gamma_{e^+e^-}}{\Gamma_T}$

THIS IS VERY LARGE COMPARED TO THE ELECTROMAGNETIC PROCESS $e^+e^- \to \mu^+\mu^-$. RECALL THAT

$$\sigma_{e^+e^- \to \mu^+\mu^-} = \frac{4\pi\alpha^2}{3s}$$

SO

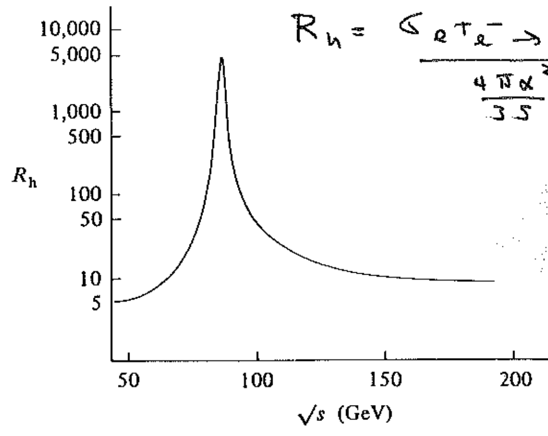
$$\frac{\sigma_{PEAK}}{\sigma_{\mu\mu}} = \frac{9}{\alpha^2} \frac{\Gamma_{e^+e^-}}{\Gamma_T} \approx 4000$$

NUMERICALLY, $\sigma_{PEAK} \sim 2.5 \times 10^{-32}\ cm^2$

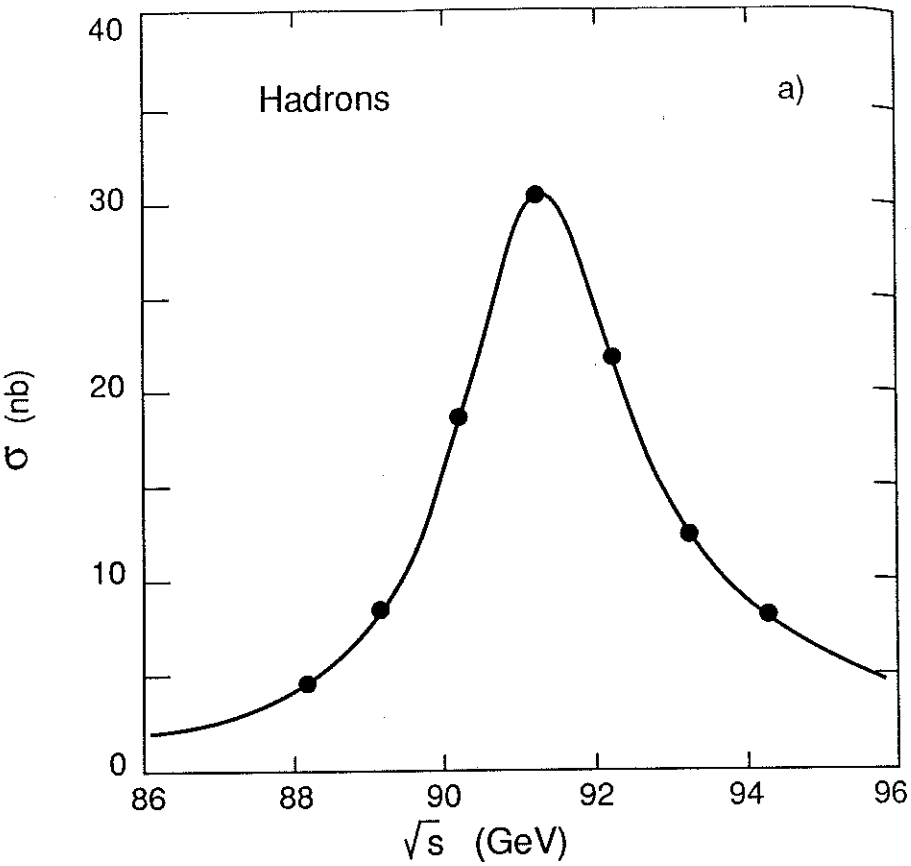
SO IF A LUMINOSITY OF $10^{31}\ cm^{-2}\ sec^{-1}$ CAN BE ACHIEVED AT THE NEW LEP e^+e^- COLLIDER, ABOUT 20000 Z^0 EVENTS WILL BE PRODUCED PER DAY!

Figure 3.12. The ratio R_h is plotted vs. $\sqrt{s}$. There is a very strong resonance at the Z^0 mass. Note the logarithmic ordinate scale. Here $\sin^2\theta_W = 0.25$ is assumed.

$$R_h = \frac{\sigma_{e^+e^- \to HADRONS}}{\frac{4\pi\alpha^2}{3s}}$$



DATA FROM THE CERN L3 EXPERIMENT (1991) ON $e^+e^- \to Z^0$.



COMBINING SUCH DATA FROM 4 EXPERIMENTS AT CERN THERE ARE ABOUT 650,000 Z^0'S. FROM THIS

$$M_{Z^0} = 91.175 \pm 0.021 \text{ GeV}$$

$$\Gamma_{Z \to ALL} = 2.487 \pm 0.010 \text{ GeV}$$

AFTER ADDING UP THE DECAY RATE TO CHARGED LEPTONS AND QUARKS, ONE CAN INFER THE RATE TO NEUTRINOS. THEN ASSUMING THAT

$$\frac{\Gamma_{Z \to \nu\bar{\nu}}}{\Gamma_{Z \to e^+e^-}} = 2 \,,$$

ONE DEDUCES THAT THE NUMBER OF (ORDINARY) NEUTRINO TYPES IS

$$N_\nu = 3.00 \pm 0.05$$

THE MASS OF THE W BOSON IS NOT DIRECTLY MEASURED IN $e^+e^- \to Z$, BUT FROM W PRODUCTION IN $p\bar{p}$ COLLISIONS, THE PRESENT RESULT IS

$$M_W = 80. \pm 0.2 \text{ GeV}$$

$$\text{THEN} \quad \sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.230 \pm 0.003$$

A SLIGHTLY DIFFERENT VALUE OF $\sin^2\theta_W$ COMES FROM ANALYSIS OF $Z^0 \to e^+e^-, \mu^+\mu^-$

THE COUPLINGS $C_L = 2\sin^2\theta_W - 1$ AND $C_R = 2\sin^2\theta_W$ ARE REWRITTEN

$$\text{AS} \quad g_V = \frac{C_L + C_R}{2} = 2\sin^2\theta_W - \tfrac{1}{2} \qquad g_A = \frac{C_L - C_R}{2} = -\tfrac{1}{2}$$

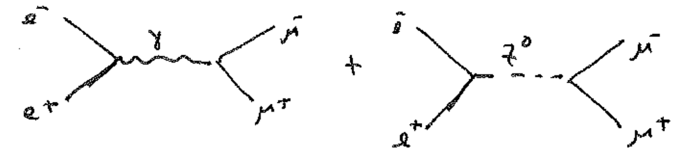
$$\left[\text{SO THAT} \quad \left(\bar{u}_e \middle| \gamma_\mu \left[C_L\left(\frac{1-\gamma_5}{2}\right) + C_R\left(\frac{1+\gamma_5}{2}\right)\right] \middle| u_e\right) \equiv \left(\bar{u}_e \middle| \gamma_\mu (C_V - \gamma_5 C_A) \middle| u_e\right)\right]$$

THE DATA INDICATE THAT $g_A^2 = 0.249 \pm 0.001$, $g_V^2 = 1.2 \pm 0.4 \times 10^{-3}$ (≈ 0)

$$\text{SO} \quad \sin^2\theta_W = \frac{1}{4}\left(1 - \frac{g_V}{g_A}\right) = 0.234 \pm 0.001 \,.$$

At present e^+e^- colliders are limited to c.m. energies $E_{cm} = \sqrt{s} \leq 40$ GeV. This is sufficient to detect a small interference between electromagnetism and the weak interaction.

Thus:



The amplitude to produce any fermion-antifermion final state may be readily written down ($f \equiv$ fermion)

$$\mathcal{M} = e^2 Q_f (\bar{v}_{e^+} | \gamma_\mu | u_e) \frac{1}{s} (\bar{v}_{\bar{f}} | \gamma_\mu | u_f)$$

$$+ \underbrace{\frac{g^2}{16\cos^2\theta_W}}_{\frac{G M_Z^2}{2\sqrt{2}}} (\bar{v}_{e^+} | \gamma_\mu [C_{L_e}(1-\gamma_5) + C_{R_e}(1+\gamma_5)] | u_e) \frac{1}{s - M_Z^2} (\bar{v}_{\bar{f}} | \gamma_\mu [C_{L_f}(1-\gamma_5) + C_{R_f}(1+\gamma_5)] | u_f)$$

With some diracology one finds

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to f\bar{f}) = \frac{\pi \alpha^2 Q_f^2}{s} \frac{1+\cos^2\theta}{2} - \frac{\alpha Q_f G M_Z^2}{8\sqrt{2}} \Big[(C_{L_e} + C_{R_e})(C_{L_f} + C_{R_f})(1+\cos^2\theta)$$

$$+ 2 (C_{L_e} - C_{R_e})(C_{L_f} - C_{R_f}) \cos\theta \Big]$$

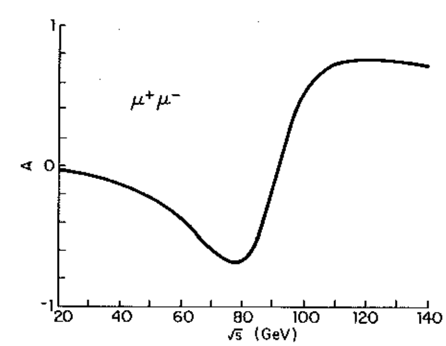
$$+ O(G^2)$$

The term in $\cos\theta$ is evidence of parity violation. To isolate this term in experiment, one considers the asymmetry

$$A = \frac{\int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta}{\int_{-1}^1 \frac{d\sigma}{d\cos\theta} d\cos\theta} = \frac{-3 G s}{16\pi \alpha Q_f \sqrt{2}} (C_{L_e} - C_{R_e})(C_{L_f} - C_{R_f})$$

if $\sqrt{s} \ll M$

The energy dependence for the asymmetry A without assuming $\sqrt{s} \ll M_Z$ is shown in the figure.



Experimentally the most accessible case is for a $\mu^+\mu^-$ final state. From p 388

$C_{L_e} = C_{L_\mu} = 2\sin^2\theta_W - 1$; $C_{R_e} = C_{R_\mu} = 2\sin^2\theta_W$

so $A = \frac{-3 G s}{16\pi\alpha\sqrt{2}} = -9.5\%$ at $\sqrt{s} = 35$ GeV

EXPERIMENTS AT THE PETRA e^+e^- COLLIDER YIELD

$A = -10.8 \pm 1.1\ \%$

AT $\sqrt{s} \simeq 35$ GeV

IT IS PERHAPS INTERESTING TO NOTE THAT FOR $\sqrt{s} \ll M_Z$ THE ASYMMETRY IS NOT EXPECTED TO DEPEND ON M_Z, Γ_Z OR Θ_W.

BUT FOR $\sqrt{s} > \frac{M_Z}{2}$

DEPENDENCE ON M_Z WILL BE OBSERVABLE.

IN THE REACTION $e^+e^- \to b\bar{b}$ THE $\gamma\ Z^0$ INTERFERENCE IS STRONGER BY $\sim 1/Q_b = 3$ $\Rightarrow$ ASYM $\sim 25\%$ AT $\sqrt{s} = 35$ GeV! SEE BARTEL ET AL PHYS. LETT 146B, 437 (1984) FOR DATA.

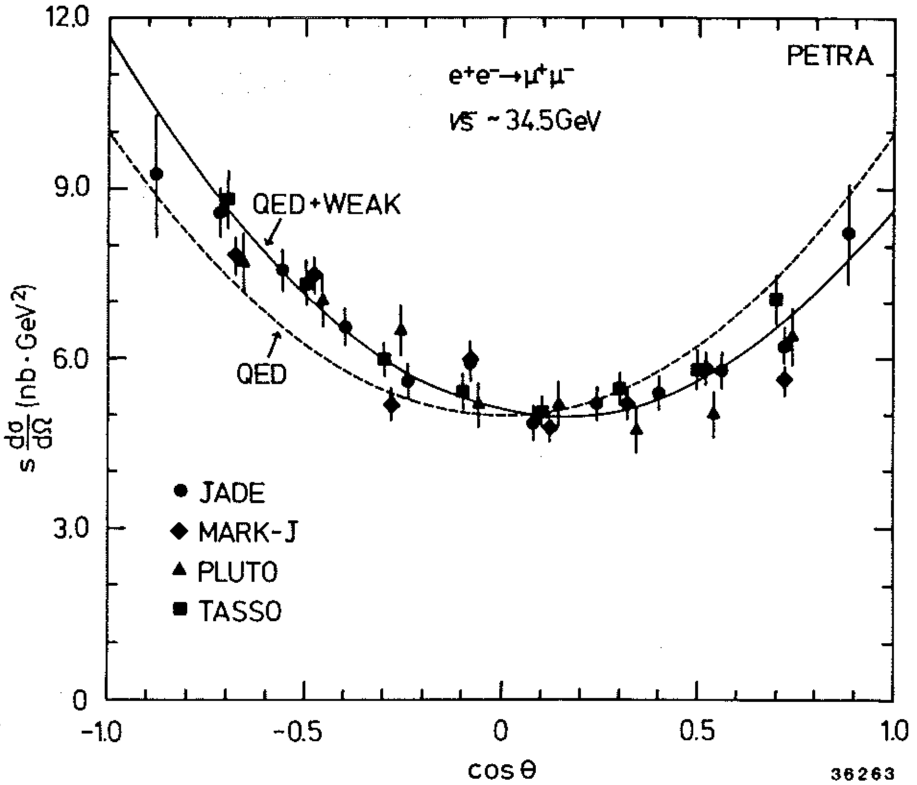
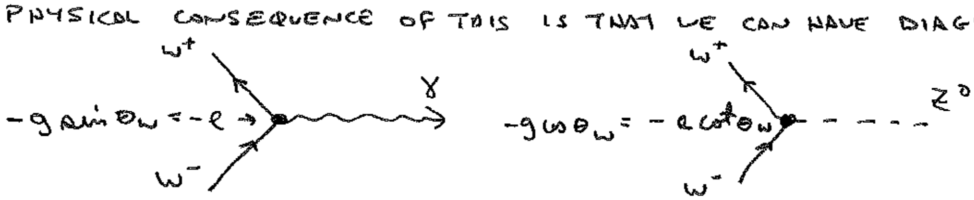


Fig. 6: Angular distributions for $e^+e^- \to \mu^+\mu^-$ for JADE, MARK J, PLUTO and TASSO at $\sqrt{s}$ 34.5 GeV. The data are corrected for QED effects up to α^3. The full curve shows a fit to the data allowing for an asymmetry, the dashed curve is the symmetric QED prediction.

7. COUPLING OF $W^\pm$ AND Z^0 BOSONS TO EACH OTHER.

THE $W^\pm$ BOSONS CARRY ELECTRIC CHARGE, WHICH IS THE CHARGE OF THE WEAK INTERACTIONS ALSO, IN THE UNIFIED THEORY. HENCE THE $W^\pm$ BOSONS CAN COUPLE TO THE PHOTON AND Z^0. THUS THE $W^\pm$ BOSONS ACT AS THE QUANTA OF THE WEAK INTERACTION AS WELL AS SOURCES OF OTHER ELECTRO-WEAK QUANTA. SUCH A POSSIBILITY WAS ANTICIPATED IN THE THEORY OF YANG & MILLS (p. 367, LECTURE 20).

INDEED, THE WEINBERG-SALAM MODEL IS A NON-ABELIAN GAUGE THEORY BECAUSE THE OPERATORS σ^1, σ^2 & σ^3 OF WEAK ISOSPIN DO NOT COMMUTE. THE PHYSICAL CONSEQUENCE OF THIS IS THAT WE CAN HAVE DIAGRAMS



THE VERTEX FACTOR FOR THESE DIAGRAMS CANNOT BE DIRECTLY READ OFF OUR EXPRESSION FOR THE COVARIANT DERIVATIVE D_μ. BUT IF WE RECALL THE ANALYSIS OF THE YANG-MILLS CASE, WE CAN WRITE THAT THE FIELDS ASSOCIATED WITH THE POTENTIALS W_μ^i ARE

$$F_{\mu\nu}^i = \partial_\nu W_\mu^i - \partial_\mu W_\nu^i - 2g\,\epsilon_{ijk} W_\nu^j W_\mu^k$$

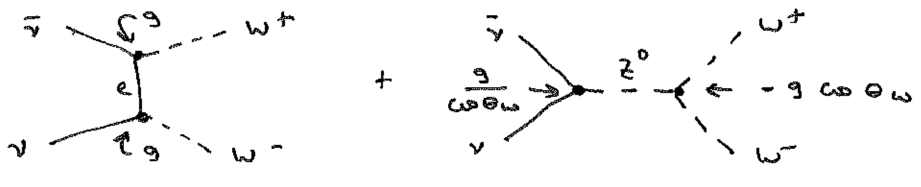
Thus the field W^3_μ couples to the product $W^1 W^2 \sim W^+ W^-$

Recall that $W^3_\mu = \cos\theta_W Z^0_\mu + \sin\theta_W A_\mu$ (p. 381)

Hence the strength of the $W^+W^-\gamma$ vertex is $-g \sin\theta_W = -e$

while the strength of the $W^+W^-Z^0$ vertex is $-g\cos\theta_W = -e \cot\theta_W$, as indicated in the diagrams on p 401.

An important conceptual application of the $W^+W^-Z^0$ coupling is to the reaction $\bar\nu\nu \to W^+W^-$. In the V−A theory with only W^+ and W^- bosons, the cross section for this reaction rises with energy in violation of unitarity bounds (p. 360, Lecture 20). Now we have 2 diagrams for this process:

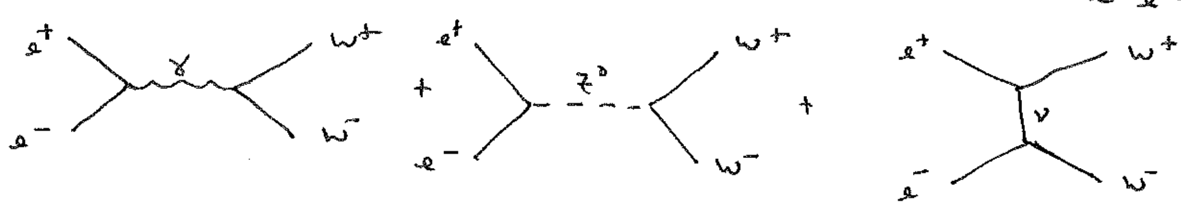


$\mathcal{M} \sim g^2(\epsilon^+\epsilon^-)$ $\qquad\qquad$ $\mathcal{M} \sim \frac{g}{\cos\theta_W}(-g\cos\theta_W)(\epsilon^+\epsilon^-) = -g^2(\epsilon^+\epsilon^-)$

↖ product of W^+ and W^- polarization vectors.

The contribution to the cross section from longitudinally polarized W's completely vanishes (in the high energy limit). Hence the Weinberg-Salam model is not plagued by the divergences of the older theory. It also turns out that the higher order weak interaction process are rendered harmless by the presence of the Z^0 – the electro-weak theory is 'renormalizable'....

An interesting practical consequence is that in the foreseeable future one should be able to observe the reaction $e^+e^- \to W^+W^-$



For this one needs $E_e \gtrsim 80$ GeV, which might become available at the LEP e^+e^- collider around 1990. However the interference among the 3 diagrams leads to a cross section which is typical of $e^+e^- \to \mu^+\mu^-$ at such energies: $\sigma \sim \frac{\alpha^2}{s} \sim 10^{-34}$ cm^2

If a luminosity of $\mathcal{L} \sim 10^{31}$ cm^{-2} sec^{-1} can be maintained, one might find 10 events a day.

8. The Higgs Particle

So far we have avoided much discussion of the physical significance of the background scalar field ϕ which gives an apparent mass to the $W^\pm$ and Z^0 bosons (p. 374 Lecture 21). This field ϕ is the analogue of the swarm of Cooper pairs in a superconductor. One readily speculates that we might be able to probe the 'vacuum' to find evidence for a particle analagous to a Cooper pair. Or perhaps there is also some interaction between pairs of a new kind of particle which leads to the field ϕ in a dynamic way, as in the case of superconductivity.... At present there is no evidence for any such new particles, but advocates of the Weinberg-Salam model feel strongly that they should exist.

We sketch one speculation, that associated with the background field ϕ there exist scalar (spin 0) particles h – the Higgs particle. Then we imagine that the total field ϕ should be written

$$\phi = \begin{pmatrix} 0 \\ \frac{f+h}{\sqrt{2}} \end{pmatrix} \qquad (p\ 379)$$

where f = constant background component

h = variable, representing an excitation of the field (= particle)

We can get a sense of some properties of the Higgs particle by recalling some arguments of Lecture 21. For example, we can substitute the new version of ϕ into the expressions for the currents associated with the potentials W_μ^i and B_μ (p.380)

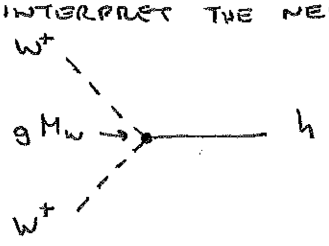
$$J_\mu^i \rightarrow \left(-\frac{g^2}{4} W_\mu^i + \frac{gg'}{4} B_\mu \delta^{i3}\right)\left(f^2 + 2fh + h^2\right)$$

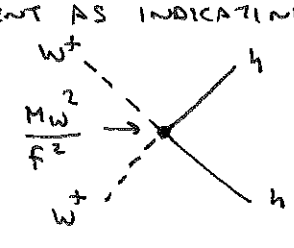
$$J_\mu^Y \rightarrow \left(\frac{gg'}{4} W_\mu^3 - \frac{g'^2}{4} B_\mu\right)\left(f^2 + 2fh + h^2\right)$$

For $i = 1, 2$, the wave equation $\Box W_\mu^i = J_\mu^i$ becomes

$$(\Box + M_W^2) W_\mu^\pm = -g\underbrace{\left(\frac{gf}{2}\right)}_{M_W} h W_\mu^\pm - \frac{g^2 h^2}{4} W_\mu^\pm = -g M_W h W_\mu^\pm - \frac{M_W^2 h^2 W_\mu^\pm}{4}$$

We interpret the new current as indicating a coupling like





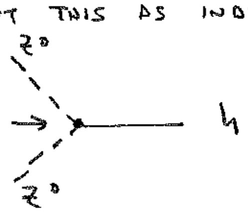
($f = 246$ GeV)
p.382

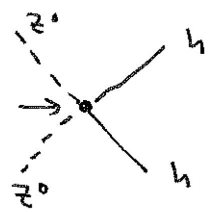
AS BEFORE WE ALSO WRITE
$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g B_\mu + g' W_\mu^3\right) \qquad \text{(p.380)}$$
$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(-g' B_\mu + g W_\mu^3\right)$$

WHICH LEADS TO $\Box A_\mu = 0$ STILL

WHILE
$$(\Box + M_Z^2)\, Z_\mu = -\frac{g^2+g'^2}{4} Z_\mu (2fh + h^2)$$
$$= -\frac{g}{\cos\theta_W} M_Z h Z_\mu - \frac{M_Z^2}{f^2} h^2 Z_\mu$$

WE INTERPRET THIS AS INDICATING THE COUPLINGS

$\frac{g M_Z}{\cos\theta_W}$ 

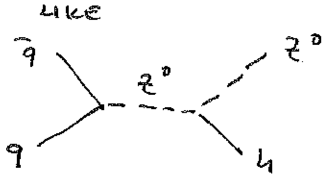
$\frac{M_Z^2}{f^2}$ 

THERE IS NO COUPLING AT ALL OF THE HIGGS PARTICLE TO THE MASSLESS PHOTON.

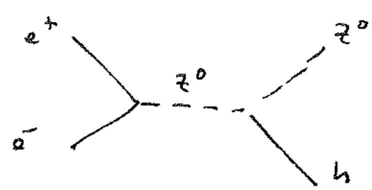
AN IMPORTANT RESULT, OF GREATER GENERALITY THAN THAT JUST SKETCHED, IS THAT THE HIGGS PARTICLE COUPLES TO OTHER PARTICLES WITH AMPLITUDE PROPORTIONAL TO THE OTHER PARTICLE'S MASS.

THUS IF WE CAN SOMEHOW PRODUCE A HIGGS PARTICLE, IT WILL DECAY PREFERENTIALLY TO A PAIR OF THE HEAVIEST PARTICLES POSSIBLE. THE EXPERIMENTAL SIGNATURE OF SUCH A DECAY MAY NOT BE TOO CLEAN, DUE TO THE CASCADE OF DECAYS: HIGGS $\to$ HEAVY $\to$ LIGHTER.
ALSO, SINCE THE HIGGS PARTICLE ONLY COUPLES TO HEAVY PARTICLES IT WILL BE HARD TO PRODUCE IT IN REACTIONS LIKE $e^+e^- \to h$ OR $\bar{p}p \to h$

PERHAPS THE BEST CHANCE TO PRODUCE A HIGGS PARTICLE IS IN REACTIONS WITH DIAGRAMS LIKE



OR

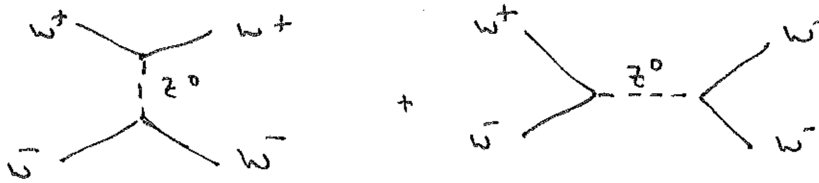


THE MASS OF THE HIGGS PARTICLE IS NOT PREDICTED IN THE WEINBERG-SALAM MODEL. THE MASS MAY BE IDENTIFIED WITH THE PARAMETER μ WHICH APPEARS IN THE EFFECTIVE POTENTIAL FOR THE HIGGS FIELD

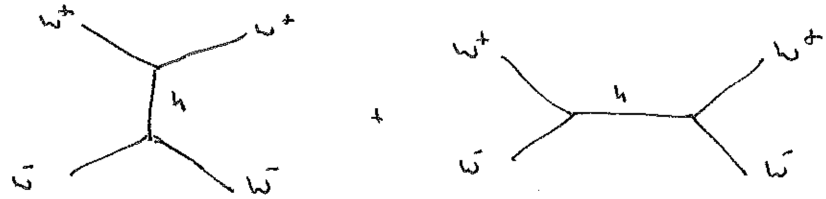
$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{2}\frac{\mu^2}{f^2}\phi^4 \qquad \text{(p.375)}$$

UNFORTUNATELY, THIS PARAMETER APPEARS NOWHERE ELSE IN THE THEORY....

THE HIGGS PARTICLE IS BELOVED IN THAT IT RESOLVES CERTAIN TECHNICAL DIFFICULTIES IN THE WEINBERG-SALAM MODEL. FOR EXAMPLE, THE REACTION $W^+W^- \to W^+W^-$ HAS A CROSS SECTION WHICH RISES WITH ENERGY DUE TO DIAGRAMS:



BUT IF WE CAN ADD THE DIAGRAMS



THE BAD BEHAVIOR IS CANCELLED.

A PATTERN IS EMERGING: WHENEVER A THEORY IS IN TROUBLE, INVENT MORE PARTICLES....