

QUANTUM CHROMODYNAMICS

REFERENCE: THE BOOK BY QUIGG

1. A GAUGE THEORY FOR THE STRONG INTERACTION

AFTER THE FORMULATION OF THE WEINBERG-SALAM MODEL AS A GAUGE THEORY PEOPLE AGAIN CONSIDERED WHETHER THE STRONG INTERACTION IS SUSCEPTIBLE TO THIS TECHNIQUE. TWO DEVELOPMENTS SINCE THE ORIGINAL WORK OF YANG & MILLS MADE PROGRESS POSSIBLE.

IT WAS REALIZED IT WAS NOT APPROPRIATE TO BASE THE THEORY ON STRONG ISOSPIN INVARIANCE, OR ITS GENERALIZATION, QUARK FLAVOR INVARIANCE. ISOSPIN INVARIANCE HAS TURNED OUT TO BE AN 'ACCIDENT' DUE TO THE NEAR EQUALITY OF u AND d QUARK MASSES. THE FACT THAT THE NEUTRON MASS IS GREATER THAN THE PROTON MASS IS A SIGN THAT THE STRONG INTERACTION ISN'T EVEN STRICTLY ISOSPIN INVARIANT ($E \& M$ CORRECTIONS ALONE WOULD GIVE $M_p > M_n$).

INSTEAD THE CONCEPT OF COLOR EMERGED AS MORE BASIC TO THE UNDERSTANDING OF THE STRONG INTERACTION. THE $SU(3)$ COLOR SYMMETRY APPEARS TO BE EXACT, AND COLOR IS A GOOD CANDIDATE FOR BEING THE 'CHARGE' OF THE STRONG INTERACTION. HOWEVER, BECAUSE THE STRONG FORCE SEEMS TO BE 'COLOR-SYMMETRIC' A PROBLEM REMAINS. WE SHOULD NOT EXPECT TO INVOLVE THE HIGGS MECHANISM HERE, AS THIS 'BREAKS' OR 'HIDES' THE SYMMETRY, AS WELL AS GIVING MASS TO THE QUANTA OF THE POTENTIALS. WITHOUT THE HIGGS MECHANISM THE QUANTA REMAIN MASSLESS AND WE STILL HAVE A THEORY OF LONG RANGE FORCES, WHICH WAS THE DIFFICULTY WITH THE YANG-MILLS CONCEPTION.

A DIFFERENT WAY OUT OF THE LONG RANGE PROBLEM WAS DISCOVERED BY GROSS & WILCZEK [P.R.L. 30, 1343 (1973)] AND POLITZER [P.R.L. 30, 1346 (1973)]. THEY CONJECTURE THAT THE STRONG INTERACTION IS SO STRONG THAT A SINGLE 'CHARGE' SURROUNDED BY ITS FREE FIELDS CANNOT EXIST. IN SOME SENSE THE FIELD MUST INTERACT WITH ITSELF IN SUCH A WAY THAT THE FREE FIELD LINES ARE FORBIDDEN, OR 'CONFINED' TO A VERY SMALL VOLUME. TWO AND THREE PARTICLE STATES ARE CONCEIVABLY CONSISTENT WITH THE CONFINEMENT OF FIELD LINES, BUT A SPHERICALLY SYMMETRIC LONG CHARGE IS NOT. FOR THE FIELDS TO BE CONFINED THERE MUST BE SOME SELF-INTERACTION OF THE FIELD, UNLIKE THE CASE OF ELECTROMAGNETISM. BUT SUCH A POSSIBILITY DOES OCCUR IN THE 'NON-ABELIAN' THEORIES PIONEERED BY YANG & MILLS.

EVEN TODAY THERE IS NO DETAILED UNDERSTANDING OF THE CONFINEMENT MECHANISM THAT KEEPS US FROM OBSERVING SINGLE QUARKS OF A DEFINITE COLOR. WHAT HAS BEEN COMPREHENDED IS THE NOTION OF 'ASYMPTOTIC FREEDOM': THE FIELDS OF A QUARK (BOUND IN A MESON OR

BARYON) APPEAR TO BE FREE FIELDS ONLY FOR VERY SMALL DISTANCES AROUND THE QUARK. AT DISTANCES ~ 1 FERMI THE FIELDS ARE BADLY DISTORTED BY THE MYSTERIOUS CONFINEMENT MECHANISM.

HENCE ONLY A PARTIAL THEORY IS AVAILABLE TODAY. WE HAVE A GAUGE THEORY OF THE 'ASYMPTOTICALLY FREE' FIELDS OF THE STRONG INTERACTION - THE SO-CALLED THEORY OF QUANTUM CHROMODYNAMICS (QCD). BUT A KEY INGREDIENT OF THE THEORY IS STILL MISSING: HOW DOES CONFINEMENT REALLY WORK? THIS INCOMPLETENESS IS PERHAPS REFLECTED IN THE FACT THAT THERE ARE ALMOST NO 'EXACT' PREDICTIONS OF THE THEORY, ONLY APPROXIMATE RESULTS WHICH HOLD IF ALL DISTANCES ARE MUCH LESS THAN 1 FERMI. AS THE STRONG INTERACTION TAKES PLACE AMONG MESONS AND BARYONS OF CHARACTERISTIC SIZE ~ 1 FERMI THIS LENGTH CANNOT BE COMPLETELY IGNORED IN A REALISTIC MODEL. BECAUSE OF THIS LIMITATION WE 'CONFINE' OURSELVES TO SUCH COMMENTS AS CAN BE MADE IN 1 LECTURE.

2. SOME FORMALISM OF THE COLOR GAUGE THEORY

IN LECTURE 14 WE GAVE AN INFORMAL INTRODUCTION TO THE MODEL OF COLORED QUARKS AND GLUONS. WE SKETCH HERE HOW THE IDEAS OF GAUGE INVARIANCE MIGHT LEAD TO THE VIEWS DISCUSSED EARLIER.

THE ONLY FERMIONS WHICH EXPERIENCE THE STRONG INTERACTION ARE THE QUARKS. THE STRONG FORCE IS FLAVOR INDEPENDENT, SO FLAVOR IS NOT THE KEY TO UNDERSTANDING THE INTERACTION. RATHER IT IS NECESSARILY THAT EACH QUARK CAN HAVE ONE OF 3 COLORS (R, G OR B (FOR RED, GREEN OR BLUE)). THE STRONG INTERACTION DOES DEPEND ON COLOR. WE WRITE THE WAVE FUNCTION OF A QUARK AS A 3 COMPONENT OBJECT

$$\psi = \begin{pmatrix} \psi_R \\ \psi_G \\ \psi_B \end{pmatrix}. \quad \text{AS IN OUR PREVIOUS EXAMPLES OF GAUGE THEORY}$$

WE CONSTRAIN THE FORM OF THE INTERACTION TO ALLOW LOCAL PHASE INVARIANCE OF THIS WAVE FUNCTION. THAT IS, WE WISH TO ^{BE ABLE TO} ALTER THE WAVE FUNCTION BY A ROTATION IN 'COLOR SPACE'. OUR PROCEDURE IS A DIRECT GENERALIZATION OF THE CASE OF ISOSPIN.

WE CONSIDER ALL 3×3 MATRIX TRANSFORMATIONS, U , OF THE 3 COMPONENT WAVE FUNCTION, ψ . RESTRICTING TO THOSE TRANSFORMATIONS WITH $\det U = 1$ WE HAVE AN $SU(3)$ GROUP. WE START WITH INFINITESIMAL ROTATIONS: $U \approx 1 + i\epsilon$. THIS TIME THERE ARE 8 INDEPENDENT TYPES OF THE BASIC ROTATIONS (IN $SU(N)$ THERE ARE $N^2 - 1$ OF SUCH 'GENERATORS' OF INFINITESIMAL TRANSFORMATIONS.)

THE FIRST 3 OF THESE, λ^1, λ^2 & λ^3 ARE RECOGNIZED AS THE PAULI SPIN MATRICES, OPERATING ON ONLY 2 OF THE 3 COLORS.

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{matrix} R \\ G \\ B \end{matrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

THE INFINITESIMAL TRANSFORMATION CAN THEN BE WRITTEN

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$U = 1 + i \epsilon^j \lambda^j \quad j = 1, \dots, 8$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

THE GENERAL TRANSFORMATION IS BUILT UP BY EXPONENTIATION

$$U = e^{i \alpha^j \lambda^j}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

WE NOW DESIRE A THEORY IN WHICH LOCAL PHASE INVARIANCE HOLDS IN THE FORM $\psi \rightarrow \psi' = e^{i \frac{g}{2} \alpha^j \lambda^j} \psi$. HERE WE INTRODUCE g AS

THE STRONG INTERACTION COUPLING CONSTANT (NO RELATION TO g OF THE WEINBERG-SALAM MODEL). FOR PHASE INVARIANCE TO BE POSSIBLE WE MUST INTRODUCE A GAUGE INVARIANT INTERACTION, DESCRIBED BY A MODIFICATION OF THE DERIVATIVE OPERATOR

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + i \frac{g}{2} \lambda^j B_\mu^j$$

THERE ARE 8 POTENTIALS B_μ^i , AND THE CORRESPONDING QUANTA ARE THE GLUONS DISCUSSED BEFORE. THE 8 KINDS OF COLOR FIELDS THEN OBEY

$$F_{\mu\nu}^j \lambda^j = \frac{1}{i g} [D_\nu, D_\mu] = \frac{1}{2} \partial_\nu B_\mu^j \lambda^j - \frac{1}{2} \partial_\mu B_\nu^j \lambda^j + \frac{i g}{4} B_\nu^k B_\mu^l [\lambda^j, \lambda^k]$$

THE λ^j MATRICES OBEY $[\lambda^j, \lambda^k] = 2i f^{jkl} \lambda^l$

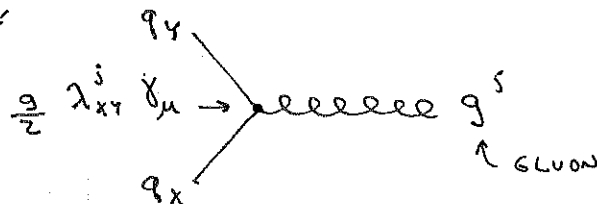
THE CONSTANTS f^{jkl} ARE A GENERALISATION OF THE ϵ^{jkl} NOTED IN CASE OF THE PAULI SPIN MATRICES. THE VALUES OF f^{jkl} FOR PERMUTATIONS OF INDICES NOT LISTED IN THE TABLE ARE JUST ± 1 TIMES THOSE LISTED, DEPENDENT ON WHETHER THE PERMUTATION IS ODD OR EVEN.

TABLE 8.1: ANTISYMMETRIC STRUCTURE CONSTANTS OF $SU(3)$

$f_{123} = 1$
$f_{147} = f_{246} = f_{257} = f_{345} = f_{316} = f_{637} = \frac{1}{2}$
$f_{458} = f_{678} = \sqrt{3}/2$

THEN FOR WHAT IT'S WORTH, $F_{\mu\nu}^j = \partial_\nu B_\mu^j - \partial_\mu B_\nu^j - g f^{jkl} B_\nu^k B_\mu^l$

THE INTERACTION OF GLUONS AND QUARKS CAN BE READ OFF THE INVARIANT DERIVATIVE!

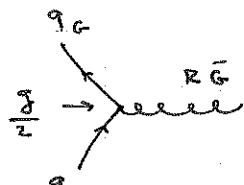


x & y ARE COLORS g_x AND g_y HAVE THE SAME FLAVOR

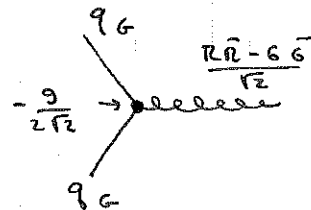
AS WHEN USING PAULI SPIN MATRICES IT IS MORE CONVENIENT TO USE RAISING AND LOWERING OPERATORS THAN THE λ_j LISTED IN THE TABLE ON P 408

$$\lambda^{\bar{R}\bar{G}} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{matrix} \bar{R} \\ \bar{G} \\ \bar{B} \end{matrix}$$

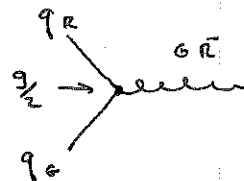
R G B



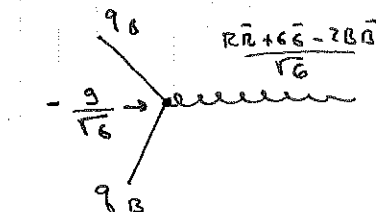
$$\lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$\lambda^{G\bar{R}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$\lambda^8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$



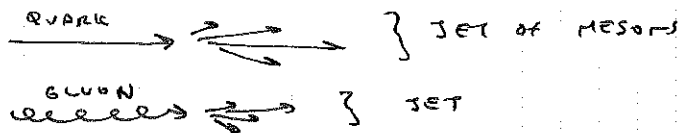
+ 4 MORE WITH OFF DIAGONAL ELEMENTS

WE RECOGNIZE THAT THESE COUPLINGS ARE EXACTLY THOSE WE WROTE DOWN ON P 260 FF, LECTURE 14 (EXCEPT FOR THE COUPLING FACTOR $g/2$). THUS OUR 'NAIVE' USE OF COLORED QUARKS AND GLUONS IS BACKED UP BY THE FORMALISM OF THE NON-ABELIAN GAUGE THEORY.

THIS WAS USED IN THE CG POTENTIAL OF P. 277.

3. EVIDENCE FOR GLUON JETS

THE GLUONS, BEING THE QUANTA OF THE COLOR FIELD, ARE PRESUMABLY 'CONFINED' ALONG WITH THE STATIC COLOR FIELD. WE CAN GET INDIRECT EVIDENCE FOR THE EXISTENCE OF GLUONS IN REACTIONS IN WHICH A GLUON IS RADIATED WITH A LARGE ENERGY. AS IS THE CASE WHEN A SINGLE QUARK IS SCATTERED WITH A LARGE ENERGY, THE LABORATORY INDICATION OF THE FUNDAMENTAL ENTITIES IS A 'JET' OF PARTICLES



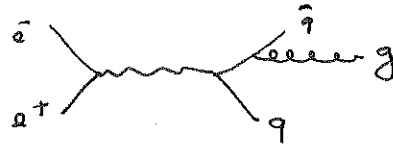
IF THE TOTAL ENERGY OF THE JET IS LARGE COMPARED TO 300 MEV (THE TYPICAL TRANSVERSE ENERGY SCALE OF THE STRONG INTERACTION), THE DECAY PARTICLES WILL APPEAR REASONABLY WELL COLLIMATED. EXAMPLES OF QUARK JETS IN $\bar{p}p$ COLLISIONS WERE SHOWN ON P 230.

A FAIRLY CLEAN TEST OF THE JET IDEA IS IN THE REACTION $e^+e^- \rightarrow$ HADRONS. OUR PICTURE IS THAT THIS ACTUALLY PROCEEDS VIA $e^+e^- \rightarrow q\bar{q}$. IN THE C.M. FRAME THE 2 QUARK JETS SHOULD BE BACK TO BACK.

WITH THE ADVENT OF e^+e^- COLLISIONS OF $E_e \geq 15$ GEV REASONABLY CONVINCING EVIDENCE OF THIS INTERPRETATION HAS BECOME AVAILABLE.

IN ADDITION, A FRACTION OF THE HADRONIC FINAL STATES APPEAR TO HAVE 3 DISTINCT JETS. THESE ARE INTERPRETED AS DUE TO GLUON EMISSION

$$e^+e^- \rightarrow q\bar{q}g$$



THE TOTAL RATE $\frac{\sigma_{e^+e^- \rightarrow q\bar{q}g}}{\sigma_{e^+e^- \rightarrow q\bar{q}}}$ SHOULD BE $\sim \alpha_s = \frac{g^2}{4\pi}$

HOWEVER THE GLUON JET CAN ONLY BE WELL IDENTIFIED IF IT HAS $E_{jet} > 300 \text{ MeV}$. SO THE 3JET RATE DOESN'T GIVE A VERY GOOD MEASURE OF α_s . (BUT SEE P. 414)

RESULTS WERE REPORTED ALMOST SIMULTANEOUSLY BY 4 EXPERIMENTS AT THE PETRA e^+e^- COLLIDER

TASSO	PHYS. LETT. 86B, 243 (1979)
MARK-J	P. R. L. 43, 830 (1979)
PLUTO	PHYS. LETT. 86B, 418 (1979)
JADE	PHYS. LETT. 91B, 142 (1980)

TASSO

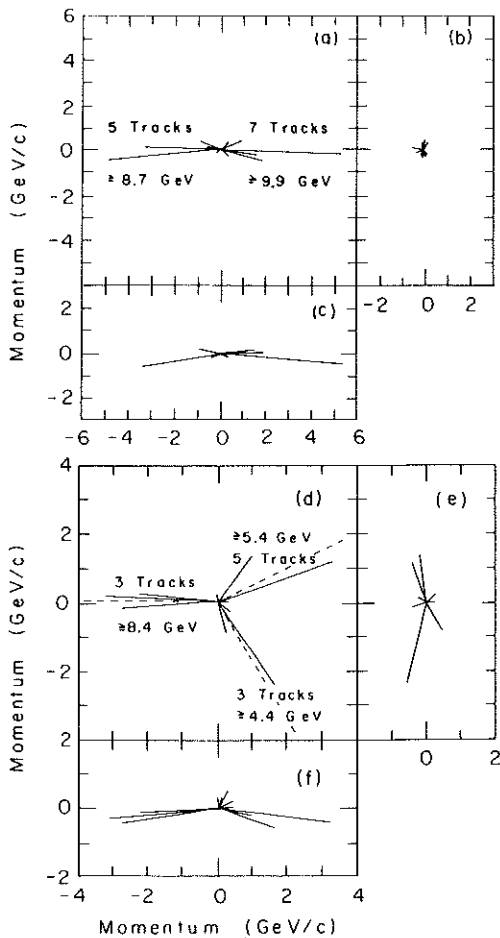


Fig. 6. Momentum space representation of a two-jet event (a)-(c) and a three-jet event (d)-(f) in each of three projections. (a), (d) $\hat{n}_2 - \hat{n}_3$ plane; (b), (e) $\hat{n}_1 - \hat{n}_2$ plane; (c), (f) $\hat{n}_1 - \hat{n}_3$ plane.

MARK-J

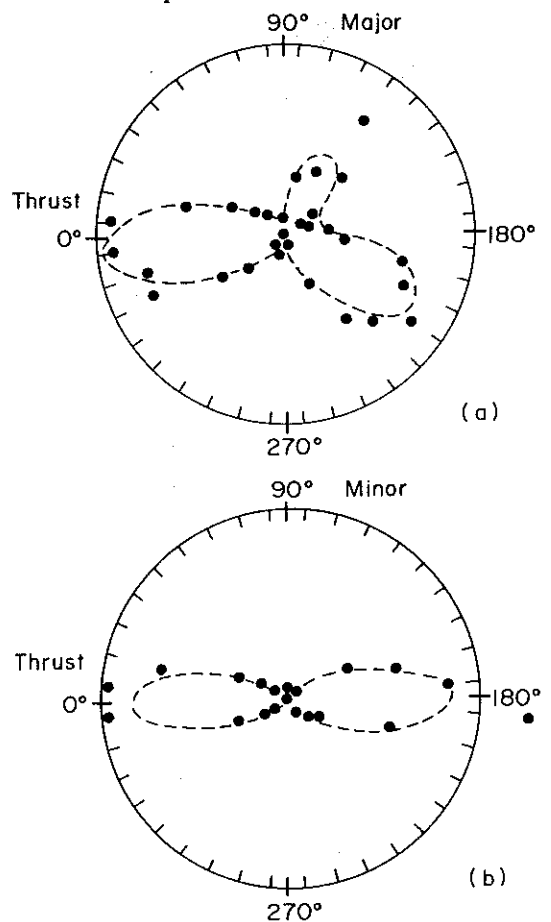


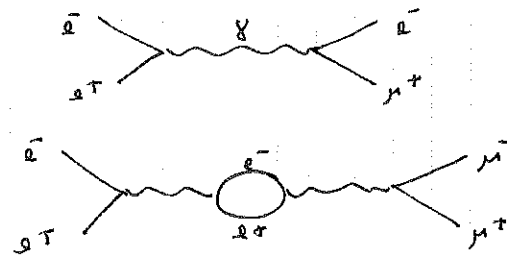
FIG. 3. (a) Energy distribution in the plane as defined by the thrust and the major axes for all the events with thrust < 0.8 and oblateness > 0.1 at $\sqrt{s} = 27.4, 30,$ and 31.6 GeV . The energy value is proportional to the radial distances. The superimposed dashed line represents the distribution calculated with use of the $q\bar{q}g$ model. (b) The measured and calculated energy distribution in the plane as defined by the thrust and the minor axes.

4. ASYMPTOTIC FREEDOM

THE REACTION $e^+e^- \rightarrow q\bar{q}g$ CAN BE THOUGHT OF AS A GLUON RADIATIVE CORRECTION TO THE SIMPLER REACTION $e^+e^- \rightarrow q\bar{q}$. THE QUESTION ARISES WHETHER AT HIGH ENERGY SUCH CORRECTIONS BECOME MORE AND MORE IMPORTANT, PERHAPS SPOILING THE SIMPLICITY OF THE QUARK MODEL VIEW. THE ARGUMENT OF GROSS, WILCEK AND POLITZER IS THAT RATHER THE REVERSE HOLDS. THE EFFECT OF GLUON RADIATION BECOMES LESS AND LESS IMPORTANT AT HIGHER ENERGIES, AS MEASURED BY AN APPARENT DECREASE IN α_s , THE FINE STRUCTURE CONSTANT OF THE STRONG INTERACTION. IN EFFECT THE STRONG INTERACTION BECOMES WEAKER AS ENERGY INCREASES. THIS IS THE PHENOMENON OF ASYMPTOTIC FREEDOM.

IT MAY BE WORTHWHILE TO CONTRAST THIS WITH THE SITUATION FOR QUANTUM ELECTRODYNAMICS, WHICH BEHAVES QUITE DIFFERENTLY. IT IS SUGGESTIVE TO CONSIDER ANOTHER KIND OF RADIATIVE CORRECTION & SO-CALLED VACUUM POLARIZATION.

THE SINGLE PHOTON REACTION



HAS A 2 PHOTON CORRECTION, AND SO ON IN HIGHER ORDER.

THE EFFECT OF THE 2ND DIAGRAM IS TO CHANGE THE APPARENT CHARGE OF THE ELECTRON AS $q^2 = E_{cm}^2$ INCREASES.

THE 1ST DIAGRAM HAS AMPLITUDE $\sim \frac{e^2}{q^2} = \frac{4\pi\alpha}{q^2}$

THE 2ND DIAGRAM CAN BE CALCULATED TO BE $\frac{4\pi\alpha}{q^2} \left(\frac{\alpha}{3\pi} \ln\left(\frac{q^2}{M_e^2}\right) + \text{SMALL} \right)$

[SEE BJORKEN & DRELL, VOL I, SEC 8.2]. ON ADDING THE AMPLITUDES WE MIGHT INTERPRET THE RESULT AS A MODIFICATION TO THE APPARENT VALUE OF α . NAMELY

AMPLI $\sim \frac{4\pi\alpha_{eff}}{q^2}$ WITH $\alpha_{eff} = \alpha_0 \left(1 + \frac{\alpha_0}{3\pi} \ln\left(\frac{q^2}{M_e^2}\right) \right)$

APPARENTLY SUMMING OVER DIAGRAMS OF ALL ORDERS LEADS TO THE RESULT

$$\alpha_{eff} = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \ln\left(\frac{q^2}{M_e^2}\right)}$$

THAT IS, THE APPARENT STRENGTH OF THE ELECTROMAGNETIC INTERACTION INCREASES WITH ENERGY. (UP TO THE VERY LARGE ENERGY $\sim 10^{150} M_e$)

THIS EFFECT WAS CONSIDERED ALREADY BY DIRAC AND WEISENBERG IN 1934. SINCE $\kappa_{eff} = \epsilon_{eff} / 4\pi$ WE INFER THAT THE APPARENT CHARGE OF THE ELECTRON BECOMES LARGER AS WE PROBE SMALLER DISTANCES WITH A HIGH ENERGY PHOTON. THIS MIGHT OCCUR IF THE VACUUM IS SOMEWHAT LIKE A DIELECTRIC MEDIUM. THE LARGE 'BARE' CHARGE OF THE ELECTRON POLARIZES THE NEARBY VACUUM. THE LIKE POLARIZATION CHARGES ARE REPELLED, LEAVING A LOWER NET CHARGE INSIDE A GIVEN RADIUS ABOUT THE 'BARE' CHARGE.

FOR AMUSEMENT WE PURSUE A CLASSICAL ANALOGY. IF THE MEDIUM HAS DIELECTRIC CONSTANT ϵ , THEN

$$\vec{D} = \epsilon \vec{E} \quad \text{AND} \quad \vec{\nabla} \cdot \vec{D} = \rho_{BARE}$$

$$\text{SO } \vec{D} = \frac{\rho_{BARE} \hat{r}}{r^2} \Rightarrow \vec{E} = \frac{\rho_{BARE} \hat{r}}{\epsilon r^2}$$

IF WE TEST THE ELECTRIC FIELD WITH A SECOND (BARE) CHARGE q_2 THE FORCE IS

$$\vec{F} = \frac{q_1 q_2 \hat{r}}{\epsilon r^2}$$

$$\text{IF WE WRITE THIS AS } \vec{F} = \frac{q_{1,eff} q_{2,eff} \hat{r}}{r^2}$$

$$\text{WE CONCLUDE } \epsilon_{eff} = \frac{\epsilon_{BARE}}{\sqrt{\epsilon}}$$

IN AN ORDINARY DIELECTRIC MEDIUM, $\epsilon \gg 1$, AND WE MAY SAY THAT THE DIELECTRIC POLARIZATION HAS 'SCREENED' THE BARE CHARGE.

IF WE MOVE THE SECOND CHARGE VERY CLOSE TO THE FIRST, r EVENTUALLY BECOMES SMALLER THAN THE SIZE OF THE MOLECULES. THEN $\epsilon \rightarrow 1$ AND THE APPARENT CHARGE GROWS LARGER, AS INFERRED FROM THE FORCE DISCUSSED ABOVE.

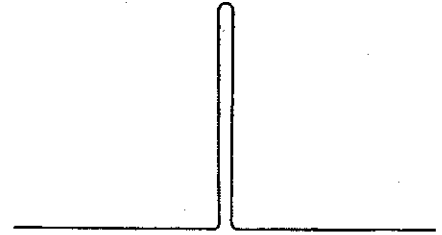


Fig. 1a. Schematic charge distribution of the electron.

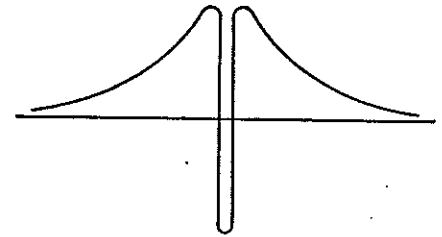


Fig. 1b. Schematic charge distribution of the vacuum electrons in the neighborhood of an electron.

FIGURE ↑ FROM WEISSKOPF, P.R. 56, 72 (1939)

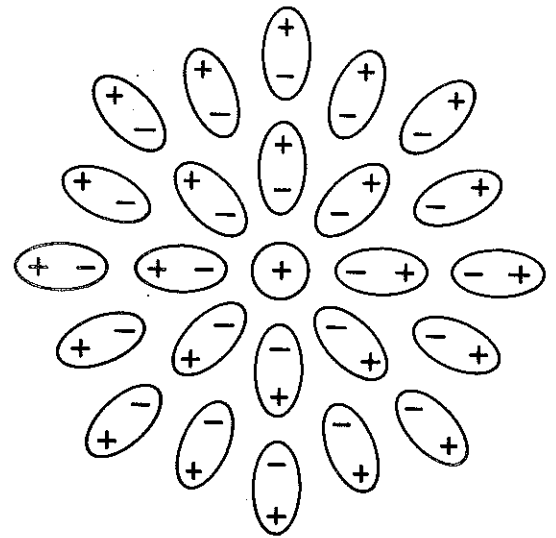


Fig. 8-8. Polarization of a dielectric medium by a test charge.

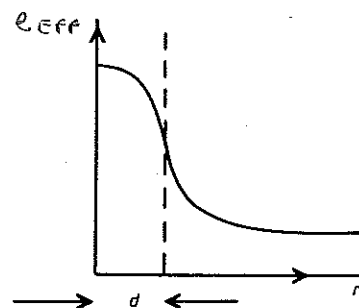


Figure 12.12 Effective charge as a function of the interparticle spacing r.

COMPARING WITH THE QED RESULT OF P. 411 WE MIGHT IDENTIFY

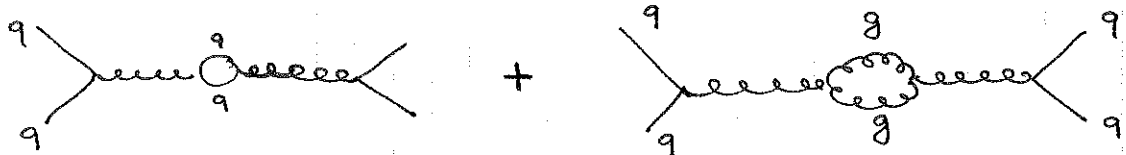
$$\alpha_{\text{EFF}} = \frac{e^2}{4\pi} = \frac{e^2}{4\pi\epsilon}$$

$$\text{SO } \epsilon \leftrightarrow 1 - \frac{\alpha_0}{3\pi} \ln\left(\frac{q^2}{m_e^2}\right)$$

AT LARGE DISTANCES (LOW q^2) $\epsilon \rightarrow 1$ AS IS APPROPRIATE FOR THE VACUUM. BUT FOR SMALL DISTANCES (LARGE q^2) ϵ DECREASES LEADING TO A LARGER EFFECTIVE CHARGE.

IN QED IT SEEMS THAT e_{BARE} IS INFINITE, AS IS THE APPARENT CORRECTION TO e DUE TO VACUUM POLARIZATION ACCORDING TO AN OBSERVER AT LARGE DISTANCES. ONLY $e_{\text{EFFECTIVE}}$ IS FINITE. THIS OF COURSE IS THE MARVELOUS SWINDLE OF RENORMALISATION.

IN THE THEORY OF QUARKS AND GLUONS VACUUM POLARIZATION EFFECTS ALTER THE APPARENT COUPLING CONSTANT α_s OF THE STRONG INTERACTION. BUT BECAUSE GLUONS CAN COUPLE TO THEMSELVES THERE IS AN ADDITIONAL TYPE OF DIAGRAM POSSIBLE.



THE DIAGRAM WITH THE GLUON LOOP HAS AN AMPLITUDE WITH THE OPPOSITE SIGN TO THAT WITH THE QUARK LOOP. THIS HAS TO DO WITH THE QUARKS BEING FERMIONS WHILE THE GLUONS ARE BOSONS. THE TASK OF SUMMING ALL SUCH DIAGRAMS WAS PERFORMED BY GROSS, WILCZEK & POLITZER, WHO FOUND

$$\alpha_s(q^2) = \frac{\alpha_s(\Lambda^2)}{1 + \frac{33 - 2N_f}{12\pi} \alpha_s(\Lambda^2) \ln\left(\frac{q^2}{\Lambda^2}\right)} \approx \frac{12\pi}{(33 - 2N_f) \ln\left(\frac{q^2}{\Lambda^2}\right)}$$

WHERE $N_f = \#$ OF QUARK FLAVORS (= 6 IF THE TOP QUARK IS THE LAST)

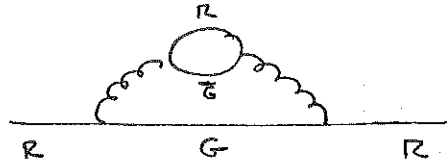
$\Lambda =$ QCD SCALE PARAMETER (PRESENTLY BELIEVED TO BE ABOUT M_{PI})

IF THERE ARE LESS THAN 17 QUARK FLAVORS THEN THE GLUON LOOP DIAGRAMS DOMINATE, AND CAUSE α_s TO DECREASE WITH INCREASING q^2 . THIS IS ASYMPTOTIC FREEDOM

ASYMPTOTIC FREEDOM VALIDATES THE INSIGHT OF BESORREN (LECTURE 8) THAT AT HIGH ENERGIES THE STRUCTURE OF HADRONIC MATTER SHOULD BE SIMPLER. FEYNMAN DIAGRAMS OF STRONG INTERACTIONS OF QUARKS AND GLUONS ARE MEANINGFUL AT

HIGH ENERGIES AS HIGHER ORDER CORRECTIONS BECOME SMALL. BUT CONVERSELY THE ANALYSIS OF STRONG INTERACTIONS AT LOW ENERGIES MAY ALWAYS REMAIN A DIFFICULT SUBJECT.

WE MAY GIVE A CRUDE PICTURE OF HOW THE COUPLING CONSTANT α_s IS DIMINISHED. BECAUSE GLUONS CARRY COLOR WE CAN HAVE A DIAGRAM FOR A RED QUARK LIKE



THIS DISPERSES THE RED COLOR AWAY FROM THE QUARK, SO ITS RED 'CHARGE' IS NOT LOCALIZED AT A POINT. A PROBE OF THE QUARK'S COLOR DISTRIBUTION WILL FIND VERY LITTLE RED AT SMALL DISTANCES. THIS PHENOMENON IS CALLED ANTI-SCREENING. IT CAN ONLY HAPPEN IN NON-ABELIAN THEORIES, IN WHICH THE FIELD QUANTA CARRY CHARGE.

WE MAY REVIEW THE EVIDENCE GIVEN THROUGHOUT THE COURSE FOR THE VARIATION OF α_s WITH ENERGY. SEVERAL RESULTS COME FROM THE ANALYSIS OF VECTOR MESON DECAY RATES. IN LECTURE 14, PP 252-254 WE FOUND $\alpha_s \sim .4$ FOR P, W, ϕ DECAYS (AFTER INSERTING THE PROPER COLOR FACTOR WHICH WAS NOT YET DONE IN THAT LECTURE). ESTIMATES FROM Ψ DECAYS (PP 276-279) GIVE $\alpha_s \sim .3$. ONE RESULT FOR Υ DECAY (P 284) SUGGESTED THAT $\alpha_s \sim .2$.

ANOTHER ESTIMATE OF α_s COMES FROM THE RATIO

$$R = \frac{\sigma_{e^+e^- \rightarrow \text{HADRONS}}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}} = 3 \sum_q \frac{1}{9} Q_q^2 \left(1 + \frac{\alpha_s}{\pi} + \dots \right)$$

THE CORRECTION α_s/π IS THE RESULT OF INCLUDING THE GLUON RADIATIVE CORRECTION $e^+e^- \rightarrow q\bar{q}g$ DISCUSSED IN SECTION 3 (WHETHER OR NOT RESULTS FROM THE PETRA STORAGE RING INDICATE $\alpha_s \sim .15$ AT $\sqrt{s} = 30.6 \text{ eV}$).

\sqrt{s}	α_s	SOURCE	Λ
1	.4	P, W, ϕ DECAY	106 MeV
3	.3	Ψ DECAY	150
10	.2	Υ DECAY	112
30	.15	$e^+e^- \rightarrow \text{HADRONS}$	75

THE PARAMETER Λ IS ESTIMATED USING $\alpha_s = \frac{12\pi}{21 \ln(s/\Lambda^2)}$

ASSUMING $N_f = 6$. PERHAPS $\Lambda \sim 100 \pm 50 \text{ MeV}$

THE TERMS ARE DISTRIBUTIVELY

5. CONFINEMENT

UNLIKE THE THEORY OF QUARKS AND GLUONS IS ASYMPTOTICALLY FREE, AT PRESENTLY REALIZABLE ENERGIES IT IS 'UN-FREE'. THAT IS, THE QUARKS, GLUONS, AND COLOR FIELDS ARE ALL CONFINED INTO VOLUMES OF RADIUS ~ 1 FERMI. FOR POSSIBLE INSIGHT WE CONTINUE THE ANALOGY THAT THE VARIATION IN THE COUPLINGS $\alpha(q^2)$ OR $\alpha_s(q^2)$ IS LIKE VARIATION OF THE DIELECTRIC CONSTANT WITH DISTANCE. THESE REMARKS FOLLOW CHAPTER 17 OF THE BOOK BY T.D. LEE.

FROM THE EXPRESSION FOR THE 'RUNNING COUPLING CONSTANT' $\alpha_s(q^2)$ GIVEN ON P 413, WE MIGHT IDENTIFY THE COLOR DIELECTRIC CONSTANT AS

$$\epsilon(q^2) = 1 + \frac{33 - 2N_f}{12\pi} \alpha_s(\Lambda^2) \ln\left(\frac{q^2}{\Lambda^2}\right)$$

THIS HAS THE PROPERTY THAT IT INCREASES WITH q^2 , AND SO INCREASES AS WE PROBE SMALLER AND SMALLER DISTANCES. IF WE LOOK TO CLASSICAL E & M FOR A POSSIBLE ANALOGY, MAGNETIC MATERIALS ARE MORE SUGGESTIVE THAN DIELECTRIC MEDIA. WE WON'T PUSH THIS RELATION TOO FAR, BUT INSTEAD IMAGINE A DIELECTRIC WITH $\epsilon < 1$ ON THE WHOLE. AGAIN AT SMALL DISTANCES AROUND A FREE CHARGE EMBEDDED IN THIS DIELECTRIC, $\epsilon \rightarrow 1$. AT LEAST THE TREND OF ϵ VS. DISTANCE IN OUR IMAGINARY DIELECTRIC IS SOMETHING LIKE THAT FOR EQCD LISTED ABOVE. PEOPLE SOMETIMES CALL THE MEDIUM WITH $\epsilon < 1$ A DIA-ELECTRIC MEDIUM IN SUPPOSED ANALOGY TO A DIA-MAGNETIC SUBSTANCE (ALTHOUGH I DON'T BELIEVE THIS BEARS CLOSE SCRUTINY).

THE POSSIBILITY OF CONFINEMENT SUGGESTS ANOTHER IMPORTANT VARIATION ON OUR CLASSICAL ARGUMENT OF P. 412. WE IMAGINE THAT A CHARGE EXISTS IN A SMALL SPHERICAL BUBBLE CREATED IN THE DIA-ELECTRIC MEDIUM. INSIDE THE BUBBLE $\epsilon = 1$ AS FOR THE IDEAL (OR ASYMPTOTICALLY FREE) VACUUM. OUTSIDE THE BUBBLE $\epsilon < 1$. OUR ARGUMENT WILL BE THAT IN THE LIMIT ϵ OUTSIDE $\rightarrow 0$ THERE IS NO STABLE SIZE FOR THE BUBBLE SURROUNDING A SINGLE CHARGE.

WE FIRST NOTE THAT IF $\epsilon < 1$ THE INDUCED POLARIZATION WAS THE OPPOSITE SIGN FROM THE CASE $\epsilon > 1$. NAMELY

$$\vec{P} = \alpha \vec{E} = (\epsilon - 1) \vec{E}$$

IF $\epsilon > 1$ THEN $\vec{P} \sim \vec{E}$, AND THE SURFACE CHARGE INDUCED ON THE INSIDE OF THE BUBBLE IS OPPOSITE TO THAT OF THE TEST CHARGE. (FIG. (b) \rightarrow)

THEN THE ATTRACTION OF THE OPPOSITE CHARGES SHRINKS

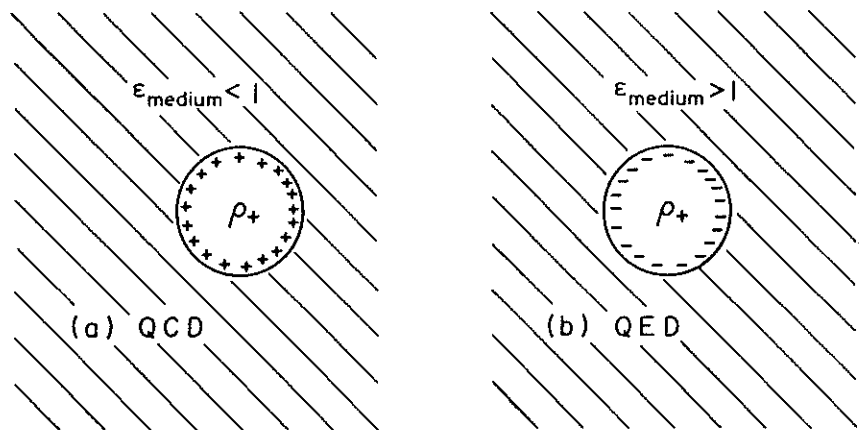


FIG. 8-37. Charge induced by a positive test charge placed at the center of a hole in a dielectric medium. (a) Dia-electric case $\epsilon_{\text{medium}} < 1$ hoped to resemble QCD. (b) Dielectric case $\epsilon_{\text{medium}} > 1$ of normal electrodynamics.

THE BUBBLE TO THE MINIMUM SIZE (\approx MOLECULAR RADIUS), BUT IF $\epsilon < 1$ THE INDUCED SURFACE CHARGE HAS THE SAME SIGN AS THE SOURCE CHARGE, AND THE BUBBLE WOULD PREFER TO GROW WITHOUT LIMIT.

WE CONTINUE THE ARGUMENT WITH ENERGY CONSIDERATIONS. THE PRESENCE OF THE DIELECTRIC OUTSIDE THE BUBBLE CHANGES THE STORED ENERGY OF THE FIELD. (WE AVOID THE QUESTION OF INFINITE SELF ENERGY BY ONLY CONSIDERING CHANGES IN U). IF r = RADIUS OF BUBBLE THEN

$$\Delta U = \frac{1}{8\pi} \int_V E^2 \left(\frac{1}{\epsilon} - 1 \right) d\text{vol} = \frac{q^2}{2r} \left(\frac{1}{\epsilon} - 1 \right) \quad q = \text{SOURCE CHARGE}$$

FOR $\epsilon > 1$ ΔU IS MINIMIZED WITH SMALL r , BUT FOR $\epsilon < 1$ BIG r IS PREFERRED. NOW WE CAN READILY IMAGINE THAT THE MEDIUM WILL OPPOSE THE CREATION OF LARGE BUBBLES INSIDE ITSELF. A MEASURE OF THIS OPPOSITION IS THE WORK NEEDED TO CREATE A BUBBLE OF SIZE r . A SIMPLE ASSUMPTION IS THAT $W \sim \text{VOLUME} = \frac{4}{3}\pi r^3 K$ $K = \text{SOME CONSTANT ENERGY DENSITY}$

THUS THE TOTAL EXTRA ENERGY ASSOCIATED WITH A CHARGE PLACED IN A BUBBLE INSIDE A MEDIUM WITH $\epsilon < 1$ IS

$$U = \Delta U + W = \frac{q^2}{2r} \left(\frac{1}{\epsilon} - 1 \right) + \frac{4\pi}{3} r^3 K$$

PRESUMABLY THE ACTUAL RADIUS r OF THE BUBBLE MINIMIZES U .

$$\text{THIS LEADS TO } r \approx \left(\frac{q^2}{8\pi \epsilon K} \right)^{1/4} \quad \text{AND } U \approx \frac{4}{3} \left(\frac{q^2}{2\epsilon} \right)^{3/4} (4\pi K)^{1/4} \quad \text{IF } \frac{1}{\epsilon} \gg 1$$

WE DRAW 2 CONCLUSIONS. IF $\epsilon < 1$ BUT ALSO $\epsilon > 0$ THERE IS A POSSIBILITY OF FINITE SIZED BUBBLES FORMING, WITH A CORRESPONDING CHARACTERISTIC MASS U/c^2 . BUT IN THE LIMIT $\epsilon \rightarrow 0$ $r \rightarrow \infty$ AND $M \rightarrow \infty$. THAT IS, IT WILL TAKE A TITENDEOUS AMOUNT OF ENERGY TO PLACE A FREE CHARGE INTO THE MEDIUM. IF WE COULD DO THIS, THE FIELDS IN THE MEDIUM.

$$E \sim \frac{q}{\epsilon r^2} \quad \text{WOULD BE VERY STRONG.}$$

IF WE IMAGINE THAT THE QCD VACUUM IS MUCH LIKE THE DIA-ELECTRIC MEDIUM SKETCHED ABOVE, IT WILL NOT BE POSSIBLE TO ISOLATE SINGLE CHARGES IN REACTIONS INVOLVING A FINITE ENERGY.

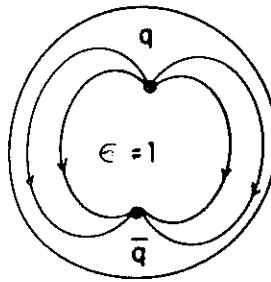
IN CONTRAST CONSIDER THE CASE OF A DIPOLE OF SOURCE CHARGE, q^+ AND q^- SEPARATED BY A SMALL DISTANCE. IF THE DIPOLE FIELD IS EXACTLY LIKE THAT OF CLASSICAL E & M WE AGAIN HAVE A CONTRADICTION AS $\epsilon \rightarrow 0$. BUT A DIPOLE FIELD DOES NOT HAVE SPHERICAL SYMMETRY, SO WE CAN IMAGINE THE FIELD LINES DISTRACT SOMEWHAT UNTIL THEY ARE ENTIRELY CONTAINED WITHIN THE BUBBLE. WHILE WE DON'T UNDERSTAND THE EXACT MECHANISM WHICH CONFINES THE LINES INSIDE THE BUBBLE, WE SEE THAT IT IS AT LEAST TOPOLOGICALLY POSSIBLE FOR A DIPOLE CHARGE DISTRIBUTION, AND SHOULD COST ONLY A FINITE ENERGY TO ARRANGE. SEE FIG. ON THE NEXT PAGE.

IN THE THEORY OF GLOUED QUARKS WE CAN ALSO IMAGINE A BUBBLE CONTAINING 3 QUARKS - IF THE QUARKS HAVE DIFFERENT COLORS. THE FIELD LINE CONFIGURATION SKETCHED TO THE RIGHT HAS NO ANALOGUE IN CLASSICAL EEM. BUT WE RECALL THAT GLUONS CAN CARRY COLOR IN THE FORM

$$a\bar{a} - b\bar{b} \quad , \text{ SO THAT}$$

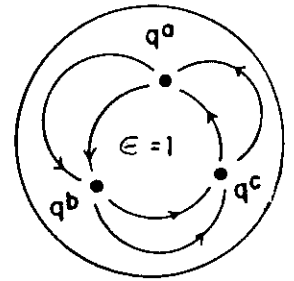
q^a CAN IN FACT BE ATTRACTED TO q^b , ETC. (LECTURE 14, P264)

OUTSIDE
 $E = 0$



MESON $\bar{q}^a q^a$

$a, b, c = \text{COLORS}$



BARYON $\epsilon_{abc} q^a q^b q^c$

Fig. 17.6. Color electric fields inside hadrons.

IN THE LITERATURE THE BUBBLES ARE USUALLY CALLED 'BAGS'. FURTHER REMARKS ON THE 'BAG MODEL' CAN BE FOUND IN THE BOOKS BY LEE, AND BY CLOSE.

6. QCD CORRECTIONS TO THE QUARK DISTRIBUTIONS OF THE NUCLEON

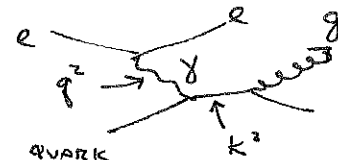
IN OUR ANALYSIS OF THE INELASTIC SCATTERING $eN \rightarrow eX$ AND $\nu N \rightarrow \mu X$ WE SUPPOSED THAT THE EXCHANGED PHOTON OR W BOSON STRIKES A SINGLE QUARK WHICH IS ESSENTIALLY FREE. IN THE THEORY OF QCD THIS IS ONLY A GOOD APPROXIMATION AT VERY HIGH ENERGIES. AT FINITE ENERGIES WE SHOULD EXPECT CORRECTIONS ASSOCIATED WITH THE RADIATION OF GLUONS.

SEMI-CLASSICALLY, IF THE QUARK IS STRUCK BY A PHOTON OR W BOSON IT RECOILS, ACCELERATING RAPIDLY FOR A SHORT TIME. THIS ACCELERATION LEADS TO RADIATION, BOTH OF PHOTONS AND GLUONS. THE GLUON RADIATION IS MORE PROBABLE AS $\alpha_s > \alpha$. AN IMPORTANT EFFECT OF THIS RADIATION IS THAT THE QUARK IS LEFT WITH LESS MOMENTUM THAN IT WOULD OTHERWISE HAVE - SO IT APPEARS TO BE AT LOWER X, WHERE X IS FRACTION OF TOTAL MOMENTUM CARRIED BY THE QUARK. WE MAY ANTICIPATE THAT AS q^2 INCREASES SO DOES THE AMOUNT OF GLUON RADIATION, WITH THE EFFECT THAT THE QUARK DISTRIBUTIONS APPEAR SHIFTED TO SMALLER X.

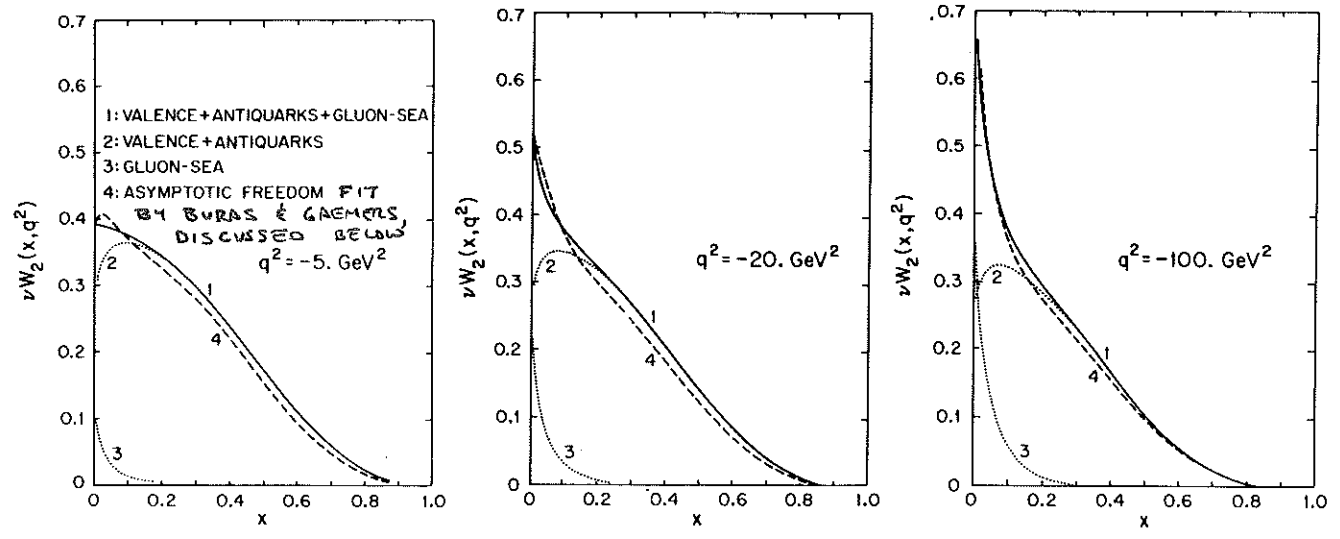
THIS SHIFT WILL VARY AS $\ln(q^2)$. WE GET A ROUGH SENSE OF WHY THIS OCCURS BY CONSIDERING A CORRECTION DIAGRAM. COMPARED TO THE DIAGRAM WITHOUT GLUON RADIATION THERE IS AN EXTRA PROPAGATOR FOR THE QUARK, WHICH LEADS TO A FACTOR

$$\sim \int_{\Lambda^2}^{q^2} \frac{dF^2}{F^2} \sim \ln\left(\frac{q^2}{\Lambda^2}\right)$$

HERE THE MASS OF THE VIRTUAL QUARK CAN VARY BETWEEN q^2 AND Λ^2 WHERE Λ IS THE SCALE PARAMETER

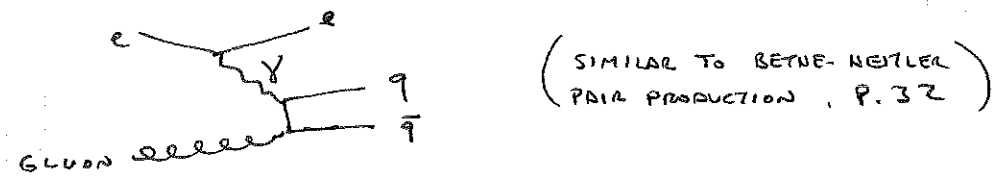


OF THE STRONG INTERACTION WHICH 'CUTS OFF' THE LOW ENERGY DIVERGENCE.



THE FIGURE [ASAD & HUMBERT, PHYS. LETT 77B, 105 (1978)] ILLUSTRATES THE SLOW EVOLUTION OF THE QUARK STRUCTURE FUNCTIONS WITH q^2 .

IT MAY BE WORTH NOTING THAT THERE IS ANOTHER PROCESS WHICH CONTRIBUTES TO THE RISE OF THE QUARK SEA AT LOW x .



A GLUON IN THE INITIAL HADRON PRODUCES A PAIR OF QUARK AND ANTIQUARKS, ONE OF WHICH THE ABSORBS TO PHOTON. IN A SENSE THIS PROCESS PROBES THE GLUON DISTRIBUTION RATHER THAN THE QUARK DISTRIBUTION, AS DISCUSSED FURTHER BELOW.

HISTORICALLY THE FIRST QCD CALCULATION WAS MADE FOR CERTAIN MOMENTS OF THE STRUCTURE FUNCTION $F_2 = \nu W_2$ [GROSS & WILCZEK, P.R. D8, 3633 (1973), GEORGI & POLITZER, P.R. D9, 416 (1974)]

THEY DEFINED
$$M_N(q^2) \equiv \int_0^1 dx x^{N-2} (F_2^p(x, q^2) - F_2^n(x, q^2))$$

AND DEMONSTRATED THAT
$$M_N(q^2) = \text{CONST.} \left[\ln(-q^2/\Lambda^2) \right]^{-P_N}$$

WHERE
$$P_N = \frac{4}{33 - 2N_f} \left(1 - \frac{2}{N(N+1)} + 4 \sum_{j=2}^N \frac{1}{j} \right)$$
 $N_f = \# \text{ OF FLAVORS.}$

IT IS HARD TO COMPARE THIS PREDICTION DIRECTLY TO EXPERIMENT, BUT IT DID REVEAL THE $\ln(-q^2/\Lambda^2)$ DEPENDENCE EXPECTED IN ANALOGY TO QED CORRECTIONS.

A FIT TO THE AVAILABLE DATA USING THIS METHOD WAS MADE BY BURAS & GAEMERS [NUC. PHYS. B132, 249 (1978)]. THEY PARAMETRIZE, FOR EXAMPLE, THE VALENCE QUARK DISTRIBUTION AS

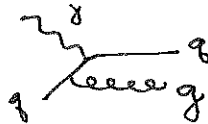
$$x V_i(x) = A_i x^{\alpha_i(\bar{s})} (1-x)^{\beta_i(\bar{s})} \quad i = \text{QUARK TYPE}$$

$$\text{WHERE } \bar{s} = \ln \left[\frac{\ln(-q^2/\Lambda^2)}{\ln(-q_0^2/\Lambda^2)} \right] ; \alpha, \beta = \text{FITTED FUNCTIONS}$$

THE MAIN POINT IS THAT THE CHANGE IN SHAPE OF THE STRUCTURE FUNCTION VARIES LIKE $\ln(\ln(-q^2)) \rightarrow$ SLOWLY!

THE FITS OF BURAS & GAEMERS ARE SHOWN AS CURVE 4 ON THE FIGURE ON P 418.

A MORE INTUITIVE WAY OF DISCUSSING THE EVOLUTION OF THE STRUCTURE FUNCTIONS WAS GIVEN BY ALTARELLI & PARISI [NUC. PHYS B126, 298 (1977)] AS ABOVE, THEY CONSIDERED THE VARIATION IN THE STRUCTURE FUNCTIONS TO BE DUE TO DIAGRAMS LIKE



THIS IS RELATED TO THE PROBABILITY THAT A QUARK OF MOMENTUM y EMITS A GLUON OF MOMENTUM $y-x$ TO BECOME A QUARK OF MOMENTUM x . CHANGES IN THE STRUCTURE FUNCTION WITH q^2 ARE THEN RELATED TO CHANGES IN THIS PROBABILITY.

AGAIN WE EXPECT THE CHANGES TO DEPEND ON $\ln(-q^2/\Lambda^2)$ RATHER THAN DIRECTLY ON q^2 . SO IT IS USEFUL TO DEFINE

$$\tau \equiv \ln(-q^2/\Lambda^2)$$

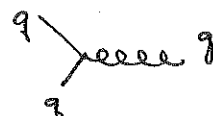
EFFECTS DUE TO GLUON EMISSION WILL OF COURSE BE PROPORTIONAL TO THE RUNNING COUPLING CONSTANT $\alpha_s(q^2) = \alpha_s(\tau)$. THE INSIGHT OF ALTARELLI & PARISI IS THAT ALL CHANGES IN THE RATE OF GLUON EMISSION WITH q^2 ARE DESCRIBED BY THE VARIATION OF α_s ITSELF. THEN IT IS QUITE REASONABLE TO ASSUME THAT THE PROBABILITY THAT A QUARK OF MOMENTUM y BECOMES A QUARK OF MOMENTUM x AFTER GLUON EMISSION IS A FUNCTION ONLY OF THE RATIO x/y , TIMES α_s , TIMES THE OVERALL FACTOR $\ln(q^2/\Lambda^2) = \tau$ FOUND ON BOTTOM OF P 417: $\text{PROB} \sim \frac{\alpha_s}{2\pi} P(x/y) \cdot \tau$.

THE VARIOUS INGREDIENTS MAY NOW BE COMBINED. A QUARK PROBABILITY DISTRIBUTION $q(x, q^2) = q(x, \tau)$ OBEYS

$$q(x, \tau + d\tau) = q(x, \tau) + \underbrace{\int_x^1 dy q(y, \tau)}_{\text{PROB OF QUARK AT } y} \underbrace{\int_0^1 dz \frac{\alpha(\tau)}{2\pi} P(z) d\tau}_{\text{CHANCE IN PROB THAT } y \rightarrow x + \text{GLUON}} \delta(x - zy)$$

$$\text{OR } \frac{d q(x, \tau)}{d \tau} = \frac{\alpha(\tau)}{2\pi} \int_x^1 \frac{dy}{y} q(y, \tau) P\left(\frac{x}{y}\right)$$

THIS IS THE ALTARELLI - PARISI EQUATION

THE FUNCTION $P(z)$ IS CALCULABLE FROM THE PROPERTIES OF THE QUARK-GLUON VERTEX  . THE EXPLICIT FORM IS GIVEN ON P 421

FOOTNOTE: WE CAN VERIFY THAT THE ALTARELLI-PARISI EQUATION IS CONSISTENT WITH THE EARLIER MOMENT ANALYSIS OF GROSS & WILCZEK. COMPARING WITH P 418, WE CAN WRITE

$$M_N = \int_0^1 dx x^{N-1} q(x, \tau)$$

SINCE THE PROBABILITY DISTRIBUTION $q(x, \tau)$ IS IDENTIFIED WITH $\frac{1}{x} F_2(x, \tau)$

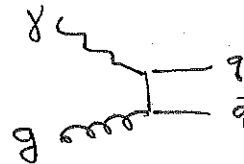
$$\begin{aligned} \text{THEN } \frac{dM_N}{d\tau} &= \int_0^1 dx x^{N-1} \frac{dq(x, \tau)}{d\tau} = \frac{\alpha_s(\tau)}{2\pi} \int_0^1 dx x^{N-1} \int_x^1 dy q(y, \tau) \int_0^1 dz P(z) \delta(x-yz) \\ &= \frac{\alpha_s(\tau)}{2\pi} A_N M_N \quad \text{WITH } A_N = \int_0^1 dz z^{N-1} P(z) \end{aligned}$$

ALSO, FROM P 418 WE HAVE $M_N \sim \tau^{-P_N} \Rightarrow \frac{dM_N}{d\tau} \sim P_N \tau^{-P_N-1} = -\frac{P_N}{\tau} M_N$

SINCE $\alpha_s(\tau) \sim \frac{12\pi}{(33-2M_f)} \frac{1}{\tau}$ (P. 413) WE HAVE THE FORM

$$\frac{dM_N}{d\tau} = \alpha_s(\tau) A_N M_N \quad \text{FROM THE MOMENT METHOD ALSO}$$

WE REMARKED EARLIER THAT THE DIAGRAM



INDICATES THAT A QUARK OF MOMENTUM x MIGHT HAVE COME FROM A GLUON OF MOMENTUM y VIA PAIR PRODUCTION. IF WE INTRODUCE

$G(x, \tau) =$ GLUON PROBABILITY DISTRIBUTION, THEN

$$\frac{dq(x, \tau)}{d\tau} = \frac{\alpha_s(\tau)}{2\pi} \int_x^1 \frac{dy}{y} \left\{ q(y, \tau) P_{q \rightarrow q} \left(\frac{x}{y} \right) + G(y, \tau) P_{G \rightarrow q} \left(\frac{x}{y} \right) \right\}$$

THIS INCLUDES ALL TYPES OF CORRECTIONS ARISING FROM SINGLE GLUON EMISSION OR ABSORPTION.

ALTHOUGH $e-p$ INELASTIC SCATTERING DOES NOT DIRECTLY PROBE THE GLUON DISTRIBUTION $G(x, \tau)$, WE READILY SEE THAT ITS EVOLUTION EQUATION HAS THE FORM:

$$\frac{dG(x, \tau)}{d\tau} = \frac{\alpha_s(\tau)}{2\pi} \int_x^1 \frac{dy}{y} \left\{ G(y, \tau) P_{G \rightarrow G} \left(\frac{x}{y} \right) + q(y, \tau) P_{q \rightarrow G} \left(\frac{x}{y} \right) \right\}$$

FOR WHAT IT'S WORTH, THE VARIOUS P_i 'S ARE

$$P_{g \rightarrow g} = \frac{4}{3} \left\{ \frac{1+z^2}{1-z} + \frac{3}{2} \delta(1-z) \right\}$$

$$P_{q \rightarrow G} = \frac{4}{3} \left\{ \frac{1+(1-z)^2}{z} \right\}$$

$$P_{G \rightarrow G} = 6 \left\{ \frac{z}{1-z} + \frac{1-z}{z} + z(1-z) + \frac{3}{4} \delta(1-z) \right\}$$

$$P_{G \rightarrow q} = \frac{1}{2} \left\{ z^2 + (1-z)^2 \right\}$$

ANYWAY THEY ARE KNOWN FUNCTIONS.

IF WE KNOW $q(x^2, \tau)$ FOR SEVERAL VALUES OF τ WE COULD INVERT THE ALTARELLI-PARISI EQUATIONS TO EXTRACT $G(x, \tau)$, THE GLUON DISTRIBUTION.

SOME OF THE MOST IMPRESSIVE DATA ON THE QCD CORRECTIONS TO THE APPARENT STRUCTURE OF THE NUCLEON COME FROM THE CDHS NEUTRINO SCATTERING EXPERIMENT [DE GROOT ET AL, PHYS. LETT. 82B, 456 (1979)]. THESE STUDIES PROVIDE AN ADDITIONAL MEASUREMENT OF THE QCD SCALE PARAMETER Λ . APPARENTLY A VALUE OF $\Lambda = 300-500$ MEV IS FOUND, SOMEWHAT HIGHER THAN OUR ESTIMATE OF 100 MEV ON P. 414.

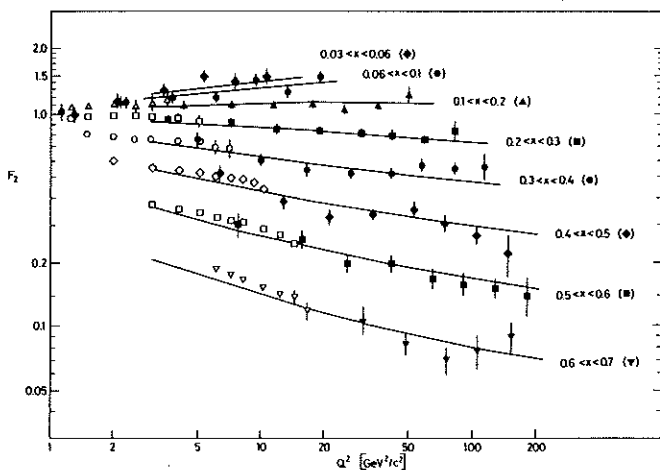


Fig. 24 Comparison of the Buras-Gaemers fit with the $F_2(x)$ data of CDHS. The open symbols represent SLAC electron-deuteron data.

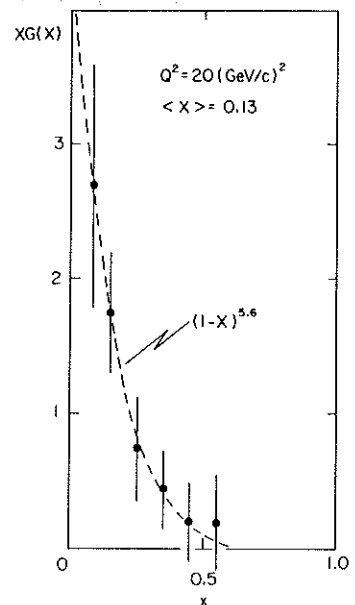


Fig. 26 The x-distribution of gluons, at $Q^2 = 20 \text{ GeV}^2$.