# INVARIANCE PRINCIPLES AND CONSERVATION LAWS

WE NOW TAKE UP THE STUDY OF THE STRONG AND WEAK INTERACTIONS, AS REVEALED BY HIGH ENERGY EXPERIMENTS IN THE 1950's ONWARDS. HERE WE HAVE NO CLASSICAL PRECONCEPTIONS AS TO THE NATURE OF THE SHORT RANGE FORCES, AND EVEN YUKAWA'S BRILLIANT INSIGHT OF HEAVY QUANTUM EXCHANGE PROVED ONLY A ROUGH GUIDE. IN THE ABSENSE OF ANY DETAILED UNDERSTANDING OF THESE FORCES, EMPIRICAL CONSERVATION LAWS PLAYED A MAJOR ROLE IN ORGANIZING OUR IMPRESSIONS. IN SEEKING EXPLANATIONS OF THESE LAWS WE ARE LED TO CONSIDER INVARIANCE OR SYMMETRY PRINCIPLES.

THESE PRINCIPLES ARE QUITE ESTHETICALLY PLEASING, AND ACCORDING TO SOME VIEWS THIS SHOULD BE SUFFICIENT TO INSURE THAT THEY ARE CORRECT. BUT NATURE TURNS OUT TO BE SELECTIVE AS TO WHICH SYMMETRY PRINCIPLES AND CONSERVATION LAWS ACTUALLY HOLD. PERHAPS THE 'SYMMETRY BREAKING' ONLY ADDS TO THE INTEREST OF THE MATTER – IT CERTAINLY ADDS TO THE RICHNESS OF NATURAL PHENOMENA THAT WE MAY OBSERVE. IN ANY CASE, WE EMPHASIZE AGAIN THAT ALL DISCUSSION OF CONSERVATION 'LAWS' IS ULTIMATELY BASED ON APPEAL TO EXPERIMENTAL FACT.

AS INDICATED ABOVE, THE EXPLANATION OF A CONSERVATION LAW IS TYPICALLY GIVEN IN TERMS OF AN INVARIANCE PRINCIPLE. THAT IS, SOME MENTALLY CONCEIVABLE QUANTITY ACTUALLY TURNS OUT TO BE UNOBSERVABLE IN PRACTICE – NATURE IS INVARIANT WITH RESPECT TO POSSIBLE VALUES OF THIS QUANTITY. A CLASSIC EXAMPLE IS ABSOLUTE POSITION IN SPACE. ALTHOUGH THE DEFINITION OF ABSOLUTE POSITION WAS CONCEIVABLE TO NEWTON, HIS 1ST LAW OF MOTION ACTUALLY CONTRADICTS THIS. CONSIDER 2 PARTICLES WHICH INTERACT ONLY WITH EACH OTHER VIA A FORCE DESCRIBED BY A POTENTIAL $V$. IF ABSOLUTE POSITION WERE MEANINGFUL, WE MIGHT WRITE $V = V(\bar{r}_1, \bar{r}_2)$. BUT IF ABSOLUTE POSITION IS UNOBSERVABLE, WE EXPECT $V = V(\bar{r}_1 - \bar{r}_2)$. IN THE LATTER CASE,

$$\bar{F}_{on\,1} = -\bar{\nabla}_1 V = +\bar{\nabla}_2 V = -\bar{F}_{on\,2}, \quad \text{so} \quad \bar{F}_{TOT} = \bar{F}_1 + \bar{F}_2 = 0$$

AND SO MOMENTUM IS CONSERVED!

THE CALCULATIONAL BENEFIT OF A CONSERVATION LAW IS AUGMENTED BY AN INSIGHT INTO THE UNDERLYING INVARIANCE PRINCIPLE. EXAMINATION OF THIS PRINCIPLE OFTEN YIELDS RESULTS NOT SIMPLY EXPRESSIBLE AS A CONSERVATION LAW.

WE MAY CATEGORIZE THE INVARIANCE PRINCIPLES INTO 5 TYPES, AS ON THE FOLLOWING PAGE.

| | Non-observable quantity | Invariance principle | Conservation law or other insight |
|---|---|---|---|
| 1. Continuous symmetries | Absolute space | $\vec{r} \to \vec{r} + \Delta\vec{r}$ | Momentum |
| | Absolute time | $t \to t + \Delta t$ | Energy |
| | Absolute direction | $\theta \to \theta + \Delta\theta$ | Angular momentum |
| | Absolute velocity | Lorentz trans. | Special relativity |
| | Absolute acceleration | Arbitrary coord. trans. | General relativity |
| 2. Exchange symmetries | Identity of identical particles | Interchange of particles | Bose-Einstein or Fermi-Dirac statistics |
| 3. Discrete symmetries | Right- or left-handedness | $\vec{r} \to -\vec{r}$ | P = parity (space inversion) |
| | Direction of time | $t \to -t$ | T = time reversal invariance |
| | Sign of electric charge | $e \to -e$ | C = charge conjugation |
| 4. Unitary symmetries | "Angle" in some internal coordinate space of a particle | U(1) - various 1 dimensional spaces | Charge, baryon number, lepton number |
| | | SU(2) - various 2 dimensional spaces | Isospin, weak isospin |
| | | SU(3) | Color |
| | | SU(n) | Quark flavor |
| | | SU(5) | Quark-lepton unification |
| 5. Gauge symmetries | Absolute strength of potentials | Gauge invariance | Form of particle interactions |
| | Bare charge and mass | Renormalization | |

We assume that symmetries of types 1 and 2 are well enough understood to be used without much further discussion. We recall that the Compton scatter experiment (1922), and more particularly the follow-up experiment of Bothe & Geiger (1925) marks the general acceptance of energy and momentum conservation in elementary particle interactions. Of course, angular

MOMENTUM CONSERVATION WAS CRITICAL TO THE INTERPRETATION OF ATOMIC SPECTROSCOPY BY LANDÉ, HEISENBERG, ETC.

WE WILL NOW FOCUS OUR ATTENTION ON THE DISCRETE SYMMETRIES, T, P & C. THE VARIOUS UNITARY AND GAUGE SYMMETRIES WILL BE EXPLORED THROUGHOUT THE REMAINDER OF THE COURSE.

THE PRESENT UNDERSTANDING OF HOW WELL NATURE OBSERVES THE CONSERVATION LAWS IS SUMMARIZED BELOW.

| LAW | INTERACTION | | |
|---|---|---|---|
| | STRONG | ELECTROMAGNETIC | WEAK |
| ENERGY-MOMENTUM<br>CPT<br>BARYON NUMBER<br>LEPTON NUMBER<br>CHARGE<br>COLOR<br>WEAK ISOSPIN | YES | YES | YES |
| T ($\leftrightarrow$ CP) | YES | YES | NO (TINY VIOLATION) |
| P<br>C | YES | YES | NO ('MAXIMAL' VIOLATION) |
| QUARK FLAVOR<br>(= STRANGENESS, CHARM...) | YES | YES | NO |
| ISOSPIN ($\equiv I$)<br>AND HIGHER QUARK SU(n) SYM. | YES | NO ($\Delta I = 0, 1$) | NO ($\Delta I = 0, \frac{1}{2}, 1$) |

EVERYDAY EXPERIENCE SUGGESTS THAT THE DISCRETE SYMMETRIES T, P & C ARE BADLY VIOLATED IN NATURE. IT IS SURPRISING THAT IN THE MICROWORLD ONLY THE WEAK INTERACTION IS KNOWN TO VIOLATE THEM, AND THIS IS A RELATIVELY RECENT DISCOVERY (1956). FOR 30 YEARS PRIOR IT WAS BELIEVED THAT ALL FUNDAMENTAL INTERACTIONS WERE T, P, & C INVARIANT, AND THAT MACROSCOPIC ASYMMETRIES ARE EITHER DUE TO THERMODYNAMIC CONSIDERATIONS, OR ACCIDENTS OF 'INITIAL CONDITIONS'. OF COURSE THE PROBLEM OF THE INITIAL CONDITION OF THE UNIVERSE IS HARDLY SOLVED TODAY.

PH 529 LECTURE 9 

# TIME REVERSAL INVARIANCE

A STORY BY T.D. LEE:

It is well-known that time-reversible dynamical equations of classical mechanics can produce time-irreversible thermodynamics in the macroscopic world. As an illustration, we may think of traveling by train between New York and Princeton. In analogy to microscopic reversibility, let us assume that for each train going from New York to Princeton, there is one in the opposite direction. Imagine now that there are no signs in the railroad station. The probability that a man starting from Princeton will end in New York is 1 if he watches the train's direction; otherwise it is $\frac{1}{2}$. But once he reaches New York, if there are also no signs in Penn Station the chance that he will succeed in taking a Princeton-bound train is extremely small; the person may end up in Boston, and go from there to Chicago, · · ·, resulting in a macroscopic irreversibility.

WE CANNOT ACTUALLY CAUSE ANY OF THE DISCRETE TRANSFORMATIONS T, P, OR C TO HAPPEN IN REAL LIFE. RATHER WE ARE CONCERNED WITH COMPARING TWO SITUATIONS WHICH CAN OCCUR, AND CAN BE RELATED IN PRINCIPLE BY A DISCRETE TRANSFORMATION

1. DETAILED BALANCE. IN THE CASE OF A 2 BODY SCATTERING PROCESS

$$a + b \to c + d$$

WE CAN SAY THAT THE REACTION $c + d \to a + b$ IS THE TIME REVERSED PROCESS. FOR THIS TO BE TRUE IN DETAIL, THE MOMENTA OF ALL PARTICLES IN ONE REACTION MUST BE THE EXACT OPPOSITES OF THOSE IN THE OTHER. IF THE PARTICLES HAVE SPIN, THE SPINS MUST ALSO BE REVERSED. BUT NOTE THAT THE HELICITY REMAINS UNCHANGED


$$T(\bar{p}) = -\bar{p}$$
$$T(\bar{\sigma}) = -\bar{\sigma}$$
$$T(\bar{\sigma}\cdot\bar{p}) = \bar{\sigma}\cdot\bar{p}$$

IF THE INITIAL STATE PARTICLES HAVE SPIN, BUT ARE 'UNPOLARISED' WE MUST BE MORE CAREFUL. BY UNPOLARISED WE MEAN THAT ALL POSSIBLE SPIN STATES ARE EQUALLY POPULATED. RECALL OUR PRESCRIPTION FOR DEALING WITH SPIN IN CROSS SECTION CALCULATIONS (p 92)

AVERAGE OVER INITIAL SPINS (IF UNPOLARIZED)

SUM OVER FINAL SPINS (IF THEY ARE NOT OBSERVED)

[THIS RULE DIFFERS SLIGHTLY FROM THAT ADVANCED BY PERKINS p 33, BUT IS EQUIVALENT.]

THEN FROM p 80, THE C.M. FRAME CROSS SECTIONS ARE

$$\frac{d\sigma}{d\Omega}(a+b\to c+d) = \frac{1}{64\pi^2 E^2}\,\frac{p_c}{p_a}\,\frac{1}{(2S_a+1)(2S_b+1)}\sum_{\text{SPINS}}|M_{a+b\to c+d}|^2$$

WHILE $\frac{d\sigma}{d\Omega}(c+d \to a+b) = \frac{1}{64\pi^2 E^2} \frac{p_a}{p_c} \frac{1}{(2S_c+1)(2S_d+1)} \sum_{\text{SPINS}} |M_{c+d\to a+b}|^2$

IF THE INTERACTION IS TIME REVERSAL INVARIANT, THEN $M_{a+b\to c+d} = M_{c+d\to a+b}$

HENCE $$\frac{d\sigma}{d\Omega}(a+b\to c+d) = \frac{p_c^2}{p_a^2}\frac{(2S_c+1)(2S_d+1)}{(2S_a+1)(2S_b+1)}\frac{d\sigma}{d\Omega}(c+d\to a+b)$$

THIS IS THE PRINCIPLE OF DETAILED BALANCING.

[THERE IS NO NEED TO INVOKE PARITY CONSERVATION, AS DONE BY PERKINS p 33, IF WE ARE CONTENT TO DESCRIBE MATRIX ELEMENTS $M_{a+b\to c+d}$ BETWEEN STATES OF HELICITY RATHER THAN $S_z$ ALONG THE $z$ AXIS.]

WE BRIEFLY REMARK ON THE FACTOR $p_c^2/p_a^2$, WHICH MEASURES THE RELATIVE PHASE SPACE AVAILABLE FOR THE FINAL STATES IN THE FORWARD AND REVERSE REACTIONS. PHASE VOLUME VARIES LIKE $p^2 dp$, FAVORING THE HIGHER MOMENTUM FINAL STATE. (IN A 2 BODY REACTION IN THE C. M. FRAME $p_a = p_b$; $p_c = p_d$). THIS MAY BE EXPRESSED PICTORIALLY:


Figure 4.4. Available phase space in parking a car and in leaving a parking space

2. TESTS OF DETAILED BALANCE FOR THE STRONG INTERACTION.

FAIRLY ACCURATE TESTS HAVE BEEN MADE AT LOW ENERGIES

$d + Mg^{24} \leftrightarrow p + Mg^{25}$ — WEITKAMP ET AL P.R. 165, 1233 (1966)

$\alpha + Mg^{24} \leftrightarrow p + Al^{27}$ — VON WITSCH ET AL P.R.L. 19, 524 (1967)

THESE COMPARISONS SHOW AGREEMENT TO WITHIN ½ % OR SO

3. SPIN OF THE $\pi^+$ MESON.

IF WE ACCEPT DETAILED BALANCE AS VALID, THEN IT CAN BE USED TO ESTABLISH ONE OF THE PARTICLE'S SPIN, THE OTHERS BEING KNOWN. THE CLASSIC EXAMPLE IS THE REACTIONS $pp \leftrightarrow d\pi^+$ WHICH COMPARISON SHOWED THAT THE $\pi^+$ MESON HAS SPIN ZERO. A QUICK APPLICATION OF THE PRINCIPLE OF DETAILED BALANCE SUGGESTS

$$\frac{d\sigma}{d\Omega}(pp\to d\pi^+) = \frac{(2S_d+1)(2S_\pi+1)}{(2S_p+1)^2}\frac{p_\pi^2}{p_p^2}\frac{d\sigma}{d\Omega}(d\pi^+\to pp)$$

HOWEVER, $d\pi^+ \to pp$ HAS A FINAL STATE WITH TWO IDENTICAL PARTICLES. HENCE PRODUCTION OF A PROTON AN ANGLE $\Theta$ IS INDISTINGUISHABLE FROM PRODUCTION AT $180° - \Theta$. IN EFFECT THE FINAL STATE PHASE SPACE IS CUT IN HALF. (IF THERE ARE $n$ IDENTICAL FINAL STATE PARTICLES THE PHASE SPACE IS REDUCED BY $1/n!$). THIS PHASE SPACE REDUCTION HOLDS WHETHER THE PARTICLES ARE FERMIONS OR BOSONS - IT IS THEIR INDISTINGUISHABILITY THAT MATTERS.

THEN, NOTING $S_p = 1/2$ WHILE $S_d = 1$, KNOWN FROM NUCLEAR PHYSICS,

$$\frac{d\sigma}{d\Omega}(pp \to d\pi^+) = 2 \cdot \frac{3}{4}(2S_\pi + 1)\frac{p_\pi^2}{p_p^2}\frac{d\sigma}{d\Omega}(d\pi^+ \to pp)$$

ALTHOUGH THE ORIGINAL EXPERIMENTS PERFORMED IN 1952-53 HAD 30% ERROR BARS, THIS IS SUFFICIENT ACCURACY TO DETERMINE THE <u>QUANTISED</u> VALUE AS $S_\pi = 0$.

4. TESTS OF DETAILED BALANCE IN THE ELECTROMAGNETIC INTERACTION.

SEVERAL REACTIONS HAVE BEEN STUDIED

$$\gamma + n \leftrightarrow p + \pi^-$$
$$\gamma + d \leftrightarrow p + n$$
$$\gamma + He^3 \leftrightarrow p + d \qquad \text{(YOUR AUTHOR'S Ph.D. THESIS)}$$

THESE WERE SHOWN TO OBEY DETAILED BALANCE TO 2-4% ACCURACY.

NOTE THAT THE SPIN WEIGHTING FACTOR FOR AN UNPOLARISED PHOTON BEAM IS $1/2$ NOT $1/3$ AS USUAL FOR A SPIN 1 PARTICLE, AS REAL PHOTONS OF $J_z = 0$ DO NOT OCCUR.

NO DETAILED BALANCE STUDIES OF THE WEAK INTERACTION HAVE BEEN MADE, ALTHOUGH, SAY, $\bar{\nu}_\mu + p \leftrightarrow n + \mu^+$ IS AT LEAST CONCEIVABLE. (WHY HASN'T THIS BEEN DONE?)

5. SPIN CORRELATION TESTS.

ANOTHER CLASS OF TESTS INVOLVES SEARCHES FOR SPIN CORRELATIONS WHICH MAY VIOLATE T INVARIANCE. NOTE HOWEVER THAT CERTAIN SPIN CORRELATIONS ALSO VIOLATE PARITY CONSERVATION.

AN INSTRUCTIVE EXAMPLE TO BEGIN WITH IS SPIN 0 - SPIN 1/2 SCATTERING, SAY $\pi^+ p \to \pi^+ p$. WE EXAMINE THE POSSIBLE FORM OF THE MATRIX ELEMENT FOR T VIOLATING TERMS. TO GET A SENSE OF THE ARGUMENT IT IS SUFFICIENT TO USE 2-COMPONENT PAULI SPINORS, RATHER THAN DIRAC SPINORS, FOR THE SPIN 1/2 PARTICLE.

Then matrix element $M = (f|\Gamma|i)$ where $\Gamma$ is a 2x2 spin operator.

Four terms suggest themselves; in that $M$ should be a scalar in ordinary space:

$$\Gamma = A\,\mathbb{1} + B\,\bar{\sigma}\cdot\bar{p}_i\times\bar{p}_f + C\,\bar{\sigma}\cdot\bar{p}_i + D\,\bar{\sigma}\cdot\bar{p}_f$$

A, B, C + D are scalar functions.

We first check the effect of the Parity operation:

$$P: \quad \bar{p}_i \to -\bar{p}_i \qquad p_f \to -\bar{p}_f \qquad \bar{\sigma}\to\bar{\sigma}$$

so $\bar{\sigma}\cdot\bar{p}_i \to -\bar{\sigma}\cdot\bar{p}_i$ ; $\bar{\sigma}\cdot\bar{p}_f \to -\bar{\sigma}\cdot\bar{p}_f$ ; $\bar{\sigma}\cdot\bar{p}_i\times\bar{p}_f \to \bar{\sigma}\cdot\bar{p}_i\times\bar{p}_f$

Thus if parity is conserved, $C = D = 0$. However $B$ can be non-zero

Now consider time reversal

$$T: \quad \bar{p}_i \to -\bar{p}_f \qquad \bar{p}_f \to -\bar{p}_i \quad \text{and} \quad \bar{\sigma}\to-\bar{\sigma}$$

$\bar{\sigma}\cdot\bar{p}_i \to \bar{\sigma}\cdot\bar{p}_f$ ; $\bar{\sigma}\cdot\bar{p}_f \to \bar{\sigma}\cdot\bar{p}_i$ ; $\bar{\sigma}\cdot\bar{p}_i\times\bar{p}_f \to -\bar{\sigma}\cdot\bar{p}_f\times\bar{p}_i = \bar{\sigma}\cdot\bar{p}_i\times\bar{p}_f$

T violation could occur if $C \neq D$, but we have just noted how parity conservation requires $C = D = 0$.

Hence if parity is conserved, there is no possible T violation in spin 0 – spin ½ elastic scattering!

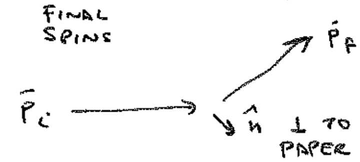
Having come this far we may still extract a useful lesson. How can one measure the spin component of a high energy spin ½ particle? The Stern-Gerlach technique doesn't work well at high energies. There are basically 2 techniques:

- If the particle decays via the weak interaction, the parity violation will lead to an asymmetry of the decay angular distribution indicative of the initial spin state.

- Scatter the particle off a target, and look for an asymmetry in the angular distribution.

To see how the 2nd case works, note $\dfrac{d\sigma}{d\Omega} \sim M^2 \sim \sum_{\text{final spins}} |(f|\Gamma|i)|^2$

$$= \sum_{f\ \text{spins}} (i|\Gamma^+|f)(f|\Gamma|i) = (i|\Gamma^+\Gamma|i)$$




It is common to define $\hat{n} = \dfrac{\bar{p}_i\times\bar{p}_f}{|\bar{p}_i\times\bar{p}_f|} \equiv$ normal to the scattering plane

The parity conserving interaction is $\Gamma = A + B\,\bar{\sigma}\cdot\hat{n}$ so $\Gamma^+ = A^* + B^*\,\bar{\sigma}\cdot\hat{n}$

and $\Gamma^+\Gamma = A^2 + B^2 + 2\,\mathrm{Re}\,A^*B\,\bar{\sigma}\cdot\hat{n}$ noting $\bar{\sigma}^+ = \bar{\sigma}$ & $(\bar{\sigma}\cdot\hat{n})^2 = 1$ (p 88)

Now $\langle i | i \rangle = 1$ and $\langle i | \bar{\sigma} | i \rangle \equiv \bar{\sigma}_i$

Thus $\frac{d\sigma}{d\Omega} \sim A^2 + B^2 + 2\,\mathrm{Re}\,A^*B\,\bar{\sigma}_i\cdot\hat{n}$

So if $\mathrm{Re}\,A^*B$ is positive, and $\bar{\sigma}_i \parallel \hat{n}$, then more particles scatter to the left than to the right

$$\frac{N_{LEFT} - N_{RIGHT}}{N_{LEFT} + N_{RIGHT}} = \underbrace{\frac{2\,\mathrm{Re}\,A^*B}{A^2+B^2}}_{\equiv \text{"ANALYZING POWER"}}\;\bar{\sigma}_i\cdot\hat{n}$$

If the 'analyzing power' term is somehow known, we can measure $\bar{\sigma}_i$ <u>statistically</u>. Many scatters of similarly polarized particles are required, and the measurements are quite lengthy...,

6. Spin correlations in $pp \to pp$ scattering.

In spin ½ – spin ½ scattering, such as $pp \to pp$ it is possible to have a T violating spin correlation which conserves parity. Such a term can exist because of the increased complexity of the matrix element. Now $M \sim \Gamma_1 \Gamma_2$ where $\Gamma_1$ is a 2x2 matrix operating on the 1st proton, and $\Gamma_2$ operates on the 2nd proton. This time we can usefully define a triad of orthogonal spatial vectors:

$$\bar{\ell} = \bar{P}_i - \bar{P}_f \qquad \bar{m} = \bar{P}_i + \bar{P}_f \qquad \bar{n} = \bar{P}_i \times \bar{P}_f$$

Then
$$M = A + B(\bar{\sigma}_1 + \bar{\sigma}_2\cdot\hat{n}) + C(\sigma_1\cdot\hat{n})(\bar{\sigma}_2\cdot\hat{n}) + D(\sigma_1\cdot\hat{m})(\sigma_2\cdot\hat{m})$$
$$+ E(\bar{\sigma}_1\cdot\hat{\ell})(\bar{\sigma}_2\cdot\hat{\ell}) + F\left[(\sigma_1\cdot\hat{\ell})(\sigma_2\cdot\hat{n}) + (\sigma_1\cdot\hat{n})(\sigma_2\cdot\hat{\ell})\right]$$

All of these conserve parity, but term F violates T-invariance (why?)

[ Actually, there are constraints on the form of A, B ··· F due to the fact that there are identical particles in this reaction. It gets a bit intricate. c.f. 'Collision Theory' by Goldberger & Watson p 398 ]

One can isolate the term $\mathrm{Im}\,F^*(D+E)$ by comparing 2 related experiments:

- Scatter fully polarized beam protons off unpolarized target protons, yielding a left-right asymmetry of the type discussed above.

- SCATTER UNPOLARIZED BEAM PROTONS OFF UNPOLARIZED TARGET PROTONS, AND MEASURE THE POLARIZATION OF THE FINAL STATE PROTONS (IN A SUBSEQUENT SCATTERING!)

IF T INVARIANCE HOLDS THEN THE MAGNITUDE OF THE ASYMMETRY AND THE POLARIZATION SHOULD BE NUMERICALLY EQUAL. THIS IS FOUND TO BE TRUE EXPERIMENTALLY TO ABOUT 1.5% ACCURACY (ABASHIAN & HAFNER, P.R.L. 1, 255 (1958).

7. SPIN CORRELATION TESTS IN THE ELECTROMAGNETIC INTERACTION.

ROCK ET AL P.R.L. 24, 748 (1970) STUDIED THE REACTION $e + P \to \Delta^+ + e$ USING A POLARIZED PROTON TARGET. AS THIS IS AN "INELASTIC" SCATTERING PROCESS WITH $M_P \neq M_\Delta$, IT TURNS OUT THAT A PIECE OF THE MATRIX ELEMENT LIKE $\bar{\sigma}_P \cdot \bar{P}_{i_e} \times \bar{P}_{f_e}$ WOULD BE T VIOLATING. NO SUCH ASYMMETRY WAS FOUND IN THE CROSS SECTION, TO ACCURACY 1.3%.

8. TESTS IN PARTICLE DECAYS VIA THE WEAK INTERACTION.

A STRIKING RESULT IS THAT THE $K^0$ DECAYS WHICH VIOLATE THE CP SYMMETRY ALSO VIOLATE T INVARIANCE, WHILE PRESERVING CPT. THE T VIOLATION IS SMALL, $\sim 10^{-3}$, AND HAS NOT BEEN OBSERVED IN ANY OTHER WEAK INTERACTION. THE DEMONSTRATION OF THE T VIOLATION IS SOMEWHAT INTRICATE, AND WILL BE DISCUSSED LATER IN THE COURSE.

THE MOST SENSITIVE SEARCH FOR T VIOLATION IN A WEAK INTERACTION OTHER THAN $K^0$ DECAY, SEEMS TO BE A PRINCETON WORK, BALTRUSAITIS & CALAPRICE, P.R.L. 38, 464 (1977). THEY LOOKED THE DECAY

$$Ne^{19} \to F^{19} + e + \nu_e$$

USING POLARIZED Ne NUCLEI. THE SEARCH WAS FOR A TERM $\bar{\sigma}_{Ne} \cdot \bar{P}_e \times \bar{P}_F$, WHICH IS T VIOLATING AS BOTH MOMENTA ARE IN THE FINAL STATE. THEIR LIMIT ON T VIOLATION IS $1 \times 10^{-3}$ IN THIS REACTION.

9. ELECTRIC DIPOLE MOMENTS.

A FINAL EXAMPLE CONCERNS POSSIBLE ELECTRIC DIPOLE MOMENTS OF ELEMENTARY PARTICLES. THE ARGUMENT HERE HAS A PECULIAR WRINKLE. A DIPOLE MOMENT IS A SPATIAL VECTOR $\bar{d}$ ASSOCIATED WITH A PARTICLE, WHICH LEADS TO AN INTERACTION WITH ELECTRIC FIELDS OF THE FORM $\bar{d} \cdot \bar{E}$. IF $\bar{d}$ IS AN ORDINARY VECTOR THIS CONSERVES BOTH T AND P

$$T(\bar{E}) = \bar{E} \quad \text{WHILE} \quad P(\bar{E}) = -\bar{E} \qquad T(\bar{d}) = d \qquad P(\bar{d}) = -\bar{d}$$

BUT THERE IS NO KNOWN ORDINARY VECTOR ASSOCIATED WITH AN ELEMENTARY PARTICLE AT REST. HOWEVER, THERE IS THE SPIN VECTOR $\bar{\sigma}$,

AN AXIAL VECTOR. SO PEOPLE (STARTING WITH PURCELL I BELIEVE) WERE LED TO ASSUME THAT IF A PARTICLE HAS AN ELECTRIC DIPOLE MOMENT, THEN

$$\bar{d} \sim \epsilon \vec{s} \qquad (\epsilon \text{ SMALL})$$

THIS WOULD VIOLATE BOTH T AND P INVARIANCE!

EXPERIMENTALLY ONE CAN SEARCH FOR ELECTRIC DIPOLE MOMENTS WITHOUT WORRYING ABOUT THIS SOMEWHAT STRANGE INTERPRETATION. THE EXPERIMENTS ARE DONE AT EXTREMELY LOW ENERGY $\sim .001$ eV! RECENT RESULTS ARE

| | d IN e-cm | |
|---|---|---|
| NEUTRON | $< 6 \times 10^{-25}$ | ALTAREV ET AL PHYS. LETT. B102, 13 (1981) |
| PROTON | $< 7 \times 10^{-21}$ | HARRISON ET AL P.R.L. 22, 1263 (1969) |
| ELECTRON | $< 3 \times 10^{-24}$ | WEISSKOPF ET AL P.R.L. 21, 1649 (1968) |

SINCE THE SIZE OF THE NEUTRON IS $\sim 10^{-13}$ cm, THE RESULT SAYS THAT THE NEUTRON CHARGE DISTRIBUTION IS SPHERICAL TO 1 PART IN $10^{11}$.

THE ONLY KNOWN INTERACTION WHICH VIOLATES BOTH T AND P IS THAT WHICH PRODUCES THE CP VIOLATION IN K DECAYS. ESTIMATES OF HOW THIS MIGHT APPLY TO THE NEUTRON DIPOLE MOMENT SUGGEST

$$d \sim 10^{-31} \text{ e-cm}$$

, QUITE SMALL INDEED.

## PARITY

STUDIES OF ATOMIC SPECTROSCOPY IN THE 1920'S LED TO LAPORTE'S RULE: ATOMIC TRANSITIONS VIA ELECTRIC DIPOLE RADIATION CAN ONLY OCCUR BETWEEN STATES OF ORBITAL ANGULAR MOMENTUM DIFFERING BY 1. WIGNER (1927) INTERPRETED THIS IN A MORE FUNDAMENTAL WAY. A STATE OF ORBITAL ANGULAR MOMENTUM $\ell$ IS AN EIGENSTATE OF THE PARITY OR SPACE INVERSION TRANSFORMATION, P DEFINED BY $P(\vec{r}) = -\vec{r}$. THIS FOLLOWS FROM THE ANGULAR BEHAVIOR OF THE WAVE FUNCTION $Y_\ell^M(\theta,\varphi)$:

$$P\left[Y_\ell^M(\theta,\varphi)\right] = Y_\ell^M(\pi-\theta, \varphi+\pi) = (-1)^\ell\, Y_\ell^M(\theta,\varphi)$$

WE LABEL THE EIGENVALUE AS $P = (-1)^\ell$ AND CALL IT THE PARITY OF THE STATE.

WIGNER THEN SURMISED THAT PARITY IS CONSERVED IN FUNDAMENTAL INTERACTIONS. THIS IS CONSISTENT WITH LAPORTE'S RULE IF WE ASSIGN $P = -1$ TO AN ELECTRIC DIPOLE PHOTON.

WIGNER'S IDEA WAS THE FIRST INDICATION OF HOW DISCRETE SYMMETRY TRANSFORMATIONS PROVIDE SIMPLIFYING INSIGHT INTO THE MICROWORLD. NOTE THAT AS WELL AS ASCRIBING A PARITY QUANTUM NUMBER TO A SPATIALLY EXTENDED SYSTEM SUCH AS AN ATOM, WIGNER ALLOWS THE POSSIBILITY THAT AN ELEMENTARY PARTICLE MIGHT HAVE AN <u>INTRINSIC PARITY</u>. THAT IS, THE PARTICLE'S WAVE FUNCTION MIGHT CHANGE SIGN UNDER THE PARITY TRANSFORMATION!

$P(\psi) = \pm\psi$. SINCE $P^2\psi = \psi$, THE POSSIBLE PARITY EIGENVALUES ARE JUST +1 AND -1.

WIGNER'S CONJECTURE THAT PARITY IS CONSERVED SEEMED VERY REASONABLE AT THE TIME. IT IS TRUE THAT THERE ARE CERTAIN LEFT-RIGHT ASYMMETRIES IN DAILY LIFE, BUT THEY SEEM 'ACCIDENTAL' RATHER THAN FUNDAMENTAL. (CONVINCE YOURSELF THAT SPACE INVERSION IS EQUIVALENT TO A LEFT-RIGHT 'MIRROR' REFLECTION, FOLLOWED BY A 180° ROTATION ABOUT AN AXIS $\perp$ TO THE MIRROR.) CONSIDER A CAR AND ITS PARITY INVERSION AS SHOWN. WHEN THE PARITY-INVERTED

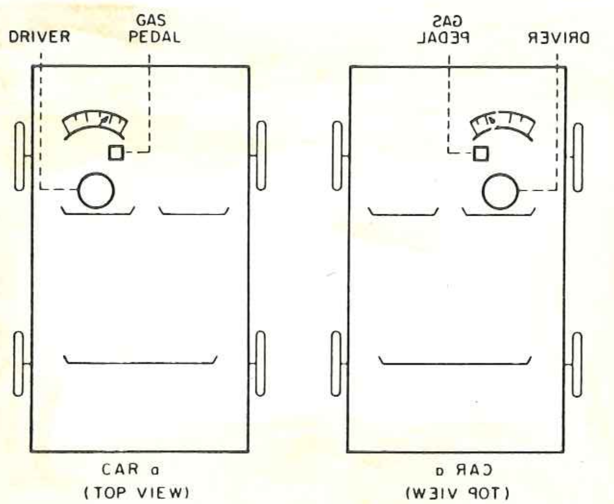



Fig. 9.2. Top view of two cars that are mirror images of each other.

PERSON STEPS ON THE GAS, WHICH WAY WILL THE P-CAR GO?

COMMON SENSE (BASED PRIMARILY ON EXPERIENCE WITH ELECTROMAGNETISM) SUGGESTS THE CAR WILL GO FORWARD. BUT IN FACT, IF THE ENGINE WERE BASED ON THE WEAK INTERACTION THE CAR WOULD GO BACKWARDS!

DESPITE THE PARITY VIOLATION OF THE WEAK INTERACTION, THE CONCEPT OF PARITY IS EXTREMELY HELPFUL IN OUR TASK OF UNDERSTANDING THE STRONG AND ELECTROMAGNETIC INTERACTIONS.

THE NOTION THAT AN ELEMENTARY PARTICLE HAS AN INTRINSIC PARITY SEEMS TO HAVE BEEN INTRODUCED BY KEMMER, PROC. ROY. SOC. LONDON <u>A166</u>, 127 (1938) AFTER NOTING THAT YUKAWA'S THEORY OF NUCLEAR FORCES WOULDN'T WORK FOR A "SCALAR" MESON, BUT COULD WORK FOR A "PSEUDOSCALAR" ONE.

# PH 529 LECTURE 9

## 1. PARITY OF THE PHOTON AND OF MULTIPOLE RADIATION.

AS INDICATED ON P 110 THE PHOTON WAVE FUNCTION CORRESPONDS TO A QUANTUM OF THE ELECTROMAGNETIC 4-POTENTIAL $\phi_\mu = (\phi, \bar{A})$. BUT BECAUSE OF GAUGE INVARIANCE AND THE LORENTZ CONDITION, ALL INFORMATION ABOUT THE PHOTON IS ACTUALLY CONTAINED IN THE TRANSVERSE PART OF THE VECTOR POTENTIAL $\bar{A}$. THEN AS $P[\bar{A}(\bar{r})] = -\bar{A}(-\bar{r})$, WE INFER THAT A PHOTON HAS NEGATIVE INTRINSIC PARITY

$$P_\gamma = -1$$

WHEN A PHOTON INTERACTS WITH A SYSTEM OF CHARGES IT WILL HAVE SOME ORBITAL ANGULAR MOMENTUM $l$. IN THIS SITUATION IT IS MORE PROPER TO SAY THAT THE PHOTON CARRIES PARITY $P = -(-1)^l = (-1)^{l+1}$, TAKING INTO ACCOUNT BOTH INTRINSIC AND ORBITAL ASPECTS OF PARITY.

THE TOTAL ANGULAR MOMENTUM OF AN INTERACTING PHOTON IS $\bar{J} = \bar{L} + \bar{S}$, WHERE $|\bar{S}| = 1$. THE TOTAL ANGULAR MOMENTUM QUANTUM NUMBER IS THEN $j = l$ OR $l \pm 1$. WE SAY THAT $j$ LABELS THE ORDER OF THE <u>MULTIPOLE</u>. CONVERSELY, A GIVEN MULTIPOLE $j$ HAS VARIOUS POSSIBLE ORBITAL ANGULAR MOMENTA $l = j, j \pm 1$

IF $l = j \pm 1$ WE HAVE THE <u>ELECTRIC MULTIPOLE</u> OF PARITY $P = (-1)^j$

IF $l = j$ WE HAVE THE <u>MAGNETIC</u> MULTIPOLE OF PARITY $P = (-1)^{j+1}$

LAPORTE'S RULE WAS BASED ON E1 TRANSITIONS $\Rightarrow j = 1$, $P = -1$.

THE ANGULAR DISTRIBUTION OF THE RADIATION IS GOVERNED BY THE APPROPRIATE SPHERICAL HARMONIC $Y_l^M$. THE COMPLETE BOOKKEEPING OF THE 'VECTOR' MULTIPOLE EXPANSION IS QUITE COMPLICATED, AND WE PROBABLY WON'T CONSIDER IT FURTHER IN THIS COURSE.

## 2. INTRINSIC PARITY OF THE PROTON, NEUTRON AND ELECTRON.

IT IS A MATTER OF CONVENTION THAT WE ASSIGN POSITIVE PARITIES TO BOTH THE PROTON AND THE NEUTRON. SINCE $p \not\to n + \gamma$ WE CANNOT USE OUR RADIATION RULES TO CHECK THE RELATIVE PARITY. THIS CONVENTION IS CERTAINLY CONSISTENT WITH THE QUARK MODEL VIEW THAT $p = uud$ AND $n = udd$ IN THE SAME SPATIAL CONFIGURATION.

ONCE THIS CONVENTION IS ESTABLISHED WE MAY DETERMINE THE INTRINSIC PARITIES OF OTHER PARTICLES WHICH CAN BE PRODUCED IN P AND N INTERACTIONS. A SIMPLE EXAMPLE IS THE DEUTERON. THIS IS A $^3S_1$ BOUND STATE OF THE PN SYSTEM, SO IT HAS EVEN PARITY, $P_d = 1$. (IF THE SPIN 1 DEUTERON HAD BEEN $^1P_1$ INSTEAD, THE PARITY WOULD HAVE BEEN ODD.)

THERE IS NO KNOWN DIRECT PARITY-CONSERVING TRANSITION BETWEEN A PROTON OR NEUTRON AND AN ELECTRON, SO WE HAVE NO CLEAR RULE FOR THE ELECTRON PARITY. INTRINSIC PARITIES ARE USEFULLY DEFINED ONLY FOR THE HADRONS, AND THE PHOTON.

## 3. PARITY OF THE $\pi^{\pm}$ MESONS.

THE INTRINSIC PARITY OF THE $\pi^-$ MESON, RELATIVE TO THAT OF THE NUCLEON, WAS INFERRED FROM THE FACT THAT THE REACTION

$$\pi^- + d \rightarrow n + n$$

OCCURS FOR PIONS CAPTURED IN AN S WAVE STATE ABOUT THE DEUTERON [PANOFSKY ET AL P.R. 81, 565 (1951).] IF SO, THE $\pi^-$ HAS THE SAME PARITY AS THE $nn$ FINAL STATE, AS THE DEUTERON HAS EVEN PARITY.

THE $nn$ PARITY DEPENDS ON THE ORBITAL ANGULAR MOMENTUM OF THE $nn$ SYSTEM. THE TOTAL ANGULAR MOMENTUM IS $J=1$, INFERRED FROM THE INITIAL STATE. HENCE THE $nn$ SYSTEM MIGHT BE IN ANY OF THE FOLLOWING STATES: ${}^3S_1$, ${}^1P_1$, ${}^3P_1$ OR ${}^3D_1$. HOWEVER THE FINAL STATE CONSISTS OF 2 IDENTICAL FERMIONS, AND SO MUST BE ANTISYMMETRIC UNDER THEIR INTERCHANGE (PAULI PRINCIPLE) THE TRIPLET SPIN STATE OF 2 SPIN 1/2 PARTICLES IS SYMMETRIC, WHILE THE SINGLET STATE IS ANTISYMMETRIC. COMBINING THE SPIN AND ORBITAL EXCHANGE SYMMETRIES TO GIVE THE OVERALL SYMMETRY, WE INFER THAT ONLY THE ${}^3P_1$ IS ANTISYMMETRIC. THIS STATE HAS ODD PARITY, AND SO DOES THE $\pi^-$.

LIKEWISE THE REACTION $\pi^+ + d \rightarrow p + p$ IS OBSERVED TO TAKE PLACE AT LOW ENERGIES, WITH THE $\pi^+ d$ SYSTEM IN AN S WAVE (BRUECKNER ET AL P.R. 81, 575 (1951)) SO WE CONCLUDE THAT THE $\pi^+$ HAS ODD PARITY ALSO.

## 4. PARITY OF A PARTICLE-ANTIPARTICLE PAIR.

FROM THE PRECEDING WE WOULD EXPECT THAT WE COULD ASSIGN A PARITY TO A PARTICLE-ANTIPARTICLE STATE SIMPLY ACCORDING TO THE ORBITAL ANGULAR MOMENTUM: $P = (-1)^{\ell}$. THIS IS FINE FOR BOSONS, SUCH AS THE $\pi^+$ AND $\pi^-$ WHICH WERE FOUND TO HAVE THE SAME INTRINSIC PARITIES.

BUT FOR A FERMION-ANTIFERMION PAIR, THE RULE TURNS OUT TO BE $P = (-1)^{\ell+1}$, AS SHOWN BELOW. WE INTERPRET THIS BY SAYING THAT AN ANTIFERMION HAS THE OPPOSITE INTRINSIC PARITY TO ITS FERMION PARTNER. WE FIRST NOTE THAT WE DON'T HAVE TO WORRY ABOUT THE SPIN PART OF THE FERMION-ANTIFERMION WAVE FUNCTION, AS THE PARITY OPERATION DOES NOT CHANGE SPINS

$$\begin{array}{ccc} \uparrow a & & \bar{a}\downarrow \\ & \xrightarrow{P} & \\ \bar{a}\downarrow & & \uparrow a \end{array}$$

WE WILL DEMONSTRATE THE RULE FOR SPIN 1/2 PARTICLES BY CONSIDERING HELICITY SPINORS. RECALL THAT THE PARITY OPERATION REVERSES HELICITY: $P(\vec{\sigma}\cdot\vec{p}) = -\vec{\sigma}\cdot\vec{p}$. SO WE EXPECT

$$P\left[u_\pm(\theta,\varphi)\right] = u_\mp(\pi-\theta,\ \varphi+\pi)$$

FOR THE SPINORS INTRODUCED ON PP 115-116. THE PARITY OPERATOR ON THE SPINOR ITSELF MUST BE A 4x4 MATRIX $\Gamma$, WHILE THE TRANSFORMATION $\vec{r}\to-\vec{r}$, $\vec{p}\to-\vec{p}$ AFFECTS THE PLANE WAVE FACTOR $e^{-ipx}$.

IN THE HIGH ENERGY LIMIT WE HAVE

$$u_+(\theta,\varphi) \sim \begin{pmatrix} \cos\theta/2\ e^{-i\phi/2} \\ \sin\theta/2\ e^{i\phi/2} \\ \cos\theta/2\ e^{-i\phi/2} \\ \sin\theta/2\ e^{i\phi/2} \end{pmatrix} \qquad u_-(\theta,\varphi) \sim \begin{pmatrix} -\sin\theta/2\ e^{-i\phi/2} \\ \cos\theta/2\ e^{i\phi/2} \\ \sin\theta/2\ e^{-i\phi/2} \\ -\cos\theta/2\ e^{i\phi/2} \end{pmatrix}$$

$$\text{SO}\quad u_+(\pi-\theta,\varphi+\pi) \sim i\begin{pmatrix} -\sin\theta/2\ e^{-i\phi/2} \\ \cos\theta/2\ e^{i\phi/2} \\ -\sin\theta/2\ e^{-i\phi/2} \\ \cos\theta/2\ e^{i\phi/2} \end{pmatrix} \qquad u_-(\theta,\varphi) \sim i\begin{pmatrix} \cos\theta/2\ e^{-i\phi/2} \\ \sin\theta/2\ e^{i\phi/2} \\ -\cos\theta/2\ e^{-i\phi/2} \\ -\sin\theta/2\ e^{i\phi/2} \end{pmatrix}$$

HENCE $\Gamma = i\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = i\gamma_0$ DOES THE JOB.

WE CAN VERIFY THAT THE DIRAC EQUATION IS INVARIANT UNDER THE PARITY TRANSFORMATION

CERTAINLY $P\left[i\gamma_\mu\partial^\mu\psi\right] = mP\psi$. WE WANT $i\gamma_\mu\partial^\mu P\psi = mP\psi$

NOW $P\,i\gamma_\mu\partial^\mu\psi = iP\gamma_\mu P^{-1}P\partial^\mu P^{-1}P\psi$

AND $P\gamma_\mu P^{-1} = \Gamma\gamma_\mu\Gamma^{-1} = i\gamma_0\gamma_\mu(-i\gamma_0) = (\gamma_0, -\vec{\gamma})$ $\qquad \left[\begin{matrix}\gamma_0^2 = 1 \\ \gamma_0\gamma_\mu = -\gamma_\mu\gamma_0\end{matrix}\right]$

WHILE $P\partial_\mu P^{-1} = P\left(\frac{\partial}{\partial t}, -\frac{\partial}{\partial \vec{r}}\right)P^{-1} = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial \vec{r}}\right)$

THUS $P\gamma_\mu\partial^\mu P^{-1} = \gamma_0\frac{\partial}{\partial t} + \vec{\gamma}\cdot\frac{\partial}{\partial \vec{r}} = \gamma_\mu\partial^\mu$ AS DESIRED.

FINALLY WE APPLY $P$ TO THE ANTI PARTICLE HELICITY SPINORS:

$$v_+(\theta,\varphi) \sim \begin{pmatrix} -\sin\theta/2\ e^{-i\phi/2} \\ \cos\theta/2\ e^{i\phi/2} \\ -\sin\theta/2\ e^{-i\phi/2} \\ \cos\theta/2\ e^{i\phi/2} \end{pmatrix} \qquad v_-(\theta,\varphi) \sim \begin{pmatrix} -\cos\theta/2\ e^{-i\phi/2} \\ -\sin\theta/2\ e^{i\phi/2} \\ \cos\theta/2\ e^{-i\phi/2} \\ \sin\theta/2\ e^{i\phi/2} \end{pmatrix}$$

YOU CAN VERIFY THAT $i\gamma_0 v_\pm(\theta,\varphi) = -v_\mp(\pi-\theta, \varphi+\pi)$

THAT IS, THE ANTI PARTICLE HAS OPPOSITE INTRINSIC PARITY FROM THE PARTICLE SPINOR. IN THE CASE OF THE ELECTRON, ITS INTRINSIC PARITY IS NOT WELL DEFINED, BUT THE POSITRON HAS OPPOSITE INTRINSIC PARITY. IN AN $e^+e^-$ STATE, THE OVERALL UNCERTAINTY IN PARITY VANISHES, AND $P = (-1)^{l+1}$!

AS A FOOTNOTE, $P\gamma_5 P^{-1} = i\gamma_0\gamma_5(-i\gamma_0) = -\gamma_5$, WHICH GIVES FORMAL VERIFICATION THAT $\gamma_5$ IS A PSEUDOSCALAR OPERATOR.

## 5. PARTICLE DECAY TO 2 PHOTONS [YANG, P.R. 77, 242 (1950)]

PRIOR TO DISCUSSING THE PARITY OF THE $\pi^0$ MESON WE DERIVE SOME SURPRISING BUT MEMORABLE RESULTS VIA TRICKY ARGUMENTS.

CONSIDER A PARTICLE AT REST WHICH DECAYS TO 2 PHOTONS, WHICH EMERGE BACK TO BACK ALONG THE Z-AXIS


THE REAL PHOTONS CAN HAVE $S_z = \pm 1$ ONLY, SO THERE ARE 4 POSSIBLE COMBINED SPIN STATES: $|\uparrow\uparrow\rangle$ ($S_z = +2$), $|\downarrow\downarrow\rangle$ ($-2$), $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ ($0$) AND $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ ($0$)

OF COURSE, $L_z = 0$ IN THIS CONFIGURATION, SO $J_z = 0$ OR $\pm 2$ ALSO. THUS IF THE ORIGINAL PARTICLE HAS SPIN LESS THAN 2, IT MUST HAVE BEEN IN AN $S_z = 0$ STATE, AND DECAYED TO EITHER THE 3RD OR 4RTH $\gamma\gamma$ SPIN STATE.

THE INSIGHT OF YANG IS THAT ALL THE $S_z = 0$ STATES ARE EIGENSTATES OF THE ROTATION OPERATOR $R_x(180°)$ = ROTATION BY 180° ABOUT THE X AXIS. FOR THE PHOTONS, EITHER STATE $|\uparrow\downarrow\rangle$ OR $|\downarrow\uparrow\rangle$ APPEARS THE SAME AFTER THE ROTATION, SO THEIR EIGENVALUE IS +1. HOWEVER, THE WAVE FUNCTION OF A PARTICLE OF SPIN S AND $S_z = 0$ TRANSFORMS LIKE $Y_S^0(\theta,\varphi) \sim P_S(\cos\theta)$. FOR ODD S THIS CHANGES SIGN UNDER A 180° ROTATION ABOUT X, SO THE EIGENVALUE IS $(-1)^S$. HENCE THERE IS NO WAY AN ODD SPIN PARTICLE CAN DECAY TO THE $S_z = 0$ 2-PHOTON STATE. FOR A SPIN 1 PARTICLE THIS IS THE ONLY POTENTIALLY ACCESSIBLE STATE, SO NO DECAY IS POSSIBLE.

IT IS USEFUL TO NOTE THE PARITY OF THE 2-PHOTON STATES


SO $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$ AND $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ ALL HAVE EVEN PARITY WHILE $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ HAS ODD PARITY.

| PARITY | PARTICLE SPIN: 0 | 1 | 2N = EVEN | 2N+1 = ODD | N = 1, 2, 3 ... |
|---|---|---|---|---|---|
| EVEN | $\lvert\uparrow\downarrow\rangle + \lvert\downarrow\uparrow\rangle$ | — | $\lvert\uparrow\uparrow\rangle, \lvert\downarrow\downarrow\rangle, \lvert\uparrow\downarrow\rangle + \lvert\downarrow\uparrow\rangle$ | $\lvert\uparrow\uparrow\rangle, \lvert\downarrow\downarrow\rangle$ | |
| ODD | $\lvert\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\rangle$ | — | $\lvert\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\rangle$ | — | |

THERE IS A PICTURESQUE WAY OF REMEMBERING THESE RESULTS.

THE WAVE FUNCTION OF THE 2 PHOTON FINAL STATE DEPENDS LINEARLY ON EACH OF THE PHOTONS POLARIZATIONS $\bar{\epsilon}_1$ & $\bar{\epsilon}_2$ (WHICH ARE 3-VECTORS ALONG $\bar{E}$). IT MAY ALSO DEPEND ON THE PHOTON MOMENTUM $\bar{k}$ (IN C.M. FRAME). THE REAL PHOTONS HAVE TRANSVERSE POLARIZATION, SO $\bar{\epsilon}\cdot\bar{k} = 0$.

FOR A SPIN ZERO PARTICLE DECAY, THE FINAL STATE PHOTONS' WAVE FUNCTION MUST BE A SCALAR IF IT HAS EVEN PARITY, OR A PSEUDOSCALAR IF IT HAS ODD PARITY.

POSSIBLE SCALARS ARE $\bar{\epsilon}_1\cdot\bar{\epsilon}_2$ OR $(\bar{k}\cdot\bar{\epsilon}_1)(\bar{k}\cdot\bar{\epsilon}_2) \equiv 0$

THE ONLY POSSIBLE PSEUDOSCALAR IS $\bar{k}\cdot\bar{\epsilon}_1\times\bar{\epsilon}_2$

HENCE WE CONCLUDE THAT IF THE STATE HAS SPIN ZERO AND EVEN PARITY, THE PHOTON POLARIZATIONS LINE UP (TO MAKE $\bar{\epsilon}_1\cdot\bar{\epsilon}_2$ BIG), BUT FOR ODD PARITY THEY ARE AT RIGHT ANGLES


CAN YOU SHOW THAT IF WE CONVERT FROM LINEAR POLARIZATION TO CIRCULAR POLARIZATION THAT $\bar{\epsilon}_1\cdot\bar{\epsilon}_2 \leftrightarrow |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$, WHILE $\bar{k}\cdot\bar{\epsilon}_1\times\bar{\epsilon}_2 \leftrightarrow |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ ?

FOR A SPIN 1 PARTICLE DECAY, THE 2 PHOTON FINAL STATE WOULD HAVE TO BE SPIN 1 ALSO. THEREFORE THE 2 PHOTON WAVE FUNCTION MUST BEHAVE LIKE A VECTOR. ALL POSSIBLE VECTORS WHICH ARE BILINEAR IN $\bar{\epsilon}_1$ & $\bar{\epsilon}_2$ ARE:

$$\bar{\epsilon}_1\times\bar{\epsilon}_2 \qquad \bar{k}\,(\bar{\epsilon}_1\cdot\bar{\epsilon}_2) \qquad \bar{\epsilon}_1(\bar{k}\cdot\bar{\epsilon}_2)=0 \qquad \epsilon_2\,(\bar{k}\cdot\bar{\epsilon}_1)=0$$

$$\bar{k}\,(\bar{k}\cdot\bar{\epsilon}_1\times\bar{\epsilon}_2) \qquad (k_1\cdot\epsilon_1)(\bar{\epsilon}_2\times\bar{k})=0 \qquad (\bar{k}\cdot\bar{\epsilon}_2)(\bar{\epsilon}_1\times\bar{k})=0 \qquad \bar{k}(\bar{k}\cdot\bar{\epsilon}_1)(\bar{k}\cdot\bar{\epsilon}_2)=0$$

ALSO, AS THE PHOTONS ARE BOSONS, THE WAVE FUNCTION MUST BE SYMMETRIC UNDER EXCHANGE OF THE 2 PHOTONS: $\bar{\epsilon}_1\to\bar{\epsilon}_2$ ; $\bar{\epsilon}_2\to\bar{\epsilon}_1$ ; $\bar{k}\to-\bar{k}$

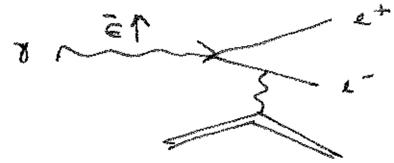
YOU CAN VERIFY THAT OUR SPIN ZERO EXAMPLES SATISFY THIS.

BUT ALL 3 REMAINING SPIN 1 WAVE FUNCTION CANDIDATES ARE ANTI-SYMMETRIC UNDER PHOTON INTERCHANGE. SO AGAIN, A SPIN 1 PARTICLE CANNOT DECAY TO 2 PHOTONS.

## 6. SPIN AND PARITY OF THE $\pi^0$ MESON.

THE DECAY $\pi^0\to\gamma\gamma$ WAS OBSERVED BY STEINBERGER ET AL, P.R. 78, 802 (1950). THE ABOVE ARGUMENTS THEN TELL US THAT THE $\pi^0$ SPIN IS 0, OR $\geq 2$. THE EARLY ARGUMENT THAT THE $\pi^0$ HAS SPIN 0 AND NOT $\geq 2$ IS BASICALLY THAT IT IS A PARTNER OF THE $\pi^+$ AND $\pi^-$ MESONS, NEEDED TO JUSTIFY THE CHARGE INDEPENDENCE OF THE NUCLEAR FORCE ($\sigma_{pp\to pp} = \sigma_{pn\to pn}$).

THE PARITY OF THE $\pi^0$ COULD BE ESTABLISHED BY OBSERVING THE POLARIZATION OF THE DECAY PHOTONS, AS MENTIONED IN SEC 5. NOW IF A PHOTON INTERACTS WITH A NUCLEUS SO AS TO PRODUCE AN $e^+e^-$ PAIR, THE PAIR TENDS TO LIE IN THE PLANE OF THE ELECTRIC POLARIZATION VECTOR $\vec{\varepsilon}$




APPARENTLY NO ONE HAS TRIED TO OBSERVE THE POLARIZATION BY CONVERTING THE PHOTONS IN EXTERNAL BLOCKS OF MATTER.

BUT THE $\pi^0$ HAS THE RARE DECAYS $\pi^0 \to \gamma e^+ e^-$ .012 OF THE TIME

$\pi^0 \to e^+e^- e^+e^-$ $3 \times 10^{-5}$ "

THESE ARE THE SO-CALLED 'DALITZ DECAYS', OR INTERNAL CONVERSION DECAYS. EACH CONVERSION COSTS A FACTOR $\sim e^2 = \alpha = 1/137$ IN THE TRANSITION RATE. HOWEVER THE PLANE OF EACH $e^+e^-$ PAIR PRESERVES SOME MEMORY OF THE PHOTON POLARIZATION $\vec{\varepsilon}$. PLANO ET AL, P.R.L. 3, 525 (1959) MEASURED THE DOUBLE DALITZ DECAY, AND INDEED CONFIRMED THAT THE $\pi^0$ HAS NEGATIVE PARITY

IN THE OLD FERMI-YANG MODEL THAT $\pi^0 \leftrightarrow p\bar{p}$ WE EXPECT THE $\pi^0$ TO HAVE NEGATIVE PARITY, AS IT CONSISTS OF A FERMION-ANTI-FERMION PAIR IN AN $\ell = 0$ STATE (SEC. 4).

## 7. PARITY OF K MESON AND $\Lambda$ HYPERON.

BEFORE 1956 THE SITUATION WAS CONFUSED. THE SO-CALLED $\theta^+$ MESON DECAYED TO $\pi^+\pi^0$, WHILE THE $\tau^+$ MESON DECAYED TO $\pi^+\pi^+\pi^-$. BUT THE MASSES OF THE $\theta$ AND $\tau$ SEEMED IDENTICAL. HOWEVER, IF THE BOTH HAVE SPIN ZERO, THE $\theta$ HAS EVEN PARITY WHILE THE $\tau$ HAS ODD PARITY, BASED ON THE DECAY MODES STATED. THE SO-CALLED DALITZ PLOT ANALYSIS (LECTURE 11) INDICATES THAT THE $\tau$ INDEED HAS SPIN ZERO.

MEANWHILE THE STRONG INTERACTION $\pi^- + p \to K^0 + \Lambda^0$ HAD BEEN OBSERVED, AND $M_K \sim M_\theta \sim M_\tau$.

THE $\Lambda^0$ HYPERON DECAYS TO $p + \pi^-$. BY OBSERVING THE DECAY ANGULAR DISTRIBUTION THE SPIN OF THE $\Lambda$ IS DETERMINED TO BE $1/2$. (HOMEWORK PROBLEM, PERKINS P112). BUT THIS ANALYSIS DOES NOT REVEAL WHETHER THE $p\pi^-$ IS IN AN S OR A P WAVE STATE, SO THE FINAL STATE PARITY WAS NOT KNOWN.

BUT BY THE MERE EXISTENCE OF THE DECAY $\Lambda \to p\pi^-$ WE CONCLUDE THAT IT IS A BARYON (3 QUARK STATE), AND BY CONVENTION WE ASSIGN IT POSITIVE PARITY.

THEN WE INFER THAT THE $K^0$ HAS NEGATIVE PARITY, IN THAT THE PRODUCTION REACTION TAKES PLACE AT LOW ENERGIES ⇒ S WAVE STATES DOMINATE.

HOWEVER, TO IDENTIFY THE K WITH BOTH THE $\Theta$ AND $\tau$ REQUIRES ACCEPTING A PARITY VIOLATION.

8. NON CONSERVATION OF PARITY.

IN 1956 LEE & YANG, P.R. 104, 254 (1956) NOTED THAT ONE ONLY HAD TO ACCEPT PARITY NON-CONSERVATION IN THE WEAK INTERACTION FOR A CONSISTENT PICTURE TO EMERGE. WE WILL STUDY THIS ASPECT OF THE WEAK INTERACTION IN GREATER DETAIL LATER. FOR NOW WE NOTE THAT SINCE 1956 NO EVIDENCE HAS BEEN FOUND FOR PARITY NON-CONSERVATION EXCEPT IN THE WEAK INTERACTION.

HOWEVER, TINY PARITY VIOLATING EFFECTS HAVE BEEN FOUND IN REACTIONS WHICH ARE OSTENSIBLY STRONG OR ELECTROMAGNETIC. THESE MAY BE INTERPRETED AS WEAK-INTERACTION CORRECTIONS TO THE OTHER 2 REACTIONS, WITHOUT CHANGING OUR CONCEPTION THAT THE LATTER ARE PARITY CONSERVING.

A REVIEW OF 20 OR SO EXPERIMENTS DEMONSTRATING WEAK-INTERACTION CORRECTIONS IS GIVEN IN CHAP. 9 OF THE BOOK BY COMMINS & BUCKSBAUM.