Why Doesn't a Picture Hang Straight?

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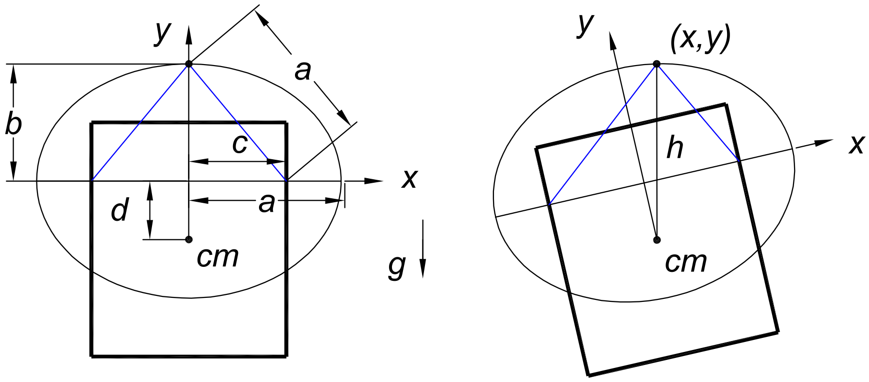
This problem was often posed by A. Shenstone on the Princeton U. Graduate Physics Preliminary Exams many years ago.

We take a picture to be a uniform rectangular plate of width $2c$ that is suspended from a single point by a wire of length $2a$. Then, when the picture is "hung straight," the point of support is distance $b = \sqrt{a^2 - c^2}$ above the horizontal line between the two ends of the wire, at the vertical edges of the picture. The center of mass of the picture is at distance $b + d$ below the support point.



If the wire slips with respect to the support point, the latter lies on an ellipse with semimajor axis a and semiminor axis b. The center of mass of the picture still lies directly below the support point, now at distance h from it. The system is unstable against such slippage if the center of mass falls as the wire slips, *i.e.*, if,

$$h > b + d, \tag{1}$$

Taking the new support point to be at (x, y) in a rectangular coordinate system with origin at the center of the ellipse, the equation of the latter is,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \tag{2}$$

So, for slippage to a new support point at small x, the new y of the support point is,

$$y = b\sqrt{1 - \frac{x^2}{a^2}} \approx b - \frac{bx^2}{2a^2}. \tag{3}$$

Then the distance h is related by,

$$h^2 = x^2 + (y + d)^2 \approx x^2 + \left(b + d - \frac{bx^2}{2a^2}\right)^2 \approx (b + d)^2 + x^2\left(1 - \frac{b(b + d)}{a^2}\right). \tag{4}$$

The system is stable only if the coefficient of x^2 in eq.(4) is positive, *i.e.*, only if,

$$a^2 - b^2 = c^2 > bd. \tag{5}$$

This condition is often not met in practice, particularly if b is small so that the support point is hidden from view behind the picture, and such pictures don't hang straight.