

# Pulses and $1/f$ Noise

Kirk T. McDonald

*Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544*

(January 24, 2015)

## 1 Problem

Discuss the relation between pulsed phenomena and  $1/f$  noise.

## 2 Solution

Low-frequency fluctuations with a  $1/f$  frequency spectrum were first noticed by Johnson [1] in thermionic-emission currents in vacuum tubes, and exist in a very wide range on phenomena.<sup>1</sup> A feature common to most of these phenomena is that they involve a quasirandom sequence of pulses on the microscopic scale.

The spectrum of energies associated with a short pulse of characteristic width  $\Delta t$  is roughly flat up to  $E_{\max} = \hbar/\Delta t$ . In general, the pulse process can be associated with quanta (such as phonons or photons) of energy  $\hbar\omega$ , so the number spectrum of these quanta varies as  $1/\omega$  up to  $\omega_{\max} = 1/\Delta t$ .

When a phenomenon consists of a sequence of closely spaced pulses, such that the macroscopic behavior is quasistatic, the microscopic behavior involve fluctuations with the frequency spectrum of the associated quanta, namely  $1/\omega$ , *i.e.*,  $1/f$ .

*While the author considers the above to be fairly “obvious”, the literature on  $1/f$  noise mainly involves much more arcane, and less satisfactory, explanations. The above view was advocated by Handel in 1975 [11, 12] (in the context of Bremsstrahlung emitted by conduction electrons in collisions with lattice ions), although it seems to be regarded with surprising (to this author) skepticism.<sup>2</sup>*

## References

- [1] J.B. Johnson, *The Schottky Effect in Low-Frequency Circuits*, Phys. Rev. **26**, 71 (1925), [kirkmcd.princeton.edu/examples/statistics/johnson\\_pr\\_26\\_71\\_25.pdf](http://kirkmcd.princeton.edu/examples/statistics/johnson_pr_26_71_25.pdf)
- [2] F.N. Hooge, *Discussion of Recent Experiments on  $1/f$  Noise*, Physica **60**, 130 (1972), [kirkmcd.princeton.edu/examples/statistics/hooge\\_physica\\_60\\_130\\_72.pdf](http://kirkmcd.princeton.edu/examples/statistics/hooge_physica_60_130_72.pdf)
- [3] W.H. Press, *Flicker Noises in Astronomy and Elsewhere*, Comm. Astro. **7**, 103 (1978), [kirkmcd.princeton.edu/examples/statistics/press\\_ca\\_7\\_103\\_78.pdf](http://kirkmcd.princeton.edu/examples/statistics/press_ca_7_103_78.pdf)
- [4] M. Gardner, *White and brown music, fractal curves and one-over- $f$  fluctuations*, Sci. Am. **238**, April, 16 (1978), [kirkmcd.princeton.edu/examples/statistics/gardner\\_sa\\_238\\_4\\_16\\_78.pdf](http://kirkmcd.princeton.edu/examples/statistics/gardner_sa_238_4_16_78.pdf)

---

<sup>1</sup>See, for example, [2]-[10].

<sup>2</sup>See, for example, sec. IIIC of [6]. Exceptions are [13, 14]. For a “fractal” view of  $1/f$  noise, see [7].

- [5] D.A. Bell, *A survey of  $1/f$  noise in electrical conductors*, J. Phys. C **13**, 4425 (1980), [kirkmcd.princeton.edu/examples/statistics/bell\\_jpc\\_13\\_4425\\_80.pdf](http://kirkmcd.princeton.edu/examples/statistics/bell_jpc_13_4425_80.pdf)
- [6] P. Dutta and P.M. Horn, *Low-frequency fluctuations in solids:  $1/f$  noise*, Rev. Mod. Phys. **53**, 497 (1981), [kirkmcd.princeton.edu/examples/statistics/dutta\\_rmp\\_53\\_497\\_81.pdf](http://kirkmcd.princeton.edu/examples/statistics/dutta_rmp_53_497_81.pdf)
- [7] P. Bak, C. Tang and K. Wiesenfeld, *Self-Organized Criticality: An Explanation of  $1/f$  Noise*, Phys. Rev. Lett. **59**, 381 (1987), [kirkmcd.princeton.edu/examples/statmech/bak\\_prl\\_59\\_381\\_87.pdf](http://kirkmcd.princeton.edu/examples/statmech/bak_prl_59_381_87.pdf)
- [8] M.B. Weissman,  *$1/f$  noise and other slow, nonexponential kinetics in condensed matter*, Rev. Mod. Phys. **60**, 537 (1988), [kirkmcd.princeton.edu/examples/statistics/weissman\\_rmp\\_60\\_537\\_88.pdf](http://kirkmcd.princeton.edu/examples/statistics/weissman_rmp_60_537_88.pdf)
- [9] E. Milotti,  *$1/f$  noise: a pedagogical review*, (Apr. 2002), <http://arxiv.org/abs/physics/0204033>
- [10] L.M. Ward and P.E. Greenwood,  *$1/f$  noise*, Scholarpedia **2**, 1537 (2007), [http://www.scholarpedia.org/article/1/f\\_noise](http://www.scholarpedia.org/article/1/f_noise)
- [11] P.H. Handel, *Quantum Theory of  $1/f$  Noise*, Phys. Lett. **53A**, 438 (1975), [kirkmcd.princeton.edu/examples/statistics/handel\\_pl\\_53a\\_438\\_75.pdf](http://kirkmcd.princeton.edu/examples/statistics/handel_pl_53a_438_75.pdf)
- [12] P.H. Handel, *Quantum approach to  $1/f$  noise*, Phys. Rev. A **22**, 745 (1980), [kirkmcd.princeton.edu/examples/statistics/handel\\_pra\\_22\\_745\\_80.pdf](http://kirkmcd.princeton.edu/examples/statistics/handel_pra_22_745_80.pdf)
- [13] A. Van der Ziel, *Unified Presentation of  $1/f$  Noise in Electronic Devices: Fundamental  $1/f$  Noise Sources*, Proc. IEEE **76**(3), 233 (1988), [kirkmcd.princeton.edu/examples/statistics/vanderziel\\_pieee\\_76\\_3\\_233\\_88.pdf](http://kirkmcd.princeton.edu/examples/statistics/vanderziel_pieee_76_3_233_88.pdf)
- [14] C.M. Van Vliet, *A Survey of Results and Future Prospects on Quantum  $1/f$  and  $1/f$  Noise in General*, Solid State Elec. **34**, 1 (1991), [kirkmcd.princeton.edu/examples/statistics/vanvliet\\_sse\\_34\\_1\\_91.pdf](http://kirkmcd.princeton.edu/examples/statistics/vanvliet_sse_34_1_91.pdf)