Wedgefall

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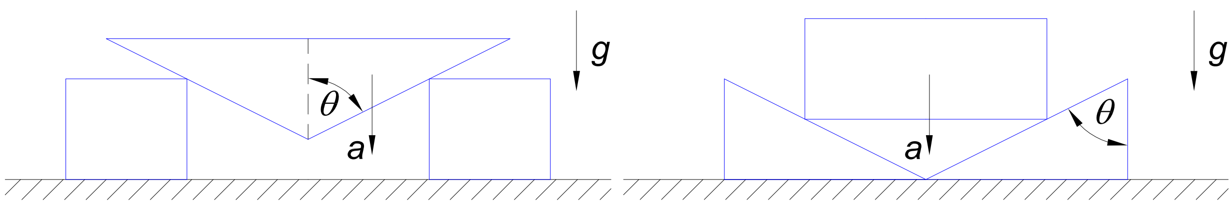
1 Problem

Deduce the vertical acceleration a of a wedge of vertical angle θ and mass $2m$ that is in contact with two rectangular blocks of mass m each, as shown in the left sketch below, assuming no friction anywhere. As the wedge falls vertically, the blocks are accelerated horizontally.



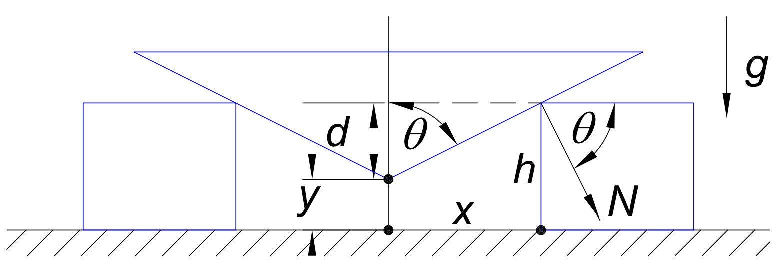
Also, what is the vertical acceleration a of a block of mass $2m$ that is in contact with two wedges of mass m and vertical angle θ, as shown in the right sketch below, again assuming no friction anywhere?

2 Solution

This problem is related to the well-known example of a rectangular block that slides on a wedge that slides on a horizontal plane, without friction. *A more complex variation has been posed at* http://kirkmcd.princeton.edu/examples/mechanics/korsunsky_pt_50_441_12.pdf

2.1 Falling Wedge

The key to this problem is the establishment of the kinematic constraint between the vertical motion of the wedge, with acceleration $a = a_y$ (positive downwards), and the horizontal motion of the blocks, with acceleration $\pm a_x$ (noting that horizontal momentum is conserved in this problem).



For this, we relate the vertical position y of the bottom tip of the wedge to the horizontal position x of the left edge of the right block relative to the symmetry axis $x = 0$, as shown in the figure above,

$$y + d = h, \qquad \frac{x}{d} = \tan\theta, \qquad y + \frac{x}{\tan\theta} = h, \qquad a_y = \frac{a_x}{\tan\theta}\,, \tag{1}$$

since a_x is the acceleration of x while a_y is minus the acceleration of y.

To apply Newton's 2nd law to this problem, we note that the force of contact $\mathbf{N}$ between the wedge and the block is normal to the surface of the wedge, and hence makes angle θ to the horizontal as shown above. The horizontal equation of motion of the right block is then,

$$N\cos\theta = ma_x = ma_y \tan\theta, \tag{2}$$

while the vertical equation of motion of the wedge is,

$$2mg - 2N\sin\theta = 2ma_y. \tag{3}$$

Thus,

$$mg = N\sin\theta + ma_y = m\tan^2\theta\, a_y + ma_y, \tag{4}$$

$$a_y = \frac{g}{1+\tan^2\theta} = g\cos^2\theta. \tag{5}$$

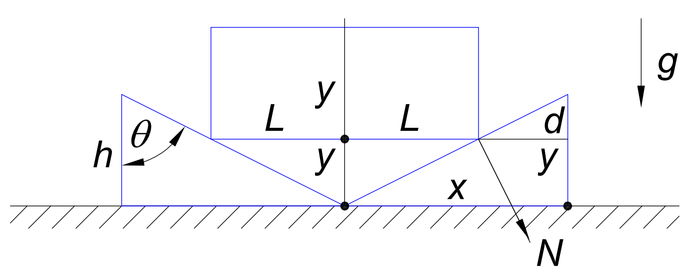
The limiting cases are $\theta = 0$ in which case the wedge is a narrow block that falls between the wall and the other block with $a_y = g$, and $\theta = 90^\circ$ in which case the wedge is a block that rests on the other block without moving ($a_y = 0$).

2.2 Falling Block

In this case, we relate the vertical position y of the bottom of the block (of total length $2L$) to the horizontal position x of the right edge of the right wedge relative to the symmetry axis, as shown in the figure below,

$$y = h - d, \qquad \frac{x-L}{d} = \tan\theta, \qquad y = h - \frac{x-L}{\tan\theta}\,, \qquad a_y = \frac{a_x}{\tan\theta}\,, \tag{6}$$

since a_x is the acceleration of x while a_y is minus the acceleration of y.



To apply Newton's 2nd law to this problem, we note that the force of contact $\mathbf{N}$ between the wedge and the block is normal to the surface of the wedge, and hence makes angle θ to the horizontal as shown above. The horizontal equation of motion of the right wedge is then,

$$N\cos\theta = ma_x = ma_y \tan\theta, \tag{7}$$

while the vertical equation of motion of the block is,

$$2mg - 2N\sin\theta = 2ma_y. \tag{8}$$

Thus,

$$mg = N\sin\theta + ma_y = m\tan^2\theta\, a_y + ma_y, \tag{9}$$

$$a_y = \frac{g}{1+\tan^2\theta} = g\cos^2\theta, \tag{10}$$

as for the case of a wedge falling between two blocks.