

Properties of High-Resistivity Carbon-Loaded Kapton

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Abstract

The results of uniformity tests on high-resistivity carbon-loaded kapton are presented.

1 Introduction

The electric field in the drift region of a time-projection chamber (TPC) is usually created by a set of conducting strips running perpendicular to the drift direction and held at regularly increasing potentials [1]. In the prototype of the low-pressure TPC designed for testing the ionization cooling of muons [2], the drift electric field will be created by covering the walls of the cylindrical drift region with carbon-loaded kapton [3] of approximately $1.4 \text{ M}\Omega/\square$ and applying a high voltage to the edge opposite from the wire grid.

If a voltage is applied across a resistive sheet folded into a cylinder, where z denotes the drift direction, and ϕ denotes the azimuthal direction, variations in the sheet's resistivity which depend only on ϕ will not affect the uniformity of the electric field inside the cylinder. On the other hand, if the resistivity varies as a function of z , or of both z and ϕ , the electric field inside the cylinder will not be uniform. Thus, in selecting the piece of resistive film, it is important to maximize the uniformity of the resistivity along the drift direction.

This note describes the results of uniformity tests performed on a sample of carbon-loaded kapton.

2 Measurement Setup

Fig. 1 shows the $V - I$ curves of samples of carbon-loaded kapton from Roll No. 1788 (left), whose resistivity was measured to be approximately $1.4 \text{ M}\Omega/\square$, and Roll No. 2356 (right), whose dynamic resistivity was measured to be approximately $100 \text{ M}\Omega/\square$ at 50 V. The $V - I$ curves were obtained by applying a voltage across a square piece of film mounted on Teflon PTFE and measuring the resulting current with a Keithley 617 Programmable Electrometer [4].

While water condensation on the film may be responsible for the nonlinearity in the $V - I$ curves, the measurements for Roll No. 1788, which was chosen for further study based on preliminary tests, display only slight curvature. The observation of different resistivities for the two samples, which presumably have the same affinity to water, suggests that the measurements are dominated by the volume resistivity of the film, and not by the surface conductivity of residues or water condensation.

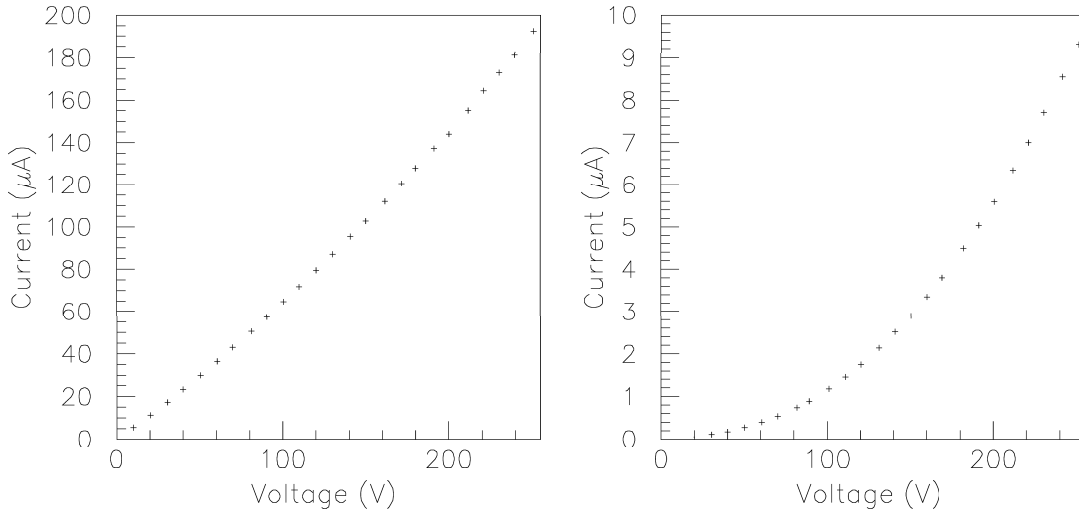


Figure 1: $V - I$ curves of square pieces of carbon-loaded kapton from Roll No. 1788 (left) and Roll No. 2356 (right). Roll No. 1788 was chosen for testing.

The tests were performed on two 6"-wide strips, referred to as strips A and B, which were cut from the roll as shown in Fig. 2. A third piece, referred to as strip C, was used to estimate the measurement error. The testing consisted in applying a high voltage across each strip as shown in Fig. 3 and measuring the voltage at a given point with a 1/1000 voltage probe connected to a Hewlett Packard 34401A Multimeter [5]. Voltages were recorded to 0.1 V accuracy. To avoid leaving marks on the kapton film, a probe with a rounded tip was chosen. The strips were mounted on paper with a

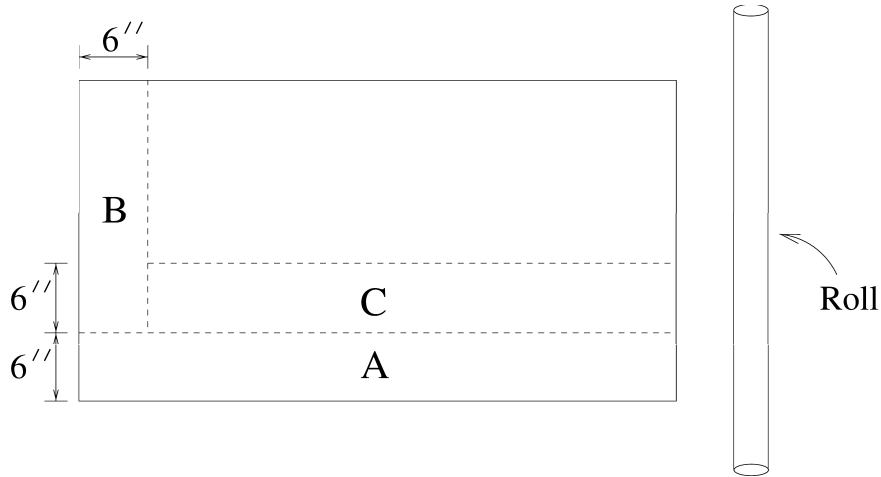


Figure 2: The original location, on Roll No.1788, of strips A and B, which were tested for uniformity, and of strip C, which was used to estimate the error in the voltage measurements.

resistivity which was measured with a Keithley 617 Programmable Electrometer [4] to be approximately $18,000 \text{ M}\Omega/\square$. By comparing the voltages measured with the film mounted on paper to those measured with the film mounted on Teflon PTFE for a piece of film of dimensions $(13 \times 4) \text{ cm}^2$, it was confirmed that the conductivity of the paper did not influence the voltages recorded.

The x -axis was marked on the mounting paper, approximately 3 cm from each edge of the film, and the y -axis was determined by laying an insulating ruler across the film. The total uncertainty in one voltage measurement was estimated to be $(0.075 \pm 0.005) \text{ V}$, as discussed in detail in the appendix.

It was found that fingerprints on the kapton could be removed by wiping the kapton with a piece of cotton dipped in ethanol, and allowing the film to dry overnight. Although the ethanol left marks on the kapton, neither the voltage measurements nor the resistivity were found to be affected within measurement error.



Figure 3: The application of high voltage and the set of axes used in testing the strips of carbon-loaded kapton.

3 Results

3.1 Strip A, Uniformity as a Function of x

The first test performed on Strip A consisted in scanning the x -axis in 1-cm intervals for a constant value of y , at a total of 76 points for $3 \text{ cm} \leq x \leq 78 \text{ cm}$. Separate scans were made for $y = 4 \text{ cm}$, 7 cm , and 11 cm . Since measurements were made by holding the ruler parallel to the y -axis, the ruler was moved for each value of x . The results for $y = 4 \text{ cm}$ are shown in Fig. 4.

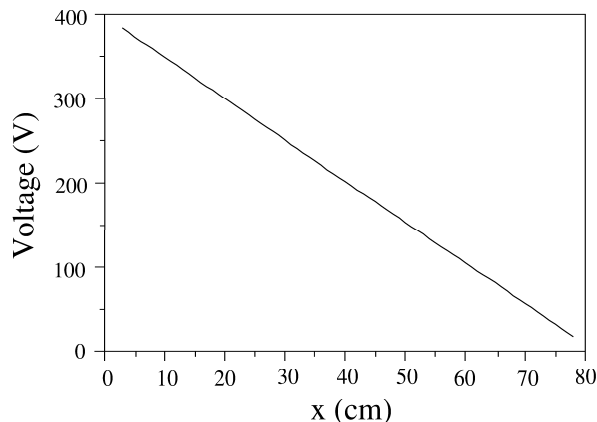


Figure 4: Measured voltages on strip A as a function of x , for $y = 4 \text{ cm}$. A voltage of $(400.2 \pm 0.2) \text{ V}$ was applied across 81 cm of film.

When the error in one voltage measurement is estimated as outlined in the appendix to be $(0.075 \pm 0.005) \text{ V}$, straight-line fits of the measured voltages for $y = 4, 7$, and 11 cm yield $\chi^2/\text{d.o.f.}$ values of 76.47, 52.34, and 35.20 respectively, indicating that the data contain further structure. The residuals of the straight-line fits are plotted in Fig. 5. In each case, as confirmed by the Fourier transform analyses of the residuals which are also shown, the measured voltage appears to oscillate about the straight line with an amplitude of about 1 V and a period of about 50 cm. Superposed on this oscillation is a suggestion of a smaller oscillation, with an amplitude of about 0.5 V, and a period of about 5 cm.

When the residuals are fit to a sinusoid, $\chi^2/\text{d.o.f.}$ values of 30.46, 21.92, and 13.66 are obtained for $y = 4, 7$, and 11 cm respectively, suggesting yet further structure. Fitting to superpositions of two and three sinusoids, however, decrease $\chi^2/\text{d.o.f.}$ values only slightly, as shown in Fig. 6 and Table 1. It thus appears that the residuals are described by more complicated functions.

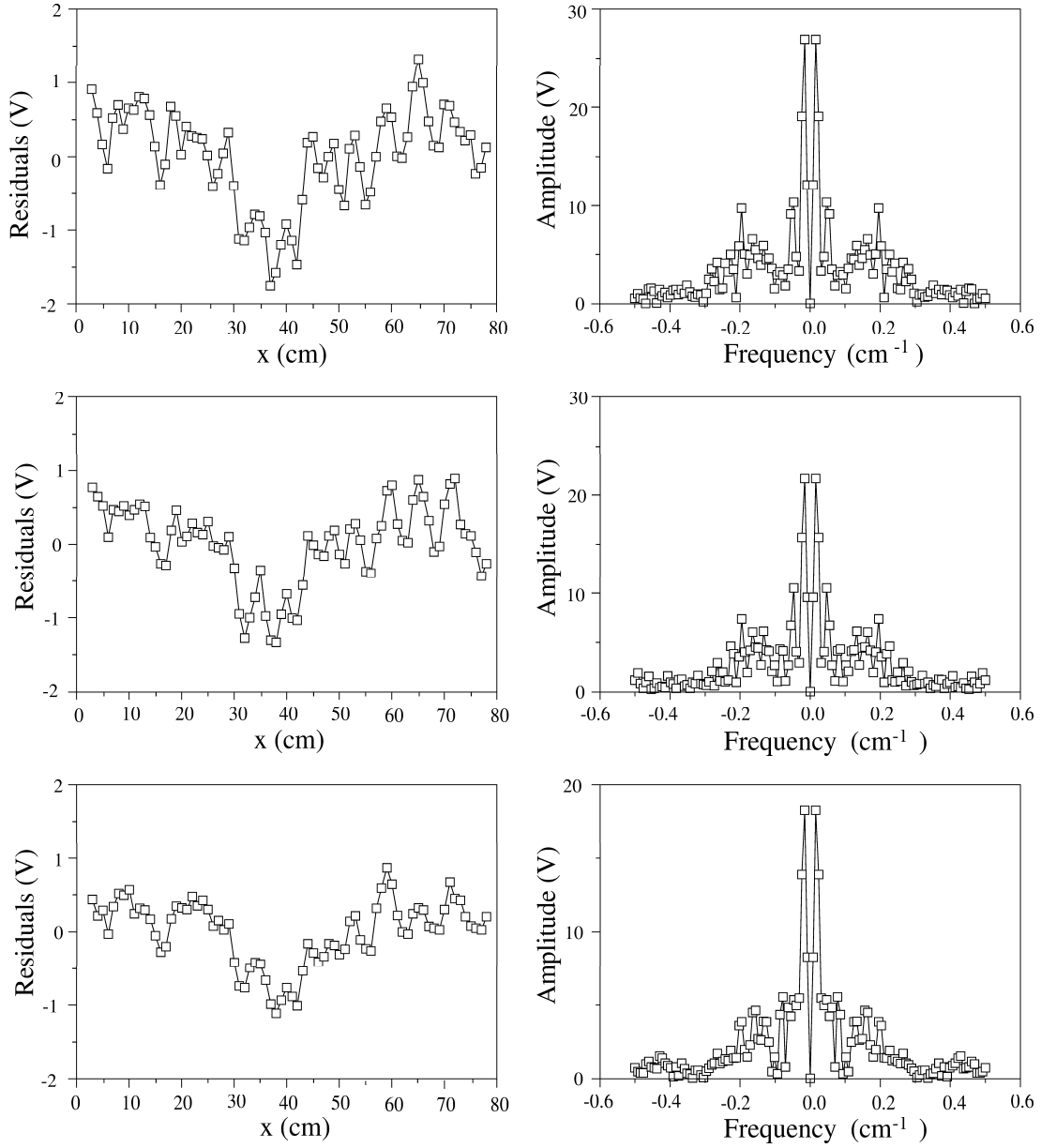


Figure 5: Left: Residuals of straight-line fits of the voltages measured (left), and Forward-Fourier Transform analyses of the residuals (right), for $y = 4$ cm (top), $y = 7$ cm (middle), and $y = 11$ cm (bottom), on Strip A.

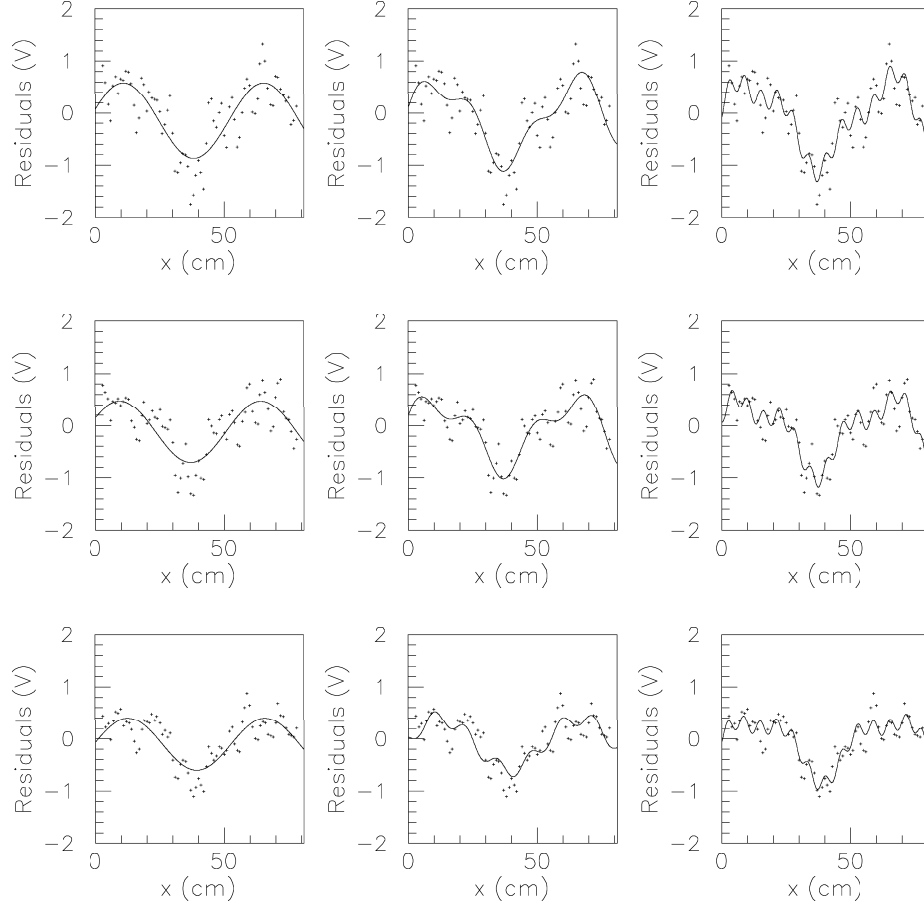


Figure 6: Residuals of straight line fits for $y = 4$ cm (top), $y = 7$ cm (middle), $y = 11$ cm (bottom), fit to one sinusoid (left), two sinusoids (middle), and three sinusoids (right).

| | One sinusoid | Two sinusoids | Three sinusoids |
|-------------|--------------|---------------|-----------------|
| $y = 4$ cm | 30.46 | 25.57 | 23.23 |
| $y = 7$ cm | 21.92 | 15.95 | 14.03 |
| $y = 11$ cm | 13.66 | 12.28 | 8.637 |

Table 1: Values of $\chi^2/\text{d.o.f.}$ values for the fits shown in Fig. 6.

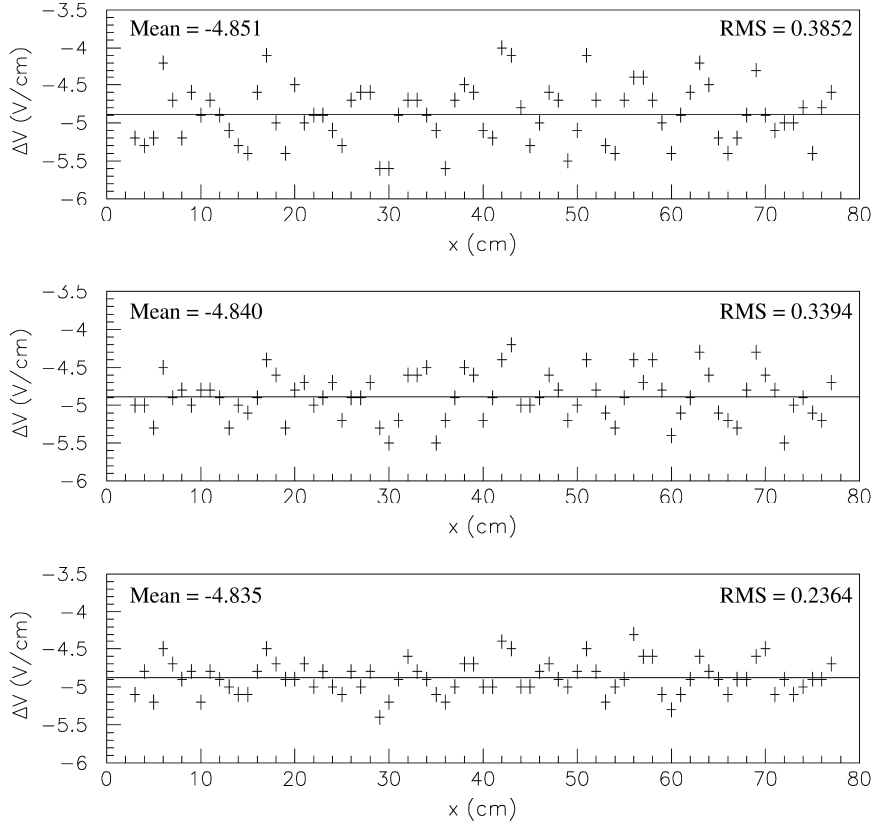


Figure 7: Gradients of measured voltages for $y = 4$ cm (top), $y = 7$ cm (middle), $y = 11$ cm (bottom), fit to horizontal straight lines.

The gradients of the measured voltages are shown in Fig. 7. Although there appears to be a suggestion of an oscillation with a period of 5 cm similar to that observed in the straight-line residuals, a Fourier Transform analysis of the gradients revealed no appreciable periodicity. For the straight-line fits shown, the $\chi^2/\text{d.o.f.}$ values corresponding to a voltage measurement error of (0.075 ± 0.005) V are (13.5 ± 1.5) , (8.6 ± 1.0) , and (4.4 ± 0.5) , respectively for $y = 4, 7$, and 11 cm. The data is therefore not consistent with a perfectly uniform voltage gradient along the x -direction. The RMS values indicated imply an upper bound on the nonuniformity of the electric field of $0.39/4.9 = 8\%$ in the case of zero measurement error.

4 Strip A, Uniformity as a Function of y

To determine the uniformity along the y -direction, Strip A was subjected to a second set of tests, in which the y -axis was scanned at 1-cm intervals for a constant value of x , at a total of 14 points, for $x = 6$ cm, 18 cm, 30 cm, 42 cm, 54 cm, 66 cm, and 78 cm. Since measurements were made by holding the ruler parallel to the y -axis, the ruler was not moved for a given value of x . The measurement error for this set of experiments therefore did not include the error associated with the placement of the ruler. Combining the electrical error and the error introduced by recording voltages to 0.1 V accuracy, both described in detail in the Appendix, the error in one voltage measurement was estimated to be (0.050 ± 0.001) V.

For each value of x , the mean of the measured voltages was calculated and subtracted from each measurement to obtain the deviation of the voltage from the mean as a function of y , which is shown in Fig. 8.

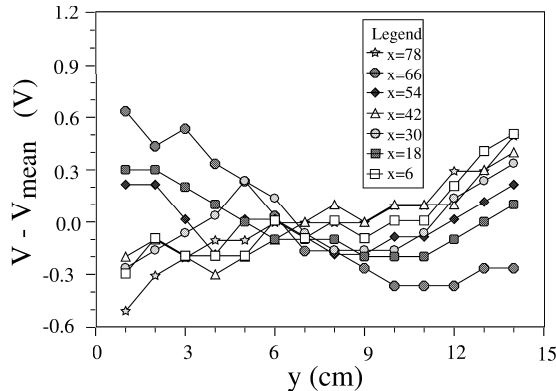


Figure 8: Left: The difference between the voltage at y and the mean of the measured voltages for the given value of x on Strip A.

The simple-polynomial behavior suggested by the curves in Fig 8 was investigated to determine whether it was a systematic effect due to the kapton film itself or a result of measurement error. As a preliminary test of reproducibility, the measurements for $x = 30$ cm and $x = 78$ cm were repeated; the same overall shape was observed in both sets of measurements, as shown in Fig. 9. Similarly, the measurements for $x = 66$ cm and $x = 78$ cm were repeated with the direction of the electric field reversed, by applying the high voltage at $x = 81$ cm and grounding $x = 0$ cm; the observed behaviour was similar to the original measurements but with the sign reversed, as shown in Fig. 10, as would be expected if the equipotentials of the film varied in the direction perpendicular to the applied electric field.

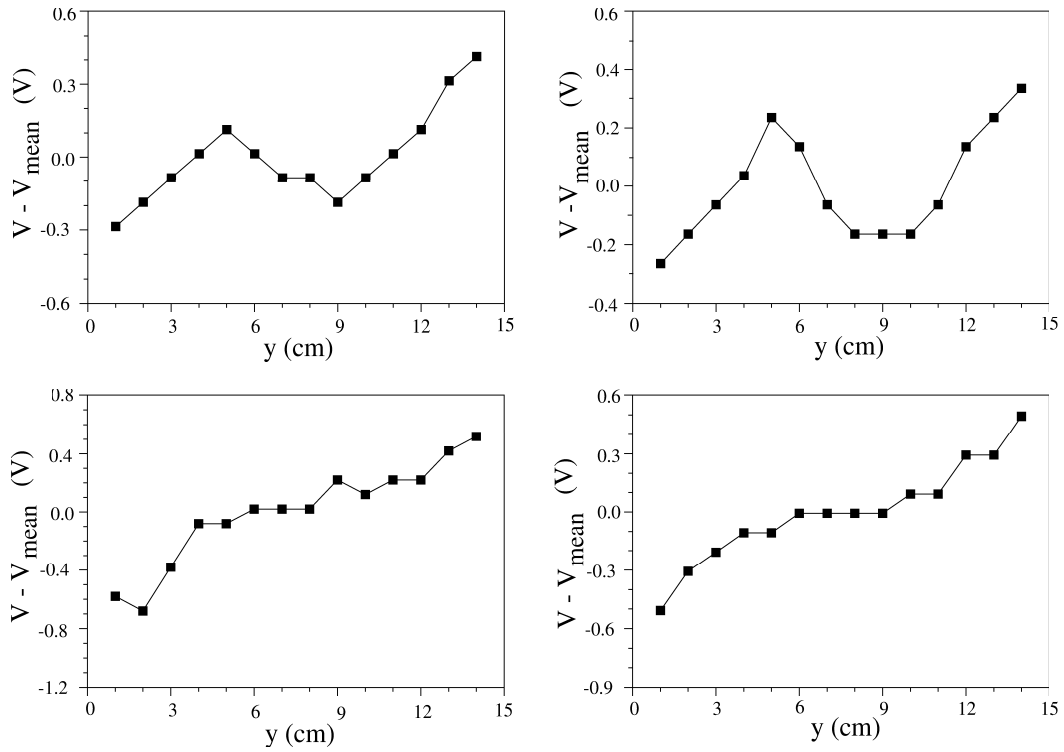


Figure 9: Left: The original measurements of the deviation of the voltage from the mean, for $x = 30$ cm (top) and for $x = 78$ cm (bottom). Right: Repeated measurements for $x = 30$ cm (top) and for $x = 78$ cm (bottom).

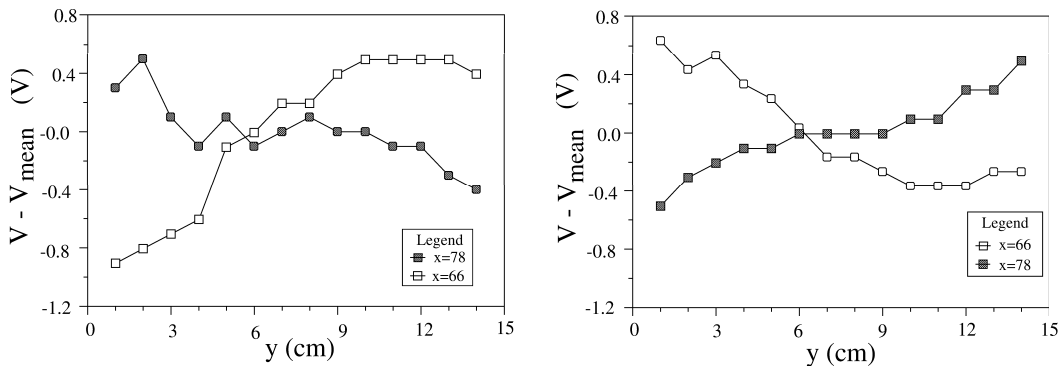


Figure 10: Right: Measurement of the deviation of the voltage from the mean for $x = 66$ cm and $x = 78$ cm. Left: The same measurements, with the direction of the applied electric field reversed.

For a measurement error of 0.050 V, the curves were fit to simple polynomials of degree 1, 2, 3, and 5. Whereas linear and quadratic fits did not appear adequate for describing all the curves, in most cases the χ^2 values were not lowered by performing quintic fits. The fits of the curves to cubic polynomials, as well as the residuals from the fits are shown in Fig. 11. The lack of any apparent structure in the residuals suggests that the cubic fits have removed all systematic effects due to variations in the film’s resistivity. The residuals are shown together in Fig. 12, and the cubic fitting functions are depicted in three dimensions in Fig. 13.

The standard deviations of the residuals, $\chi^2/\text{d.o.f.}$ values for the fits assuming a measurement error of 0.050 V, and corresponding confidence levels for 10 degrees of freedom are listed in Table 2. Although the mean of the standard deviations of the residuals is 0.06 V, inconsistent with the estimated measurement error of (0.050 ± 0.001) V, the cubic fits are in agreement with the curves within two standard deviations.

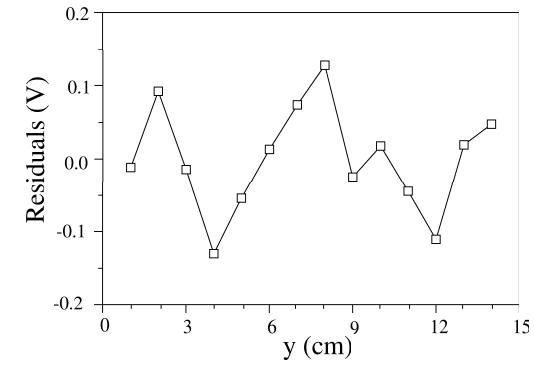
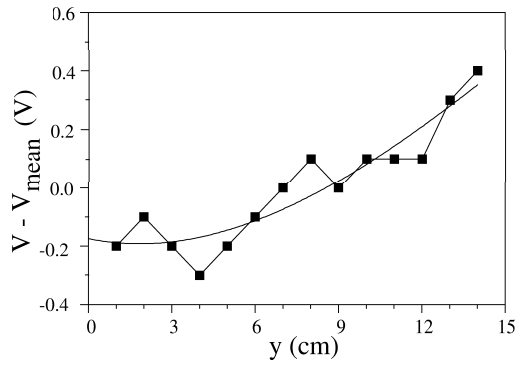
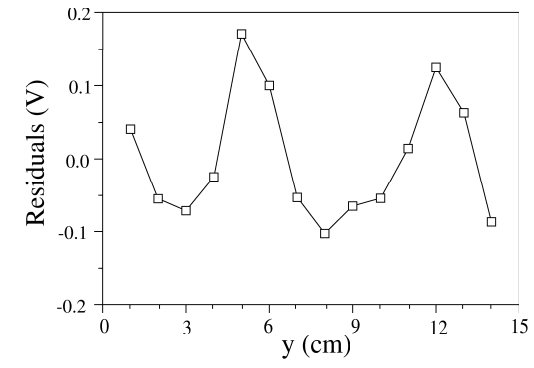
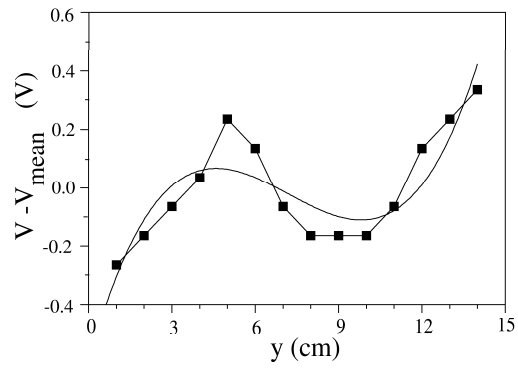
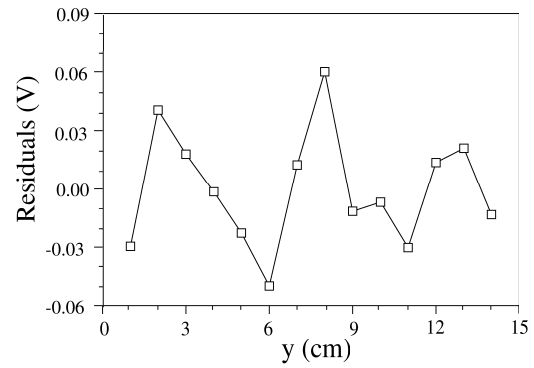
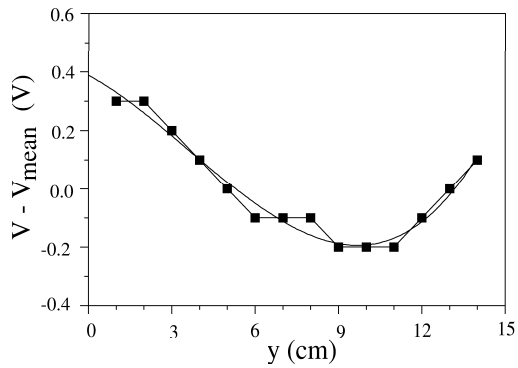
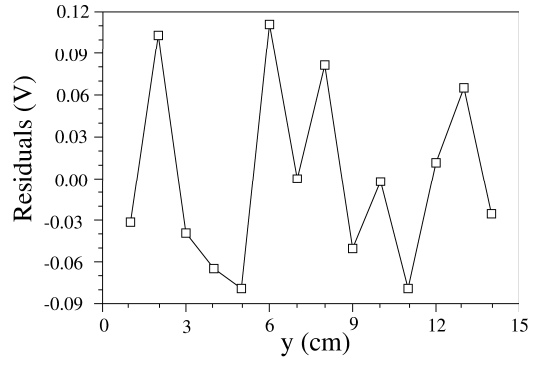
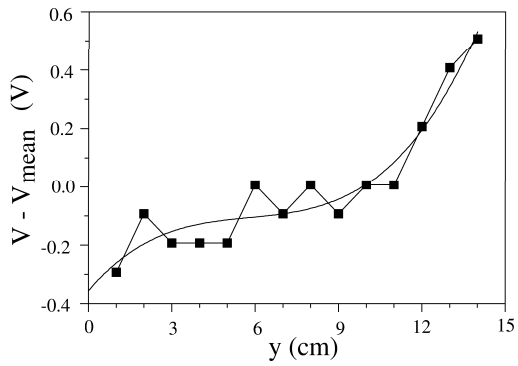
| x (cm) | Standard Deviation (V) | $\chi^2/\text{d.o.f.}$ | Confidence level (%) |
|----------|------------------------|------------------------|----------------------|
| 6 | 0.066 | 2.258 | 1.20 |
| 18 | 0.030 | 0.4613 | 91.55 |
| 30 | 0.087 | 3.899 | 0.003 |
| 42 | 0.073 | 2.734 | 0.23 |
| 54 | 0.072 | 2.710 | 0.25 |
| 66 | 0.056 | 1.608 | 9.74 |
| 78 | 0.034 | 0.5984 | 81.66 |

Table 2: Standard deviations of the residuals, $\chi^2/\text{d.o.f.}$ values, and confidence levels of the cubic fits shown in Fig. 11

4.1 Strip B

In a set of measurements similar to those made on Strip A, the x -axis on Strip B was scanned in 1-cm intervals for a constant value of y , at a total of 48 points for $3 \text{ cm} \leq x \leq 50 \text{ cm}$. Two separate scans were made, for $y = 4 \text{ cm}$ and $y = 11 \text{ cm}$. The measured voltages and residuals of straight-line fits are shown in Fig. 14. The overall uniformity along the x -direction on Strip B is clearly inferior to that observed on Strip A, as can be verified by comparing Fig. 14 to Figs. 4 and 5.

Since most of the nonuniformity of strip B appears to occur in the region



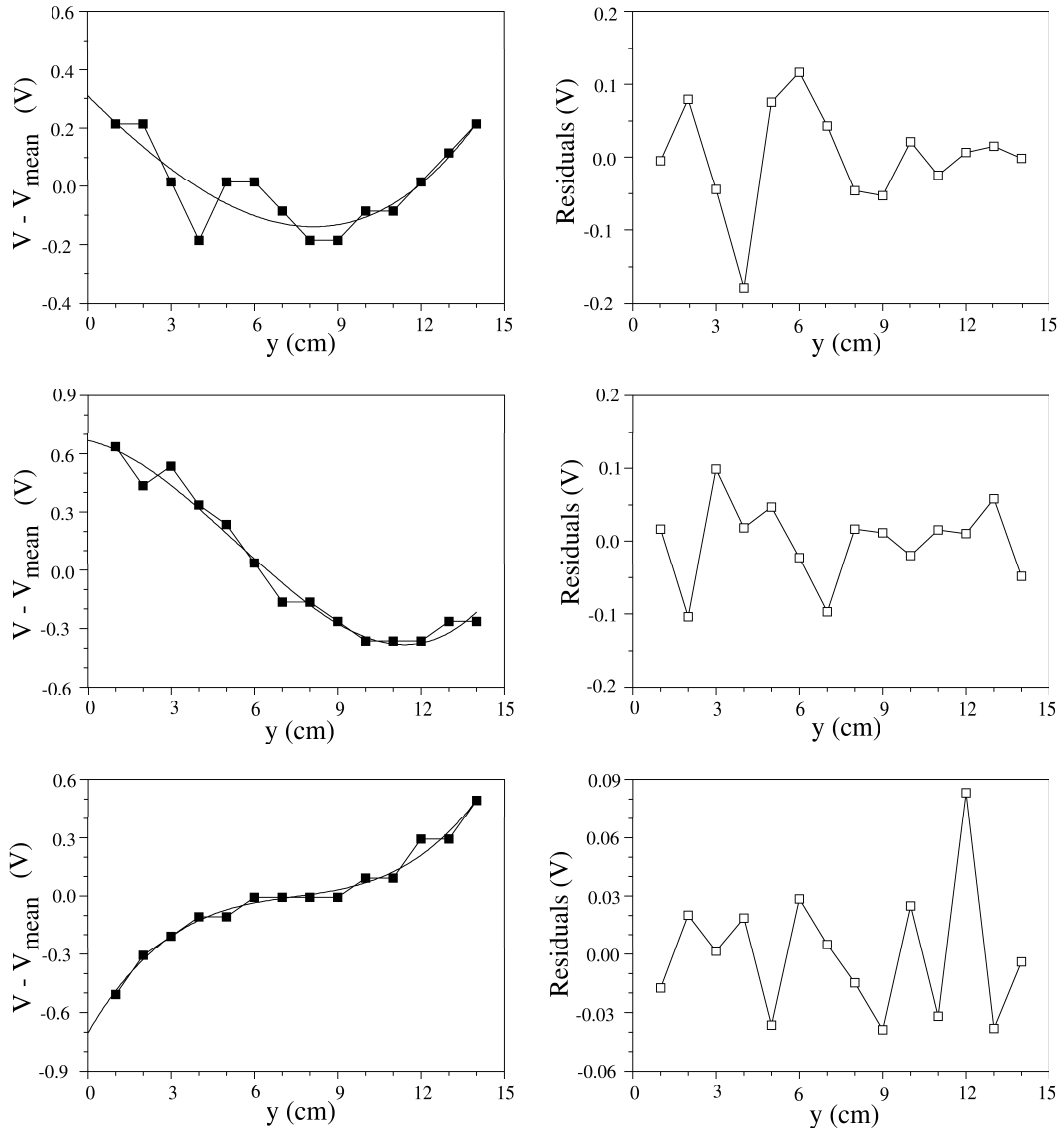


Figure 11: The cubic fits (left column) and the residuals of the cubic fits (right column) of the curves shown in Fig. 8, for (from top to bottom) $x = 6$ cm, 18 cm, 30 cm, 42 cm, 54 cm, 66 cm, 78 cm.

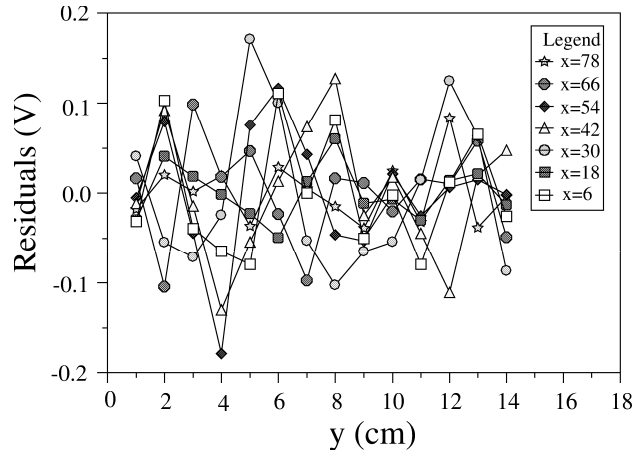


Figure 12: The residuals of the cubic fits shown in Fig 11.

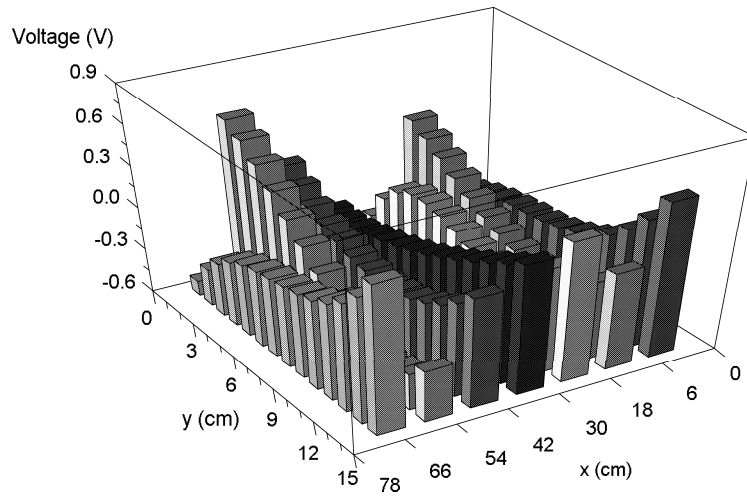


Figure 13: The cubic-fit functions for the curves plotted in Fig. 8, shown in 3-D.

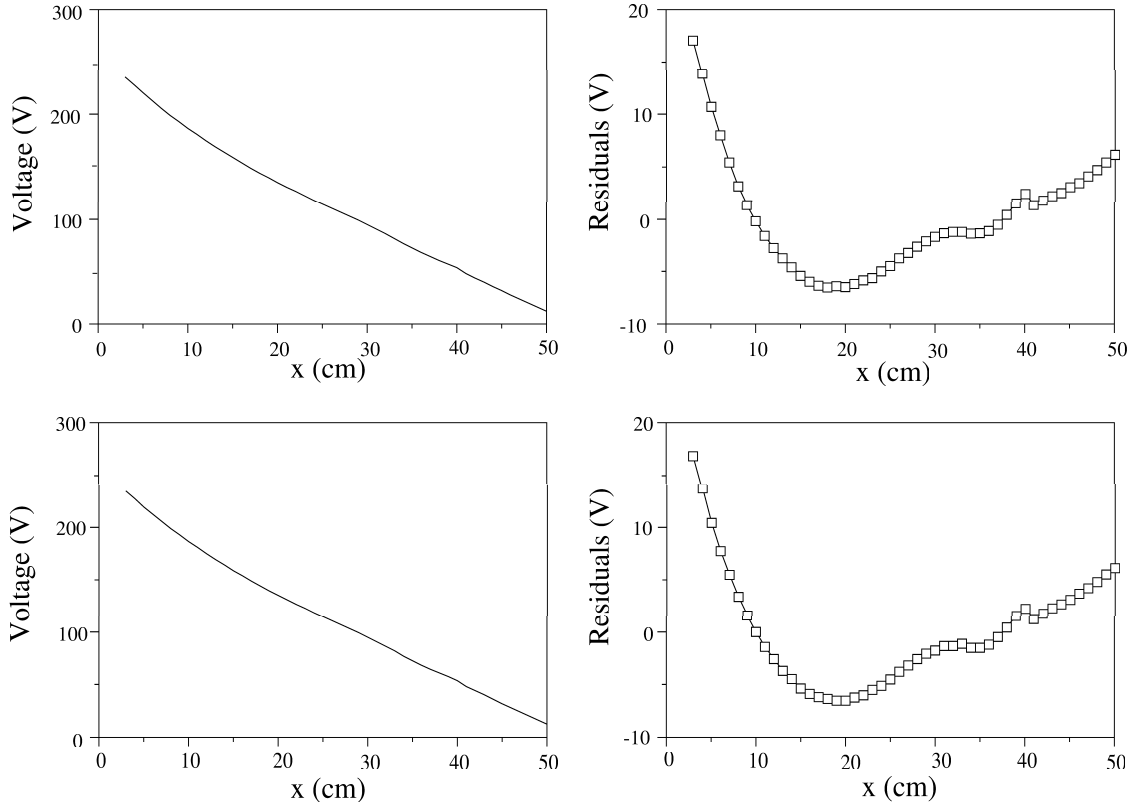


Figure 14: Top Left: Voltages measured on Strip B along $y = 4$ cm. A voltage of 262.2 ± 0.2 V was applied across 53 cm of film. Top Right: Residuals of a straight-line fit of the voltages for $y = 4$ cm. Bottom Left: Voltages measured on Strip B along $y = 11$ cm. Bottom Right: Residuals of a straight-line fit of the voltages for $y = 11$ cm.

$x < 20$ cm, the voltages along $y = 4$ cm for $x > 20$ cm were separately fit to a straight line. The results are shown in Fig 15. Although strip A and the region $x > 20$ cm of strip B both deviate from a straight line by approximately 1 V, a larger region of high uniformity can be obtained from strip A than from strip B.

The voltage gradients for Strip B are shown in Fig 16. Since the ratio of applied high voltage to film length is 4.9 V/cm, same as for the previous measurements on Strip A, the estimated error in one voltage measurement is again (0.075 ± 0.005) V, as discussed in the Appendix. Since the corresponding error in one gradient is (0.106 ± 0.007) V, the nonuniformity in the voltage gradients for Strip B clearly cannot be accounted for by measurement error.

It is interesting to note that although strip B showed poor uniformity along

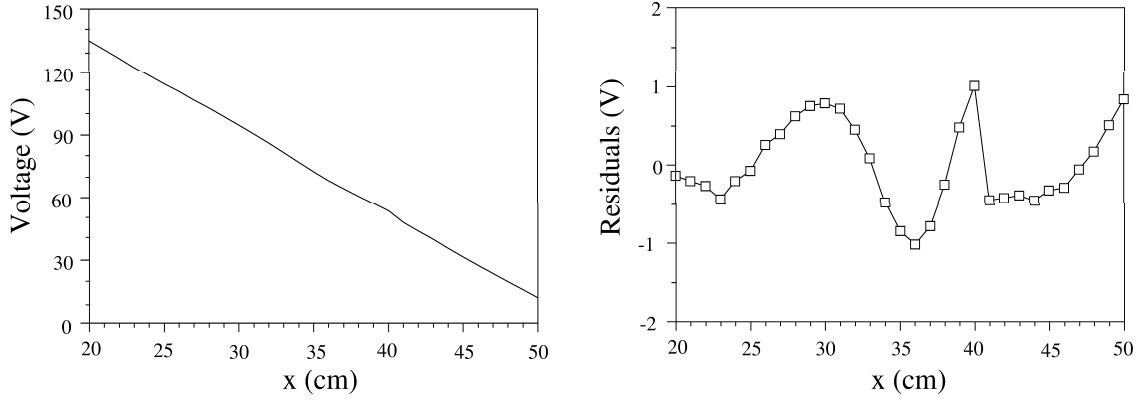


Figure 15: Measured voltages and residuals of the straight-line fit along $y = 4$ cm for $x > 20$ cm on Strip B.

the direction of the electric field, its uniformity in the direction perpendicular to the electric field was superior to that of strip A. Comparing Fig. 17, which shows the deviation from the mean of the voltages measured as a function of y at $x = 12$ cm, 24 cm, 36 cm, and 48 cm on Strip B, and Fig. 8, which is the corresponding plot for Strip A, the largest voltage difference between the two extreme values of y for any given value of x is 0.5 V for the former, and 0.9 V for the latter.

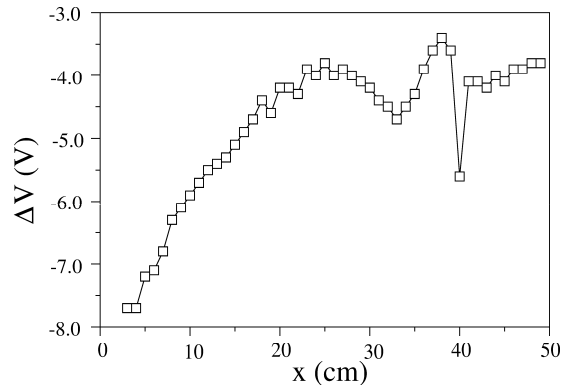


Figure 16: Gradients of the measured voltages for for strip B.

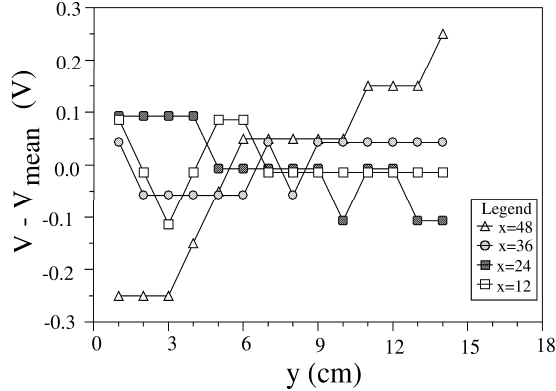


Figure 17: The difference between the voltage measured at y and the mean of the measured voltages for the given value of x on Strip B.

5 Strip A, $0 \text{ cm} \leq x \leq 30 \text{ cm}$

Based on the voltage measurements made on Strips A and B, it is plausible that the uniformity along the direction of the applied electric field depends on the direction in which the voltage is applied across the film. To test this hypothesis, the section of Strip A between $x = 0 \text{ cm}$ and $x = 30 \text{ cm}$ was subjected to a further test¹.

A voltage gradient of 10 V/cm was first applied parallel to the x -direction by holding $x = 0 \text{ cm}$ at $(299.8 \pm 0.1) \text{ V}$ and grounding $x = 30 \text{ cm}$. The voltages measured as a function of x for $y = 7 \text{ cm}$, and as a function of y for $x = 15 \text{ cm}$ are shown in the top two plots of Fig. 18. Second, the same voltage gradient was applied parallel to the y -direction by holding $y = 14.7 \text{ cm}$ at $(147.4 \pm 0.1) \text{ V}$ and grounding $y = 0 \text{ cm}$. The voltages measured as a function of y for $x = 15 \text{ cm}$, and as a function of x for $y = 7 \text{ cm}$ are shown in the bottom two plots of Fig. 18. As expected, the uniformity along the direction of the applied electric field was found to be better when the voltage was applied parallel to the x -direction than when it was applied parallel to the y -direction. On the other hand, the uniformity in the direction perpendicular to the applied electric field was better when the voltage was applied parallel to the y -direction than when it was applied parallel to the x -direction. Since, as discussed in the Introduction, high uniformity is desired along the direction of the applied electric field for the time projection chamber, it was decided that the electric field should be applied perpendicular to the roll (See Fig. 2).

¹Note that in this section only, the determination of the coordinate system is independent of the direction in which the voltage is applied; the axes referred to are the same as those used in the previous measurements on Strip A.

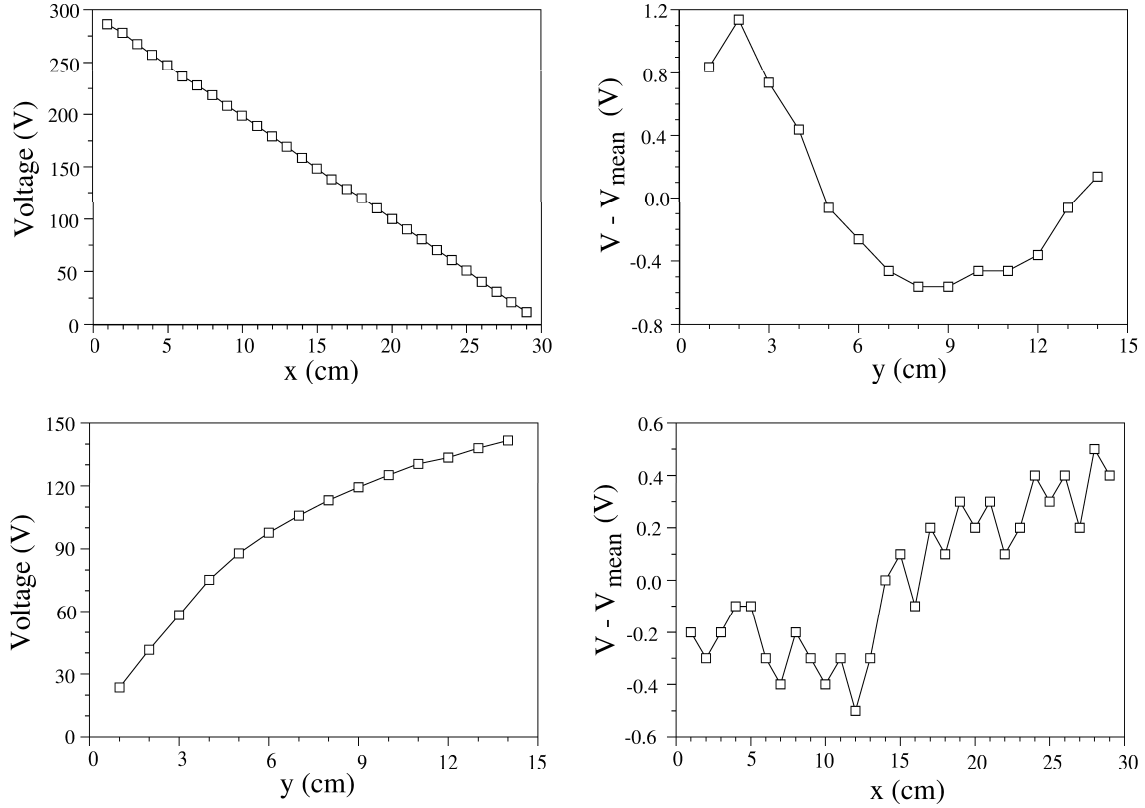


Figure 18: Top: With the electric field applied parallel to the x -axis on Strip A, voltages measured as a function of x for $y = 7$ cm (left), and as a function of y for $x = 15$ cm (right). Bottom: With the electric field applied parallel to the y -axis on Strip A, voltages measured as a function of y for $x = 15$ cm (left), and as a function of x for $y = 7$ cm (right).

Residuals and voltage gradients for a complete voltage map of the region $0 \text{ cm} \leq x \leq 30 \text{ cm}$ on Strip A for an applied voltage gradient of 10 V/cm is shown in Fig. 19.

6 Conclusion

The uniformity of the resistivity of the sample of carbon-loaded kapton which was tested was found to depend on the direction in which voltage was applied across it. To attain higher uniformity along the drift direction, which is desirable in a cylindrical time projection chamber, it was found that the electric field should be applied perpendicular to the direction of the particular roll(See Fig. 2).

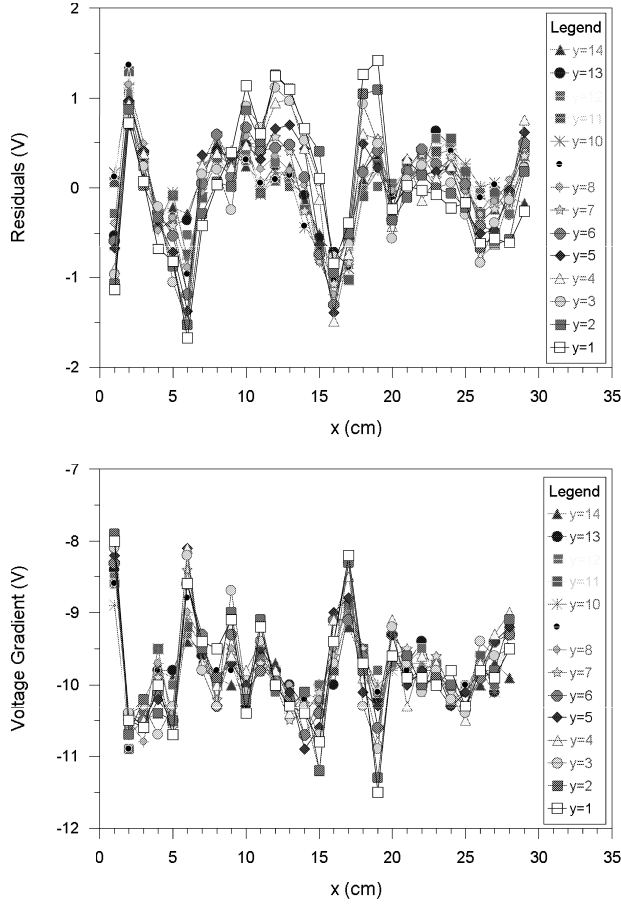


Figure 19: Top: Residuals of straight-line fits along the x -direction for $0 \text{ cm} < x < 30 \text{ cm}$ on strip A. A voltage of $(299.8 \pm 0.1) \text{ V}$ was applied. Bottom: The voltage gradients as a function of x for each value of y for the same set of measurements.

When a strip of film referred to as Strip A was tested by applying an electric field in the favored direction, the voltages measured at various points along the direction of the applied electric field were found to be inconsistent with a straight line within measurement error but to deviate from the straight line with an amplitude of about 1 V. An upper limit of 8% was set on the nonuniformity of the voltage gradient. In the direction perpendicular to the applied electric field, a voltage variation of about 1 V over 14 cm was observed and was approximated within two standard deviations by cubic functions, suggesting approximately cubic-shaped equipotentials.

A Estimating the Statistical Error

Two types of measurement error were considered: the first, referred to as “ruler error”, introduced by misplacing the ruler along the x -axis, was characterized by σ_t and σ_b , the error at the top and bottom ruler marks, respectively; the second, referred to as “electronic error”, σ_e , included errors due to causes other than ruler misplacement, such as tilting of the probe, misplacement of the probe along the y -axis, and temperature variation. The voltage gradient on the film was assumed to be much greater along the x -axis than along the y -axis.

A series of voltage measurements were made on Strip C in order to estimate σ_t , σ_b and σ_e . For each of 25 equally-spaced values of x between $x = 3$ cm and $x = 78$ cm, the y -axis was scanned at 1-cm intervals, at a total of 14 points for $1 \text{ cm} \leq y \leq 14$ cm. The ruler position was kept fixed for a given scan. For each value of x , two separate scans were made. A voltage of 390.6 ± 0.2 V was applied across 79.0 cm of film. The applied voltage was constant to within 0.05 V for any pair of scans with equal x .

A.1 Estimating σ_t and σ_b

For a given x , the voltages measured in the second scan were subtracted from the corresponding voltages measured in the first scan to obtain the voltage differences Δx_i , where i is an integer between 1 and 14 representing the y -coordinate of each measurement. Δx_i were fit to the straight-line function $\Delta x_t \left(\frac{y_i - y_b}{y_t - y_b} \right) + \Delta x_b \left(\frac{y_t - y_i}{y_t - y_b} \right)$, where Δx_t and Δx_b are voltage differences which would be measured at the top and bottom ruler marks if the film extended to the ruler marks, and y_t , y_b , and y_i are, respectively, the y -coordinates at the top and bottom ruler marks and for the i -th measurement. The fit was performed by minimizing:

$$\chi^2 = \sum_{i=1}^{14} \frac{\left[\Delta x_i - \Delta x_t \left(\frac{y_i - y_b}{y_t - y_b} \right) - \Delta x_b \left(\frac{y_t - y_i}{y_t - y_b} \right) \right]^2}{\sigma_e^2} \quad (1)$$

with respect to Δx_t and Δx_b , assuming χ^2 to be fixed with respect to y_t , y_b and y_i . The resulting matrix:

$$\begin{pmatrix} \sum_{i=1}^{14} \left(\frac{y_i - y_b}{y_t - y_b} \right) \Delta x_i \\ \sum_{i=1}^{14} \left(\frac{y_t - y_i}{y_t - y_b} \right) \Delta x_i \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^4 \left(\frac{y_i - y_b}{y_t - y_b} \right)^2 & \sum_{i=1}^4 \left(\frac{y_i - y_b}{y_t - y_b} \right) \left(\frac{y_t - y_i}{y_t - y_b} \right) \\ \sum_{i=1}^4 \left(\frac{y_i - y_b}{y_t - y_b} \right) \left(\frac{y_t - y_i}{y_t - y_b} \right) & \sum_{i=1}^4 \left(\frac{y_t - y_i}{y_t - y_b} \right)^2 \end{pmatrix} \begin{pmatrix} \Delta x_t \\ \Delta x_b \end{pmatrix} \quad (2)$$

was solved to obtain Δx_t and Δx_b for the 25 values of x .

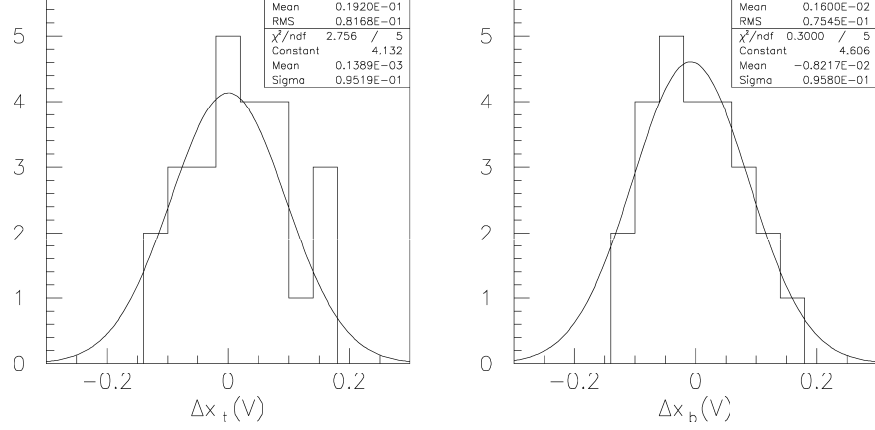


Figure 20: Histograms of Δx_t (left) and Δx_b (right), fit to gaussians. Statistics of the distributions and attributes of the fits are shown.

The values of Δx_t and Δx_b calculated from the measurements are histogrammed and fit to gaussians in Fig. 20. As expected, the mean of each distribution is small compared to the standard deviation. Expressing Δx_t and Δx_b for a given value of x as:

$$\Delta x_{t,b} = x_{t1,b1} - x_{t2,b2} \quad (3)$$

where $x_{t1,b1}$ and $x_{t2,b2}$ are the differences of the measured voltages from the “true” voltages at the top and bottom ruler points for the first and second scans, σ_t and σ_b are given in terms of the RMS values of the distributions in Fig. 20 by:

$$\sigma_{t,b} = \frac{\text{RMS}_{\Delta x_{t,b}}}{\sqrt{2}}. \quad (4)$$

The values of σ_t and σ_b thus obtained, as well as the corresponding distances from the top and bottom ruler points in an electric field of 4.9 V/cm along the x -axis, are listed in Table 3.

| | $\text{RMS}_{\Delta x_{t,b}}$ | $\sigma_{t,b}$ | $\sigma_{t,b}/(4.9 \text{ V/cm})$ |
|-----|-------------------------------|-------------------------------|-----------------------------------|
| t | $(0.082 \pm 0.012) \text{ V}$ | $(0.058 \pm 0.008) \text{ V}$ | $(0.012 \pm 0.002) \text{ cm}$ |
| b | $(0.075 \pm 0.011) \text{ V}$ | $(0.053 \pm 0.008) \text{ V}$ | $(0.011 \pm 0.002) \text{ cm}$ |

Table 3: Standard deviations $\text{RMS}_{\Delta x_{t,b}}$ of the distributions in Fig. 20, the errors $\sigma_{t,b}$, and the corresponding distance errors at the top and bottom ruler marks.

A.2 Estimating σ_e

To estimate the electronic error, which would account for the deviation of the measured voltage differences Δx_i from $\Delta x_t \left(\frac{y_i - y_b}{y_t - y_b} \right) + \Delta x_b \left(\frac{y_t - y_i}{y_t - y_b} \right)$, the value of:

$$\delta_i = \Delta x_i - \Delta x_t \left(\frac{y_i - y_b}{y_t - y_b} \right) - \Delta x_b \left(\frac{y_t - y_i}{y_t - y_b} \right) \quad (5)$$

was computed for each of the $14 \times 25 = 350$ measurements of voltage difference. The RMS of the approximately gaussian distribution was 0.0570 V, with an error of 0.0022 V for a sample size of 350. The electrical error on one voltage measurement is therefore $(0.0570 \pm 0.0022) \text{ V} / \sqrt{2} = (0.040 \pm 0.002) \text{ V}$. Since all measurements were recorded to 0.1 V accuracy, this represents an upper limit on the electronic error σ_e .

To check the validity of this estimate, a second estimate of the electrical error was performed by substituting into Eq. 1 the values of Δx_t and Δx_b calculated in Section A.1 and solving for σ_e^2 for each value of x , taking $\chi^2 = 14 - 2 = 12$. The resulting distribution is histogrammed in Fig. 21. From the mean and RMS of the distribution, the electrical error in one voltage measurement was calculated to be $(0.044 \pm 0.005) \text{ V}$, in agreement with the previous estimate.

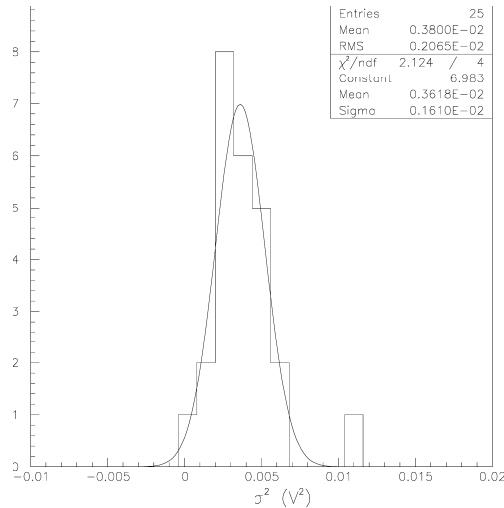


Figure 21: The distribution of σ_e^2 obtained by substituting Δx_t and Δx_b into Eq. 1. A gaussian fit is also shown.

A.3 The Total Statistical Error

By taking the mean of σ_t and σ_b , found in Section A.1 to be, respectively, (0.058 ± 0.008) V and (0.053 ± 0.008) V, the ruler error in one measurement was estimated as (0.0555 ± 0.00566) V. Similarly, by taking the weighted mean of the two estimates of σ_e discussed in Section A.2 and measured to be (0.0403 ± 0.0015) V and (0.0436 ± 0.0047) V, the electronic error in one measurement was estimated as (0.0406 ± 0.0014) V. Combining the ruler error and the electrical error in quadrature, the total error in one voltage measurement was estimated to be (0.069 ± 0.005) V.

Finally, including the effective error of 0.0289 V introduced by recording voltage measurements to only 0.1 V accuracy, the best estimate of the total measurement error was taken to be (0.075 ± 0.005) V.

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