EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH
EUROPEAN LABORATORY FOR PARTICLE PHYSICS

INTERNAL REPORT CERN/TIS-RP/IR/98-???

Radiological hazard due to neutrinos from a muon collider

Colin Johnson, Gigi Rolandi and Marco Silari

Abstract

This paper is intended to provide a first estimate of the radiological hazard posed by the neutrino radiation generated by decays of high energy muons circulating in a future $\mu^+\mu^-$ collider installed in the CERN region. Values of off-sites annual dose equivalent due to the neutrino radiation in equilibrium with its secondaries are calculated for muon energies from 1 to 10 TeV and for various depths at which the accelerator may be installed. A comparison is made with similar data available from the literature. For CoM energies exceeding 4 TeV some countermeasure must be adopted to limit the radiation dose; possible solutions are briefly discussed.

CERN, Geneva, Switzerland
1 December 1998

Radiological hazard due to neutrinos from a muon collider

Colin Johnson, Gigi Rolandi and Marco Silari

1. Introduction

This note is intended to provide a first estimate of the radiological hazard posed by the neutrino radiation generated by decays of high energy muons circulating in a future muon collider installed at CERN.

Due to their extremely low interaction cross section, only recently the possibility of radiological consequences induced by neutrinos was raised. Collar has suggested that ionising radiation due to the intense neutrino flux from stellar collapses (an astronomical event much more frequent than supernova explosions) might be the responsible for the extinction of some biological species in the history of Earth [1]. This hypothesis, which is based - amongst other assumptions - on the estimate of the biological effectiveness of neutrino-induced recoils in generating cell transformation and radiocarcinogenesis, is actually debated [2,3]. Cossairt and co-workers have presented a method for conservatively estimating the dose equivalent due to neutrinos over a wide energy range, from the MeV domain (solar neutrinos) to TeV (muon colliders) [3]. These are the only data presently available in the open literature (at least to the best of our knowledge). Four processes are considered, of increasing importance with increasing neutrino energy, namely scattering from atomic electrons, scattering from nuclei and scattering from nucleons, with the neutrino beam either unshielded or shielded. The latter case, which becomes important for $E_\nu > 0.5$ GeV, is the most common situation encountered with particle accelerators. The fluence to dose equivalent conversion coefficients as a function of neutrino energy, taken from ref. 3, are listed in Table A.1 of the Appendix, for both the unshielded and the shielded case. Column 3 actually includes contributions from all processes, the first two being important only at low energies, and therefore coincides with the total value of the conversion coefficient.

Dose equivalent rates due to solar and atmospheric neutrinos and to neutrinos from present day accelerators are insignificant. Expected dose equivalent rates for the neutrino beams planned for future long and short baseline neutrino experiments, namely the CERN/Gran Sasso beam and the NuMI project at Fermilab, are also negligible [3-5], as shown in Table 1. However, the neutrino flux generated in a muon collider is much higher. The radiological hazard is in actual fact much larger (up to three orders of magnitude at TeV energies) if the neutrino beam is shielded than if it is left unshielded, because of the secondary radiation (mainly hadrons, electrons and muons) produced in the shielding material (in practice, earth, if the collider is installed underground). The secondaries with the longest range are the muons. The maximum energy of these secondary muons cannot obviously exceed the energy of the collider. The collider must obviously be shielded and the shield must be thick enough to absorb the full muon beam circulating in the ring in case of a beam loss. It follows that the shield must be thicker than the maximum range of all secondaries, i.e. the neutrino radiation emerging from the shield is in equilibrium with its secondary radiation. The data to be used for the present assessment are therefore those of column 3 in Table A.1.

Table 1. Expected annual dose equivalent from natural and accelerator neutrino sources (Short and Long Baseline neutrino experiments) [3-5].

		Annual dose equivalent (μSv)
Solar neutrinos (E_ν ~ 1 - 10 MeV)		10^{-7}
Atmospheric neutrinos (E_ν ~ 100 MeV - 2 GeV)		2×10^{-9}
Neutrino experiments (E_ν ~ 10 – 100 GeV):		
Fermilab (NuMI)	SBL, 1 km distance	10
	LBL, 730 km distance	8.5×10^{-6}
CERN/Gran Sasso	SBL	10
	Gran Sasso	5×10^{-5}

2. Neutrino fluence expected from a muon collider

Let us assume that bunches of $N^0=2\text{x}10^{12}$ muons are produced at a repetition period F=15 Hz. The average rate of neutrino production therefore is:

$$r_\nu = 2\ N^0\ F= 6\ 10^{13}\ s^{-1}$$

The time structure of the neutrino rate is bunched since the muon lifetime in the laboratory frame is typically shorter than the repetition period of muon production:

$$\tau_\mu = 2.2\ 10^{-2}\ s\ \ (E_0/1\ TeV)\quad \text{versus}\quad 1/F = 6\ 10^{-2}\ s$$

where E_0 is the energy of the muon beam. The average radius R of the machine is determined by the beam energy and the magnetic field:

$$R = 420\ m\ \ (8\ T/B)\ \ (E_0/1\ TeV)$$

The divergence of the neutrino beam (the opening half-angle) induced by the decay (expressed in radians) is the inverse of the relativistic factor:

$$\theta = 1/\gamma = 10^{-4}\ \ (1\ TeV/E_0)$$

At a distance of 10 km this divergence produces a spot size of 2 m (1 TeV/E_0).

The neutrino fluence rate is mainly concentrated in the plane of the machine. At a distance L from the centre of the machine the average fluence rate is:

$$\Phi_\nu = r_\nu\gamma / (2\ \pi\ L^2) = 10^5\ cm^{-2}\ s^{-1}\ (10\ km/L)^2\ \ (E_0/1\ TeV) \qquad (1)$$

A straight section of length ls will concentrate the fluence by a factor:

$$\gamma\ \mathrm{ls}/R = 500\ (B/8\ \mathrm{T})\ \ (\mathrm{ls}/20\ \mathrm{m})$$

Therefore an inter-magnet gap of 0.5 m will increase the neutrino fluence by one order of magnitude and a comparatively short straight section of a few tens metres (e.g., for injection) by three orders of magnitude. On the other hand, at the interaction point (along the associated straight sections within and near to the detector) the beam divergence is given by:

$$(\varepsilon \times (1 + \alpha^2)/\beta)^{1/2}$$

where ε is the beam emittance and α and β are the local Twiss parameters. The low β and/or large α values reduce the fluence by an order of magnitude. Further away from the interaction point, but still within the interaction point lattice insertion, dipole fields must be introduced to avoid hot spots in regions of locally low beam divergence. For the present purpose we will assume that elsewhere, on average, a straight section in the regular collider lattice will enhance the neutrino fluence by a factor of 10.

The spectrum of the neutrinos from muon decay in the muon reference system can be approximated by the expression:

$$\frac{dN_\nu}{dE^*_\nu} = \frac{8E^*_\nu}{m^2_\mu}$$

This is a good approximation for the muon neutrinos and a decent approximation for the electron antineutrinos. The spectrum in the laboratory system, averaged over all production angles, is:

$$\frac{dN_\nu}{dE_\nu} = \Phi_0\,\frac{2}{E_0}\left(1 - \frac{E_\nu}{E_0}\right) \tag{2}$$

which is the expression we will use below. Note that at TeV energies the transverse broadening of the hadronic and leptonic showers is comparable to the overall neutrino radiation opening-angle and this partly justifies averaging the neutrino energy spectrum over all production angles. A correct integration of the neutrino spectrum will be included in any detailed design.

3. Shielding

The collider has to be installed underground to shield the muon beam in case of a beam loss. The energy loss of a muon is:

$$dE/dx = 0.6\ \mathrm{TeV/km}\ \ (\rho\ /3\ \mathrm{g\ cm^{-3}}) \tag{3}$$

which means that a 5 TeV muon beam is dumped in less than 10 km of earth and a 10 TeV beam in less than 20 km.

On the other hand, the interaction cross section σ_ν of neutrinos is extremely small, of the order of 10^{-35} cm^2 (E_ν/1 TeV). The attenuation length is then:

$$\lambda = A/(\rho N_A \sigma_\nu) = 1/(N\sigma_\nu) = 0.5\ 10^6\ \text{km}\ (1\ \text{TeV}/\ E_\nu)\ (3\ \text{g cm}^{-3}/\rho) \qquad (4)$$

in which A and ρ are the atomic number and the density of the medium, N_A is the Avogadro's number and N is the number of atoms per unit volume. From expression (4) one sees that the attenuation length is very long, i.e. the neutrino fluence is not attenuated at all while traversing the shield. Neglecting local effects, i.e. approximating the Earth as a sphere, it can be easily shown that, for a machine situated at a depth d, the exit point of the neutrino beam is at a distance L given by:

$$L = \sqrt{2dR_t - d^2} \approx \sqrt{2dR_t} = 36\ km\ \sqrt{d/100\ m} \qquad (5)$$

in which R_t = 6400 km is the radius of the Earth. For the purpose of muon shielding, it would be sufficient that a 5 TeV collider is placed at a depth of 10 to 20 m, but we shall see below that this is not sufficient for the neutrino radiation.

4. Neutrino dose equivalent

Starting from the above assumptions we can estimate the radiological hazard which can be posed by the neutrinos generated by decays of high-energy muons in the collider. The dose equivalent rate is obtained by folding the neutrino spectrum dN_ν/dE_ν with the conversion coefficients $C(E_\nu)$ of Table A.1, column 3:

$$\dot{H} = \int_0^{E0} \frac{dN_\nu}{dE_\nu} C(E_\nu) dE_\nu \qquad (6)$$

where dN_ν/dE_ν is given by expression (2). If E_ν is in GeV, dN_ν/dE_ν in $cm^{-2}\ s^{-1}\ GeV^{-1}$ and $C(E_\nu)$ in $\mu Sv\ cm^2$, $\dot{H}$ is in $\mu Sv\ s^{-1}$. In the neutrino energy range from 0.5 GeV to 10 TeV, the fluence to dose equivalent conversion coefficients can be fitted by the expression:

$$\log_{10} C(E_\nu) = 2 \log_{10} (E_\nu) - 15 \qquad (7)$$

Let us consider a collider placed at two different "reasonable" depths, d = 100 m and d = 200 m, and at a more problematic depth d = 500 m. From expression (5) it follows that the exit point of the neutrino beam is at distances

L = 36 km, L = 51 km and L = 80.5 km, respectively. The integral neutrino fluence rate Φ_0 emerging from the earth is calculated from expression (1) and is given in Table 2 for a few representative values of the collider energy E_0.

Table 2. Integral neutrino fluence rate F_0 *at L = 36 km, L = 51 km and L = 80.5 km.*

Collider energy (TeV)	Φ_0 (cm^{-2} s^{-1})		
	L = 36 km	L = 51 km	L = 80.5 km
1	8×10^3	3.8×10^3	1.6×10^3
2	1.6×10^4	7.7×10^3	3.1×10^3
5	4×10^4	1.9×10^4	7.8×10^3
10	8×10^4	3.8×10^4	1.6×10^4

The divergence of the neutrino beam induced by the decay (from 100 μrad for a collider energy of 1 TeV to 10 μrad at 10 TeV) is such that even at the shortest of the above distances from the source the beam is large enough (at 36 km the spot radius is 3.6 m and 0.36 m, respectively) that a whole body exposure to the radiation should be considered. The annual dose equivalent expected for operation of the collider for 180 days/year operation (1.56×10^7 s) is given in Table 3 and in Figs. 1-3. Estimates are given for neutrino radiation emitted from a bending section and from a straight section, assuming an enhancement factor of 10, as discussed above. The dose scales with E_0^3.

Table 3. Estimated annual dose equivalent at 36 km, 51 km and 80.5 km distances from 1, 2, 5 and 10 TeV muon colliders, for operation of 180 days per year. The neutrino fluence from a straight section (SS) is supposed to be 10 times more intense than from a bending section.

	Annual dose equivalent (μSv)					
	d = 100 m, L = 36 km		d = 200 m, L = 51 km		d = 500 m, L = 80.5 km	
	Arc	SS	Arc	SS	Arc	SS
E_0 = 1 TeV	20	200	10	100	4	40
E_0 = 2 TeV	160	1,600	80	800	32	320
E_0 = 5 TeV	2,500	25,000	1,250	12,500	500	5,000
E_0 =10 TeV	20,000	200,000	10,000	100,000	4,000	40,000

In can be shown that the neutrino radiation produced in case of a loss of the circulating muon beam has an average energy of 100 MeV (3 g cm$^{-3}/\rho$) (E_0/1 TeV), too low to represent any radiological hazard.

5. Discussion

An assessment of the radiological risk due to neutrinos from a muon collider has been made at Fermilab but it is still unavailable [6]. Some results given in ref. 7 are in agreement with the present estimate (which is possibly not surprising as data of

Table A.1 come from Fermilab). Conversion coefficients for neutrinos in equilibrium with their secondaries, calculated at Fermilab by Mokhov with the Monte Carlo code MARS [8], are still unpublished but were provided by the author as a private communication to G.R. Stevenson. These values, shown in Fig. 10 of ref. 4, are somewhat higher than those listed in Table A.1 for energies up to about 10 GeV, but approximately the same above. As the most important contributions to the dose equivalent come from the highest energies, the difference in using either set of data in this context should be very small.

The present estimates are also in substantial agreement with those of B. King for various collider parameters [9], who has also calculated that for a 4 TeV collider the distance at which the neutrino dose from an arc is within the US limit of 1 mSv per year is 34 km [10]. The present estimates are also in agreement with those of ref. [11] which, for a 3 TeV CoM (i.e., E_0 = 1.5 TeV) collider placed at a depth d = 500 m and a muon current close to the value used in the present paper, predict an off-site annual dose in the plane of a bending dipole of 11 µSv.

The value of ambient dose equivalent caused by ionising radiation emitted by CERN beyond the boundaries of its site must not exceed 1.5 mSv per year [12]. The radiological impact on the environment of a muon collider built at or nearby CERN will therefore have to comply with this limit. If the collider is built at a sufficient depth to guarantee a minimum distance of 30 to 40 km from the surface exit points of the neutrino-induced radiation, a problem exists only if the collider CoM energy exceeds 4 TeV. For higher energies some countermeasure must be adopted.

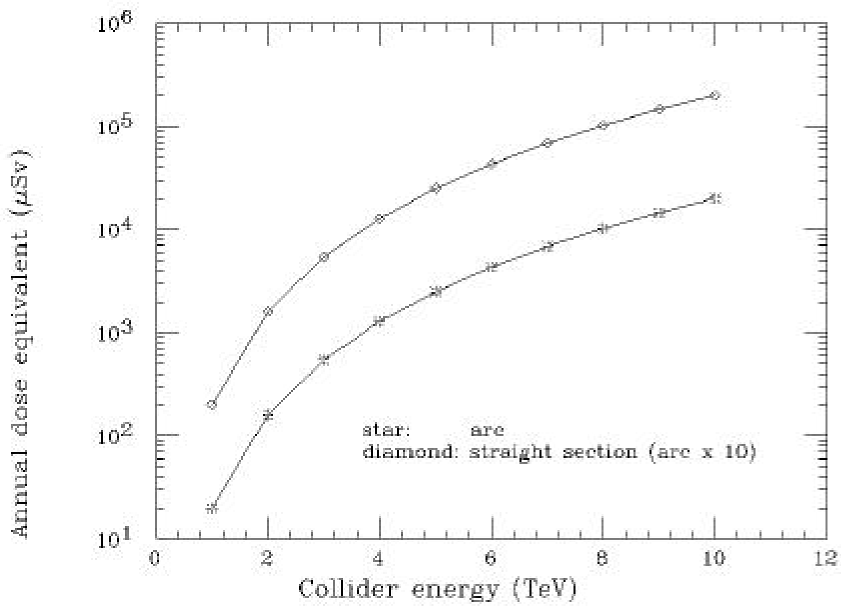

Fig. 1. Dose equivalent due to neutrino radiation at 36 km distance (collider at 100 m depth)

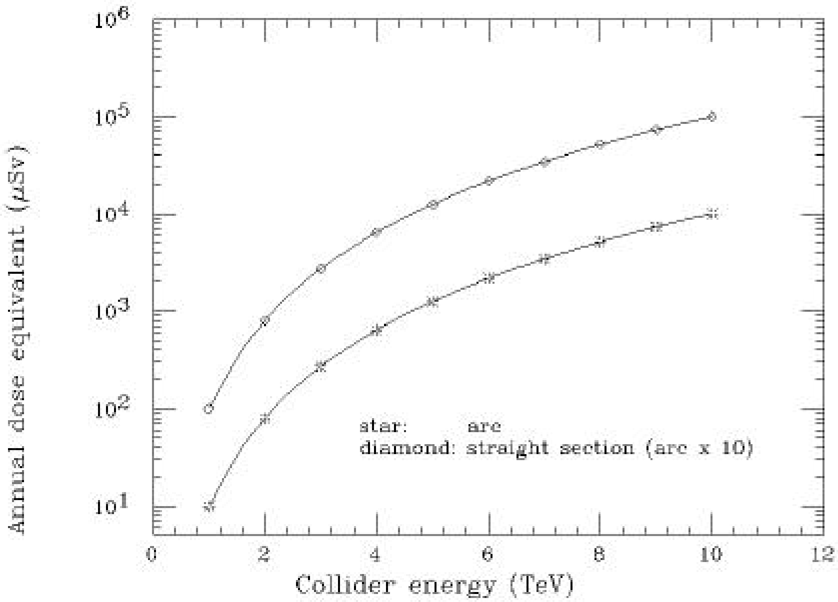

Fig. 2. Dose equivalent due to neutrino radiation at 51 km distance (collider at 200 m depth)

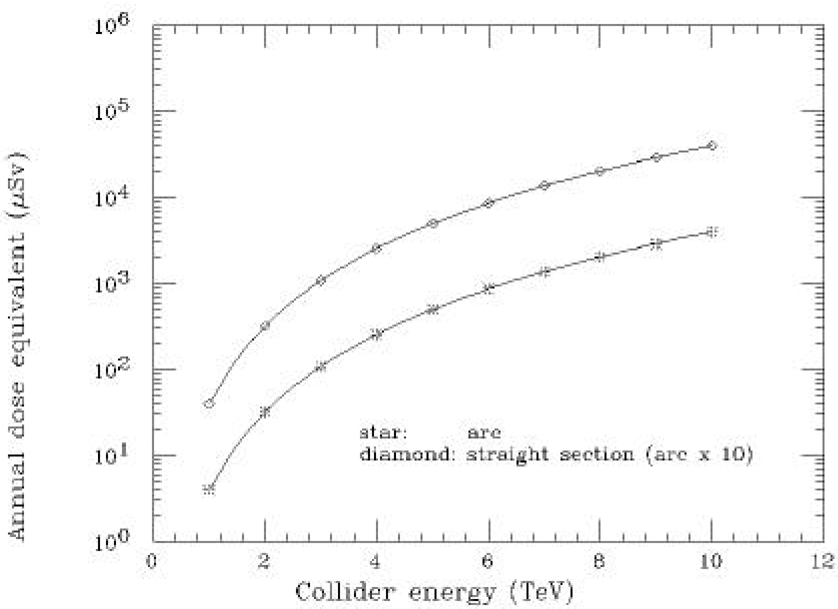

Fig. 3. Dose equivalent due to neutrino radiation at 80.5 km distance (collider at 500 m depth)

To distribute the radiation and lower the average dose equivalent, Fermilab has proposed to vary the production direction of the neutrino beam by instituting a vertical wave in the collider ring [7]. To limit the number of "hot spots" King has suggested to decrease the number of straight sections by designing a magnet lattice with combined function magnets, where bending and focussing of the beam is achieved in the same magnet thus avoiding the straight sections in-between dipoles and quadrupoles [10]. However, this solution cannot possible avoid the need for a few long straight sections.

In case a number of radiation "hot spots" are unavoidable, one can think to fence off the area where the neutrino beam emerges from the ground. At the location where the neutrinos emerge from the earth, at a distance L given by expression (5), the radial extent of the region traversed by the radiation is (Fig. 4):

$$b = a/\phi$$

in which $a \approx 2\theta L$, θ is the opening half-angle of the neutrinos and ϕ is the angle subtended by L with respect to the Earth centre. The height of the radiation fan above ground is a function of the distance z from the point where the radiation cuts the Earth surface:

$$h \approx z \tan\phi$$

The relevant geometrical parameters for a muon collider of increasing energy placed at increasing depth are given in Table 4.

Table 4. Geometrical parameters for a few representative cases of muon colliders of increasing energy installed underground at increasing depth.

E_0 (TeV)	d (m)	L (km)	ϕ (rad)	z (km)	h (m)	θ (μrad)	a (m)	b (m)
1	100	36	5.6×10^{-3}	10	56	106	7.6	1360
2	100	36	5.6×10^{-3}	10	56	53	3.8	680
5	200	51	8×10^{-3}	10	80	21	2.1	260
10	500	80.5	12.5×10^{-3}	10	125	11	1.8	145

Another solution which can perhaps be conceived is to transport the collimated neutrino beam from a "hot spot" in a beam pipe for the last tract (let us say one kilometre) before it emerges from the ground. In this way the secondary radiation produced in the upstream material would be absorbed in the earth shield surrounding the pipe. Evacuating the pipe to a modest vacuum of 1 torr would prevent even the small production of secondaries which occurs in air (already a factor 10^{-3} with respect to the earth shielding). One has of course to make sure that the "pure" neutrino beam emerging from the Earth surface does not interact with any other material (such as buildings) before lifting to a sufficient height from the ground.

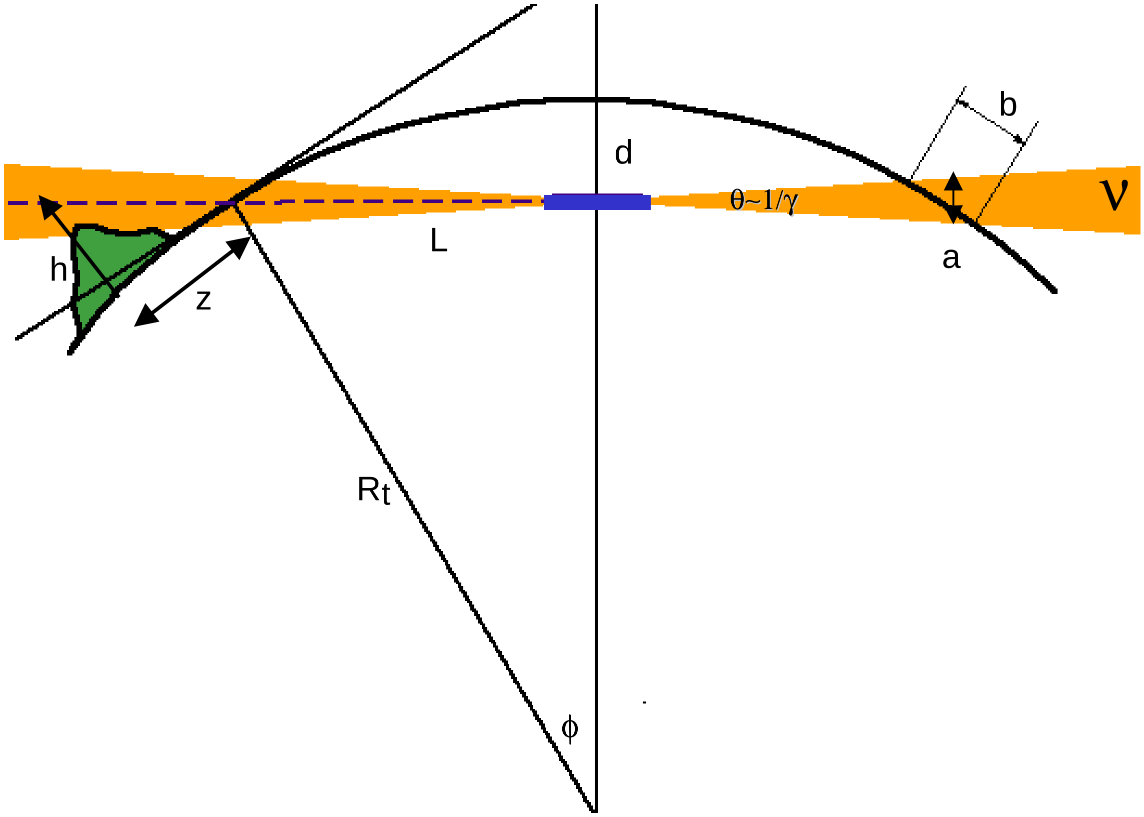

Fig. 4. Some typical geometrical features of the neutrino radiation from an underground muon collider: $L^2 = 2R_t d - d^2$, $\sin\phi = L/R_t$, $h \approx z \tan\phi$, $\theta \approx 1/\gamma$, $a \approx 2\theta L$, $b \approx a/\phi$. R_t is the radius of the Earth.

The last, obvious, solution to decrease the neutrino radiation dose is to decrease the muon current in the ring. This would imply changes to the machine parameters requiring substantial R&D work. The use of Optical Stochastic Cooling and/or beam-beam tune-shift compensation [13] are speculative proposals to this end. But the study of parameter sets for muon colliders in the CoM energy range of 5 TeV and above still offers much scope for invention.

It should be recalled that the present estimates only represent a first approach. A more comprehensive evaluation of the problem may require a detailed Monte Carlo calculation by a code treating neutrino transport, which at present is only provided by MARS [8]. In addition to the collider energy, other relevant parameters to be considered are the number, location and length of the straight sections. The enhancement factor of the neutrino fluence due to a straight section is a critical issue which needs to be carefully assessed. Important is also the choice of orientation, positioning and possible tilting of the collider ring, as well as the site selection of the accelerator complex. Disregarding "exotic" solutions such as installing the collider on top of a mountain (in order that the radiation halo is above ground level) or at a few hundred metre depth in the sea, in the case of CERN the site selection is limited to the French region presently housing the SPS and LEP. The actual orography of the region must be taken into account, as locally there may be significant deviations in the inclination of the ground from the spherical approximation used for the present assessments.

References

[1] J.I. Collar, Biological effects of stellar collapse neutrinos, Phys. Rev. Lett. 76 (1996), 999-1002.

[2] J.D. Cossairt and E.T. Marshall, Comment on “Biological effects of stellar collapse neutrinos”, Phys. Rev. Lett. 78 (1997), 1394-1395.

[3] J.D. Cossairt, N.L. Grossman and E.T. Marshall, Assessment of dose equivalent due to neutrinos, Health Phys. 73 (1997), 894-898.

[4] G.R. Stevenson and H.H. Vincke, Initial estimates of radiological parameters of environmental interest for the CERN/INFN Gran-Sasso neutrino project, CERN Internal Report TIS-RP/IR/98-13 (1998).

[5] T. Rancati, Deep inelastic scattering e dose da neutrini, Scuola di Specializzazione in Fisica Sanitaria, Università di Milano, A.A. 1997/98.

[6] N.V. Mokhov and A. Van Ginneken, Muon collider neutrino radiation, to be published.

[7] C.J. Johnstone and N.V. Mokhov, Shielding the muon collider interaction region, Fermilab Internal Note.

[8] O.E. Krivosheev and N.V. Mokhov, New MARS and its applications, Proceedings of the Third Workshop on Simulating Accelerator Radiation Environments (SARE3), KEK, Tsukuba, Japan, 7-9 May 1997, KEK Proceedings 97-5 (1997), pp.12-20.

[9] B.J. King, Discussion on muon collider parameters at center of mass energies from 0.1 TeV to 100 TeV, Proc. of the 6th European Particle Accelerator Conference, Stockholm, Sweden, 22-26 June 1998 (in press, see Bruce King's Web page at http://pubweb.bnl.gov/people/bking/).

[10] B.J. King, A characterisation of the neutrino-induced radiation hazard at TeV-scale muon colliders, to be published (see Bruce King's Web page at http://pubweb.bnl.gov/people/bking/).

[11] C.M. Ankenbrandt et al., Status of muon collider research and development and future plans (draft), BNL-65-623, Fermilab-PUB-98/179, LBNL-41935 (1998).

[12] CERN Radiation Safety Manual, Safety Code F (1996).

[13] A. Skrinski, Towards ultimate luminosity polarized muon colliders (problems and prospects), Workshop on the Physics Potential and Development of $\mu^+\mu^-$ colliders, AIP Conf. Proc. 441 (1997), pp 249-264.

Appendix

Table A.1. Fluence to dose equivalent conversion coefficients for neutrinos. Data are extracted from Fig. 1 of ref. [3]. In the neutrino energy range from 0.5 GeV to 10 TeV, the fluence to dose equivalent conversion coefficients $C(E_n)$ can be fitted by the expression:

$$\log_{10} C(E_n) = 2 \log_{10} (E_n) - 15$$

Energy (GeV)	Dose equivalent per unit fluence (µSv cm^2)	
	Unshielded	Total (≈ shielded)
2 x 10^{-4}		10^{-27}
5 x 10^{-4}		10^{-26}
1 x 10^{-3}		5 x 10^{-26}
2 x 10^{-3}		2 x 10^{-25}
5 x 10^{-3}		3 x 10^{-24}
1 x 10^{-2}		2 x 10^{-23}
2 x 10^{-2}		3 x 10^{-22}
5 x 10^{-2}	2 x 10^{-21}	1 x 10^{-20}
1 x 10^{-1}	8 x 10^{-20}	2 x 10^{-19}
2 x 10^{-1}	2.5 x 10^{-18}	4 x 10^{-18}
5 x 10^{-1}	8 x 10^{-17}	2.5 x 10^{-16}
1	1.5 x 10^{-16}	1 x 10^{-15}
2	4 x 10^{-16}	4 x 10^{-15}
5	2 x 10^{-15}	2.5 x 10^{-14}
10	4 x 10^{-15}	1 x 10^{-13}
20	8 x 10^{-15}	4 x 10^{-13}
50	4 x 10^{-14}	2.5 x 10^{-12}
1 x 10^{2}	1 x 10^{-13}	1 x 10^{-11}
2 x 10^{2}	3 x 10^{-13}	4 x 10^{-11}
5 x 10^{2}	1.5 x 10^{-12}	2.5 x 10^{-10}
1 x 10^{3}	4 x 10^{-12}	1 x 10^{-9}
2 x 10^{3}	1 x 10^{-11}	4 x 10^{-9}
5 x 10^{3}	5 x 10^{-11}	2.5 x 10^{-8}
1 x 10^{4}	1.5 x 10^{-10}	1 x 10^{-7}