

(Some)
Accelerator Physics of a Muon Collider

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Seminar at the Princeton Plasma Physics Laboratory

Muon Collider main page:

http://www.cap.bnl.gov/mumu/mu_home_page.html

Muon Collider R&D Status Report:

http://www.cap.bnl.gov/mumu/status_report.html

Princeton Muon Collider page:

<http://puhep1.princeton.edu/mumu/>

What is a Muon Collider?

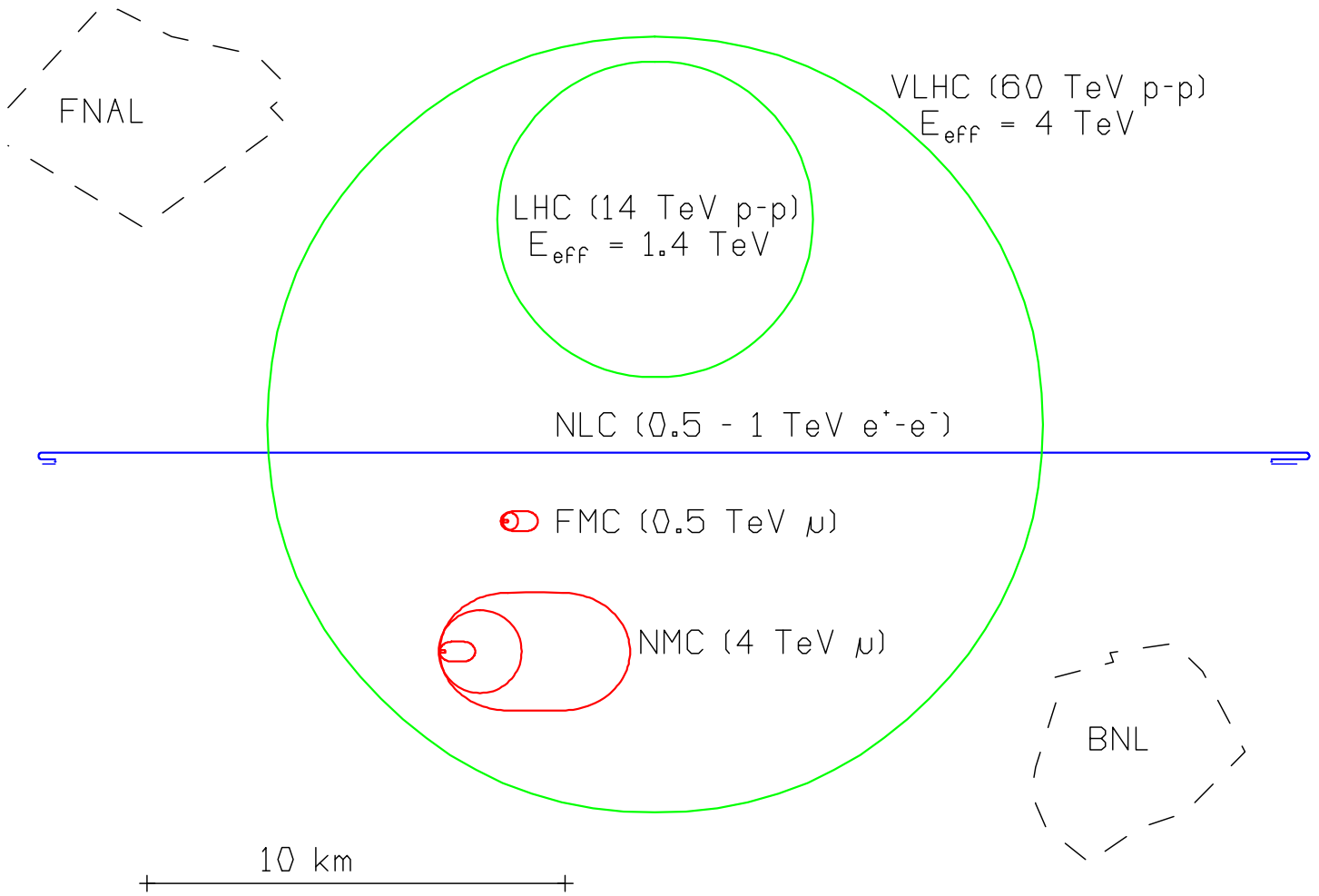
An accelerator complex in which

- Muons (both μ^+ and μ^-) are collected from pion decay following a pN interaction.
- Muon phase volume is reduced by 10^6 by ionization cooling.
- The cooled muons are accelerated and then stored in a ring.
- $\mu^+\mu^-$ collisions are observed over the useful muon life of ≈ 1000 turns at any energy.
- Intense neutrino beams and spallation neutron beams are available as byproducts.

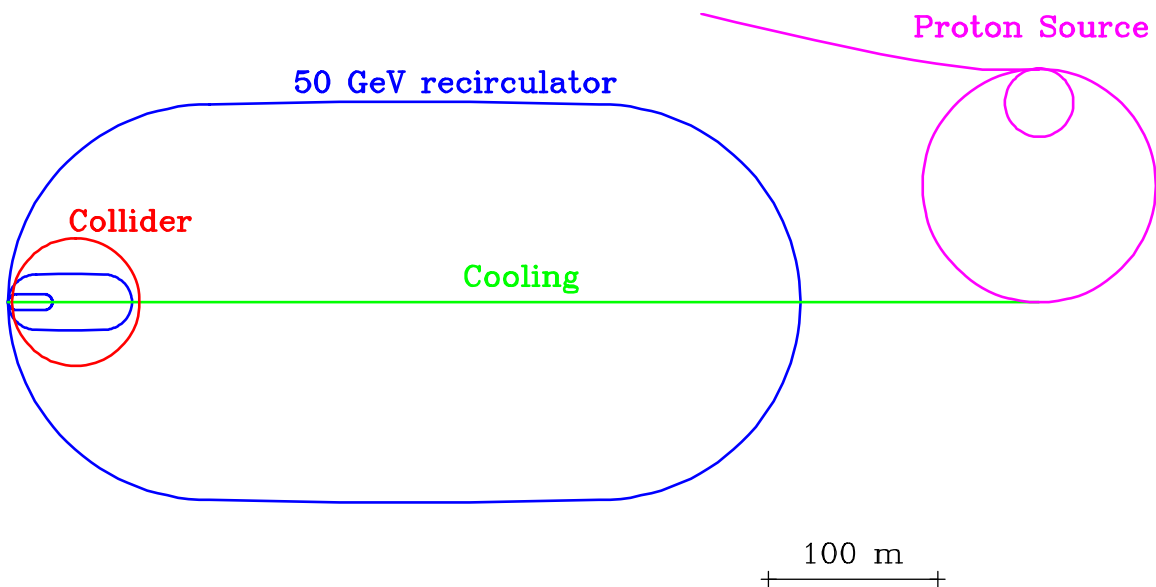
Muons decay: $\mu \rightarrow e\nu \quad \Rightarrow$

- Must cool muons quickly (stochastic cooling won't do).
- Detector backgrounds at LHC level.
- Potential personnel hazard from ν interactions.

Footprints

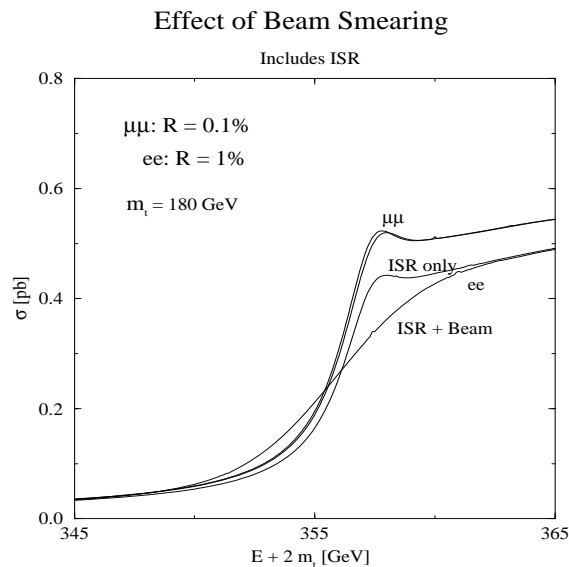


A First Muon Collider to study light-Higgs production:



The Case for a Muon Collider

- More affordable than an e^+e^- collider at the TeV (LHC) scale.
- More affordable than either a hadron or an e^+e^- collider for (effective) energies beyond the LHC.
- Precision initial state superior even to e^+e^- .



- Initial machine could produce light Higgs via s -channel:
 Higgs coupling to μ is $(m_\mu/m_e)^2 \approx 40,000 \times$ that to e .
 Beam energy resolution at a muon collider $< 10^{-5}$,
 \Rightarrow Measure Higgs width.
 Add rings to 3 TeV later.
- Neutrino beams from μ decay about 10^4 hotter than present.

Future Frontier Facilities

(Will the U.S. have one?)

- **Hadron collider** (LHC, SSC): \approx \$100k/m [magnets].
 \approx 2 km per TeV of CM energy.
Ex: LHC has 14-TeV CM energy, 27 km ring, \approx \$3B.
- **Linear e^+e^- collider** (SLAC, NLC(?)): \approx \$200k/m [rf].
 \approx 20 km per TeV of CM energy;
But a lepton collider needs only \approx 1/10 the CM energy to have equivalent physics reach to a hadron collider.
Ex: NLC, 1.5-TeV CM energy, 30 km long, \approx \$6B (?).
- **Muon collider**: \approx \$1B for source/cooler + \$100k/m for rings
Well-defined leptonic initial state.
 $m_\mu/m_e \approx 200 \Rightarrow$ Little beam radiation.
 \Rightarrow Can use storage rings.
 \Rightarrow Smaller footprint.
Technology: closer to hadron colliders.
 \approx 6 km of ring per TeV of CM energy.
Ex: 3-TeV muon collider \approx \$3B (?).

The Muon Collider Collaboration

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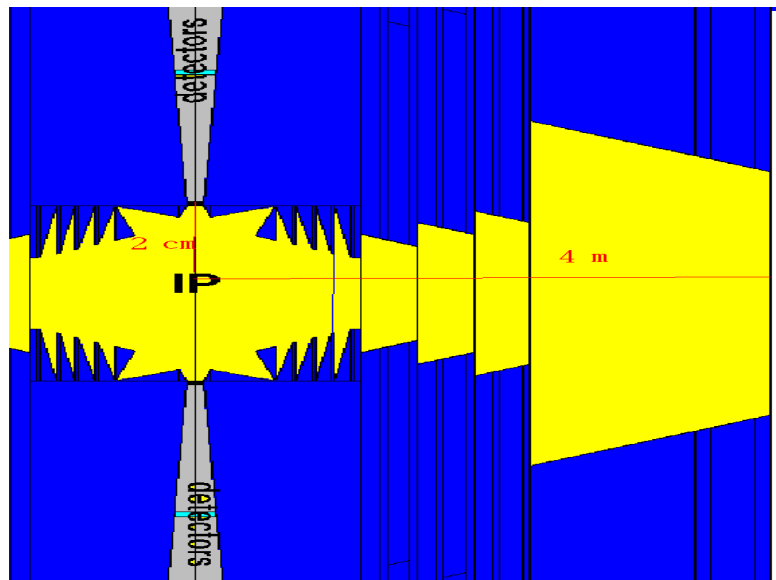
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Spokesperson: R.B. Palmer

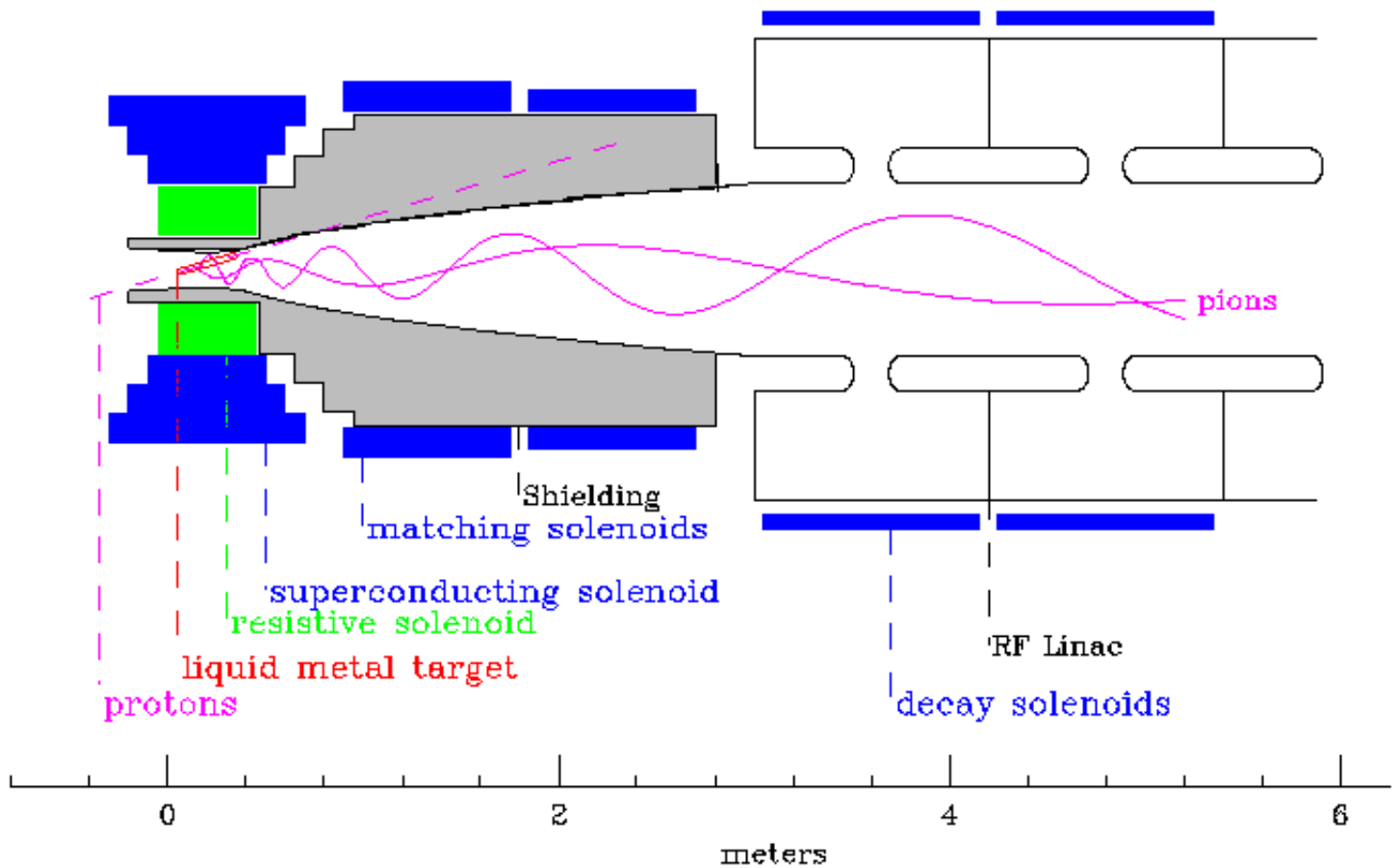
Technical Challenges

- 16-GeV proton driver, 15 Hz, 4-MW beam power, 1-ns bunch length.
- **Targetry and Capture**
- **Muon Cooling**
- Acceleration
- Storage rings
- Interaction region and detector design

A muon's view of the interaction region:

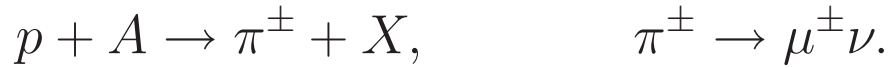


Overview of Targetry for a Muon Collider



- $1.2 \times 10^{14} \mu^\pm/\text{s}$ via π -decay from a 4-MW proton beam.
- Cooling jacket around stationary target would absorb too many pions.
- Liquid-metal jet target: Ga, Hg, or solder (Bi/In/Pb/Sn).
- 20-T capture solenoid followed by a 1.25-T π -decay channel with phase-rotation via rf (to compress energy of the muon bunch).

Magnetic Bottle Around the Target

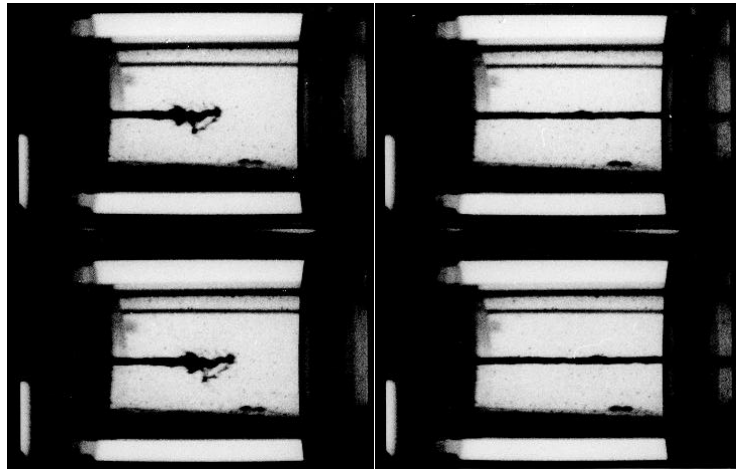


- In high-energy interactions, pions are produced with average transverse momentum $P_\perp \approx 200 \text{ MeV}/c$.
- Goal: Capture pions with $P_\perp < 225 \text{ MeV}/c$.
- Solution: Surround the target with a 20-T solenoid magnet, whose field drops to 1.25 T in the pion-decay channel.
- Peak field at upstream end of target \Rightarrow (some) backwards pions reflected off the high-field region of the **magnetic bottle**.
- As pions advance into the weak-field region, their P_\perp drops, \Rightarrow Confined to smaller radii than if produced in a weak-field.
- Adiabatic invariant: $\Phi = \pi r^2 B$ as B drops from 20 to 1.25 T.
- $r = P_\perp / eB =$ radius of helix.

$$\Rightarrow \frac{P_{\perp,f}}{P_{\perp,i}} = \sqrt{\frac{B_f}{B_i}} = 0.25, \quad [\text{and} \quad P_{\parallel,f} > P_{\parallel,i}].$$

Targetry Issues

- 1-ns beam pulse \Rightarrow shock heating of target.
 - Resulting pressure wave may disperse liquid (or crack solid).
 - Damage to target chamber walls?
 - Magnetic field will damp effects of pressure wave.
- Eddy currents arise as metal jet enters the capture magnet.
 - Jet is retarded and distorted, possibly dispersed.
 - Hg jet studied at CERN, but not in beam or magnetic field:



High-speed photographs of mercury jet target for CERN-PS-AA (laboratory tests)
4,000 frames per second, Jet speed: 20 ms⁻¹, diameter: 3 mm, Reynold's Number:>100,000
A. Poncet

- Targetry area also contains beam dump.
 - Need 4 MW of cooling.
 - Harsh radiation environment for magnets and rf.

Effect of a Short Beam Pulse on a Liquid?

Will shock heating disperse the target violently?

Simple model to estimate magnitude of shock pressure wave:

Beam energy heats liquid (no heat flow);

Liquid expands causing strain (shock wave);

Liquid 'tears' if pressure exceeds tensile strength.

Fact: tensile strength (T_S) is about $0.002E$ (Young's modulus) in most metals.

$$\Delta U[\text{J/gm}] = C\Delta T = \frac{C\Delta l}{\alpha l} = \frac{CP}{\alpha E} \approx 0.002\frac{C}{\alpha},$$

when $P = T_S$:

Ex: Gallium: $\alpha \approx 2 \times 10^{-5}/\text{K}$; $C_P \approx 0.3 \text{ J/gm-K}$, tears when

$$\Delta U \approx (0.002)(0.3)/(2 \times 10^{-5}) \approx 30 \text{ J/gm}.$$

This is roughly the nominal energy deposition in the target!

Magnetohydrodynamics

Field \mathbf{E}' inside a conductor with velocity $v \ll c$ in field \mathbf{B} :

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \quad (\text{MKSA}).$$

$$\begin{aligned} \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, & \nabla \times \mathbf{B} &= \mu_0 \mathbf{j}, & \mathbf{j} &= \sigma \mathbf{E}' = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \\ \Rightarrow & & \frac{\partial \mathbf{B}}{\partial t} &= \frac{\nabla^2 \mathbf{B}}{\mu_0 \sigma} + \nabla \times (\mathbf{v} \times \mathbf{B}). \end{aligned}$$

\Rightarrow Field diffusion time into long cylinder: $\tau = \mu_0 \sigma r^2$.

Ex: $\sigma_{\text{Hg}} = \sigma_{\text{copper}}/50$, $r = 1$ cm,

$\Rightarrow \tau \approx 4\pi \times 10^{-7} \cdot 10^6 \cdot (10^{-2})^2 \approx 10^{-4}$ s.

Magnetic Reynolds number : $\mathcal{R} = \frac{\tau v}{D} \approx \frac{10^{-4} \text{s} \cdot 10 \text{m/s}}{0.3 \text{m}} = 0.003$,

for motion through a solenoid of diameter $D = 0.3$ m.

\Rightarrow The liquid is a “poor” conductor, and the field penetrates quickly.

Eddy Current Effects on Conducting Liquid Jets

- In frame of jet, changing magnetic field induces eddy currents.
- Lenz: Forces on eddy current oppose motion of jet.
- Longitudinal drag force \Rightarrow won't penetrate magnet unless jet

has a minimum velocity: $\sigma = \sigma_{\text{Cu}}/60$, $\rho = 10 \text{ g/cm}^3$, \Rightarrow

$$v_{z,\text{min}} \approx \frac{\sigma r^2 B_0^2}{6\rho D} \approx 60 \text{ m/s} \left[\frac{r}{1 \text{ cm}} \right] \left[\frac{r}{D} \right] \left[\frac{B_0}{20 \text{ T}} \right]^2.$$

Ex: $B_0 = 20 \text{ T}$, $r = 1 \text{ cm}$, $D = 20 \text{ cm}$, $\Rightarrow v_{\text{min}} = 3 \text{ m/s}$.

- Drag force is larger at larger radius \Rightarrow planes deform into cones:

$$\frac{\Delta z(r)}{r} \approx \frac{\sigma r^2 B_0^2 \alpha}{12\rho v_z} \approx -3\alpha \left[\frac{r}{1 \text{ cm}} \right] \left[\frac{B_0}{20 \text{ T}} \right]^2 \left[\frac{10 \text{ m/s}}{v} \right].$$

Ex: $\alpha = L/D = 2$, $r = 1 \text{ cm}$, $v = 10 \text{ m/s}$ $\Rightarrow \Delta z = 6 \text{ cm}$.

- Radial pressure: compression as jet enters magnet, expansion as it leaves:

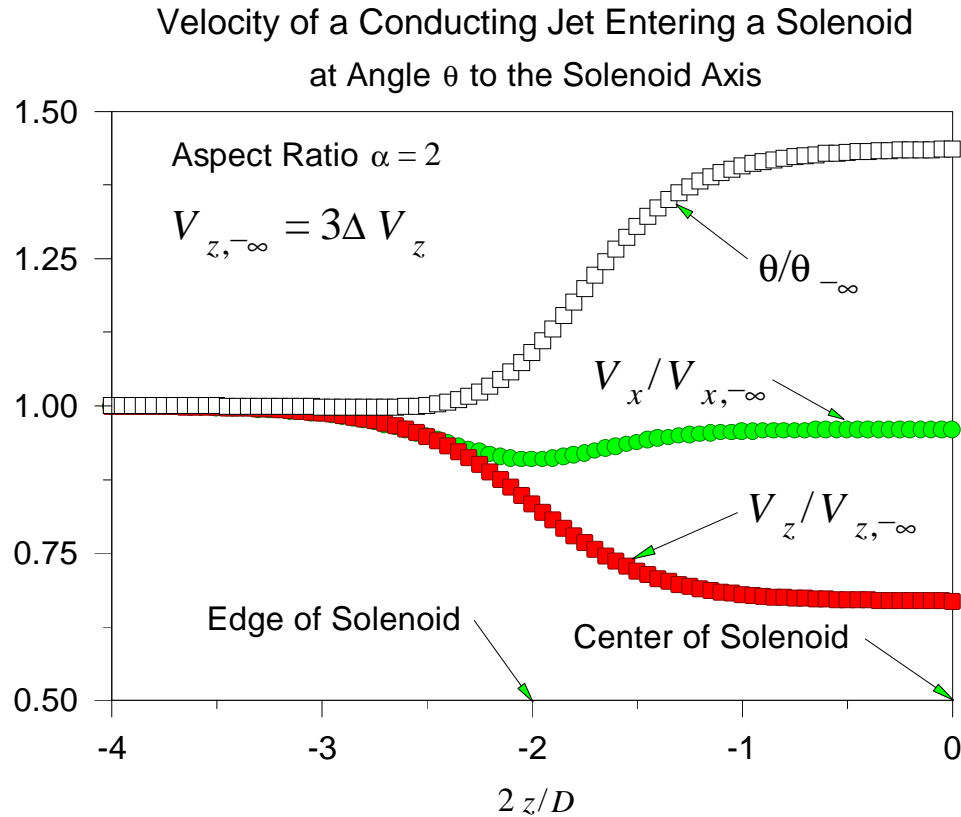
$$P_r \approx \frac{\sigma r^2 B_0^2 v_z}{8D} \approx 50 \text{ atm.} \left[\frac{r}{1 \text{ cm}} \right] \left[\frac{r}{D} \right] \left[\frac{B_0}{20 \text{ T}} \right]^2 \left[\frac{v}{10 \text{ m/s}} \right].$$

Ex: $P = 2.5 \text{ atm}$ for previous parameters.

- Will the jet break up into droplets?

- Jet at angle θ to magnet axis \Rightarrow transverse drag.

But, $\Delta v_x = \Delta v_z/8$.



$\Rightarrow \theta$ increases as jet enters magnet.

Ex: $\alpha = 2, v = 3\Delta v_z \Rightarrow \theta_{in} = 1.5\theta_{out}$.

- Drag and shear are smaller for larger initial velocity, but pressure rises with velocity.
- Is there a safe working regime?
- Need both FEA analysis and **lab tests**.

Magnetic Damping of Radial Perturbations

If jet blows apart radially, the flux thru rings of metal changes,
 \Rightarrow Eddy current damping.

$$\Rightarrow \Delta P_{r,\text{damp}} \approx \sigma r v_r B_0^2.$$

Ex: Radial pinch $\Rightarrow v_r \approx \frac{\sigma r B_0^2}{4\rho}$, $\Rightarrow P_{r,\text{damp}} \approx \frac{\sigma^2 r^2 B_0^4}{4\rho} \gtrsim P_{r,\text{pinch}}$.

Ex: If beam shock $\Rightarrow v_r \approx 1,000$ m/s,

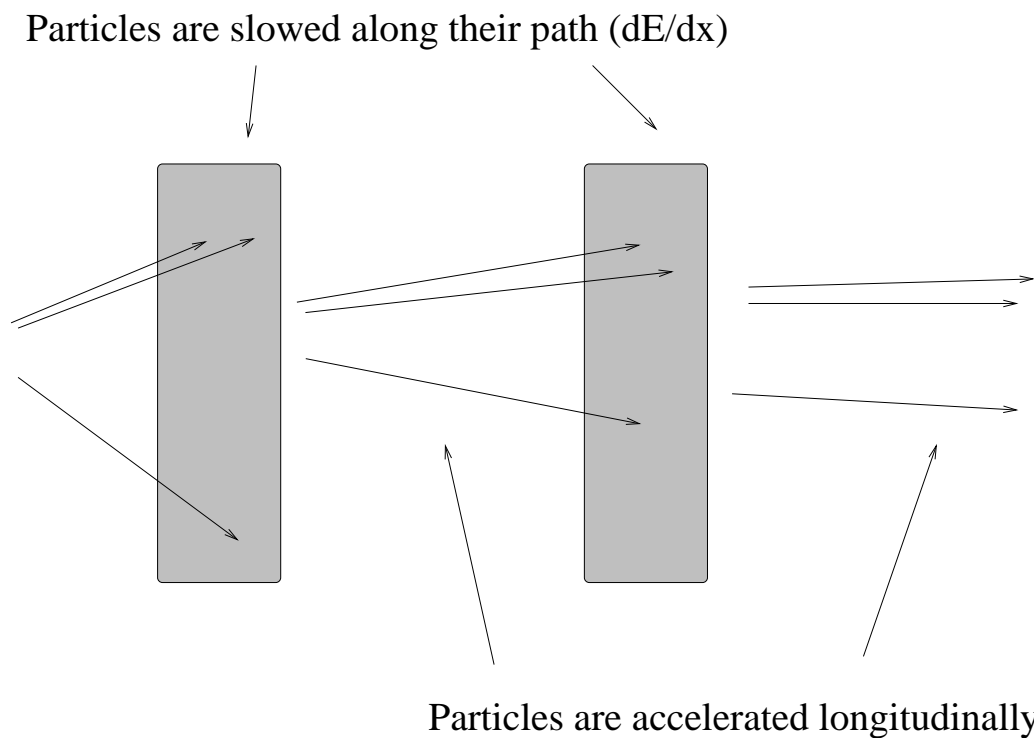
then $P_{r,\text{damp}} \approx 4$ GPa $\approx T_{S,\text{steel}}$.

Also, a strong magnetic field damps the Rayleigh instability (breakup of a jet into droplets due to surface tension) [Chandrasekhar].

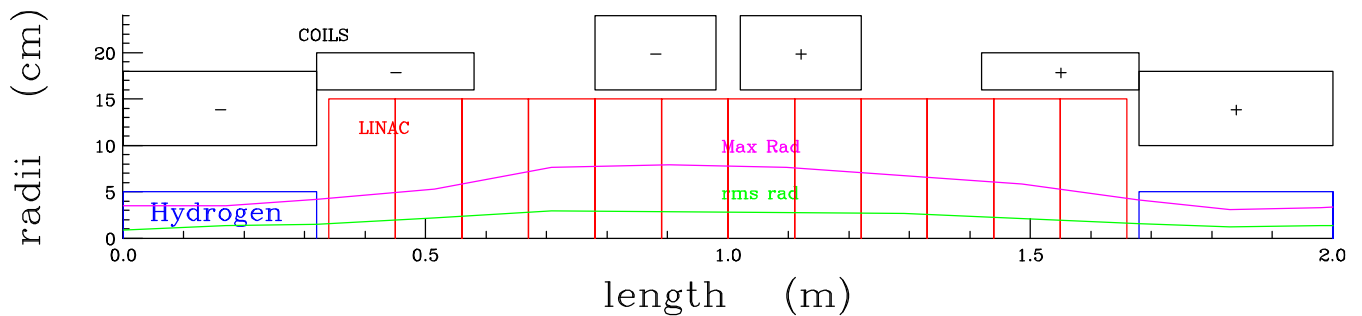
Will test liquid jets in proton beam at Brookhaven National Lab,
and in 20-T magnet at National High Magnetic Field Lab.

Ionization Cooling

- Ionization: takes momentum away.
- RF acceleration: puts momentum back along z axis.
- \Rightarrow Transverse “cooling”.

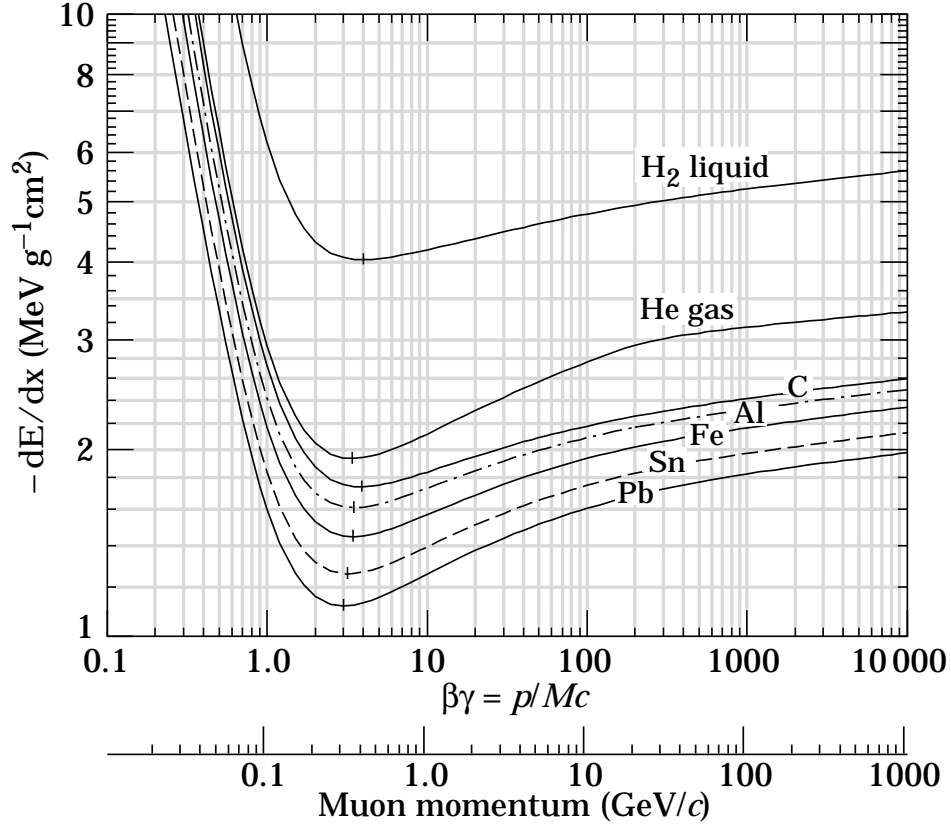


- Use channel of LH_2 absorbers, rf cavities and alternating solenoids (to avoid buildup of angular momentum).



Ionization Cooling Theory

Transverse cooling by ionization, heating by multiple scattering:



$$\frac{d\epsilon_{N,\perp}}{ds} = -\frac{1}{\bar{\beta}^2} \frac{dE_\mu}{ds} \frac{\epsilon_{N,\perp}}{E_\mu} + \frac{\beta_\perp (0.014)^2}{2\bar{\beta}^3 E_\mu m_\mu L_R},$$

$\epsilon_{N,\perp} = \sigma_x \sigma_{P_x} / m_\mu c =$ normalized transverse emittance,

$$\bar{\beta} = \bar{v}/c, \quad \bar{\gamma} = 1/\sqrt{1 - \bar{\beta}^2}$$

$\beta_\perp = \sigma_x / \sigma_{x'}$ = Betatron function at the absorber,

$$\epsilon_\perp = \frac{\epsilon_{N,\perp}}{\bar{\gamma}\bar{\beta}}, \quad \sigma_x = \sqrt{\epsilon_\perp \beta_\perp}, \quad \sigma_{x'} = \frac{\sigma_{P_x}}{P} = \sqrt{\frac{\epsilon_\perp}{\beta_\perp}},$$

L_R = Radiation length of absorber.

$$\Rightarrow \text{Equilibrium } \epsilon_{N,\perp} \propto \frac{\beta_{\perp}}{\bar{\beta} L_R (dE_{\mu}/ds)}.$$

\Rightarrow Low- Z absorber (liquid hydrogen is best),

\Rightarrow Put absorber at low- β_{\perp} (beam-waist) where angles are large, so multiple scattering hurts less.

\Rightarrow Need strong focusing (15-T solenoids, Li lens...).

Economics favor $\bar{\beta} < 1$, $\bar{\gamma} \approx 1$, since must restore the beam energy ($\propto \bar{\gamma} - 1$) many times.

However, $\beta(dE_{\mu}/ds) \propto \beta^{-2/3}$ for low β , so cooling is less effective at smaller β .

Present scenario: Cool at $\bar{\beta} = 0.86$, $P_{\mu} = 180$ MeV/ c ,
KE = 100 MeV.

The Angular Momentum Problem

A solenoid with field B_z has vector potential $A_\phi = rB_z/2$.

The **canonical momentum**, $\Pi = \mathbf{P} + e\mathbf{A}/c$, is conserved.

The **canonical angular momentum** L is also conserved:

$$\mathbf{L} = \mathbf{r} \times \Pi = \mathbf{r} \times (\mathbf{P} + e\mathbf{A}/c).$$

$$\Rightarrow L_z = r\Pi_\phi = rP_\phi + er^2 B_z/2c.$$

So, if the mechanical transverse momentum, P_ϕ , has been “cooled” to zero inside the solenoid, the charge will emerge with

$$L_{z,\text{out}} = L_{z,\text{in}} = er^2 B_{z,\text{in}}/2c.$$

\Leftrightarrow The fringe field of the solenoid imparts an undesirable kick.

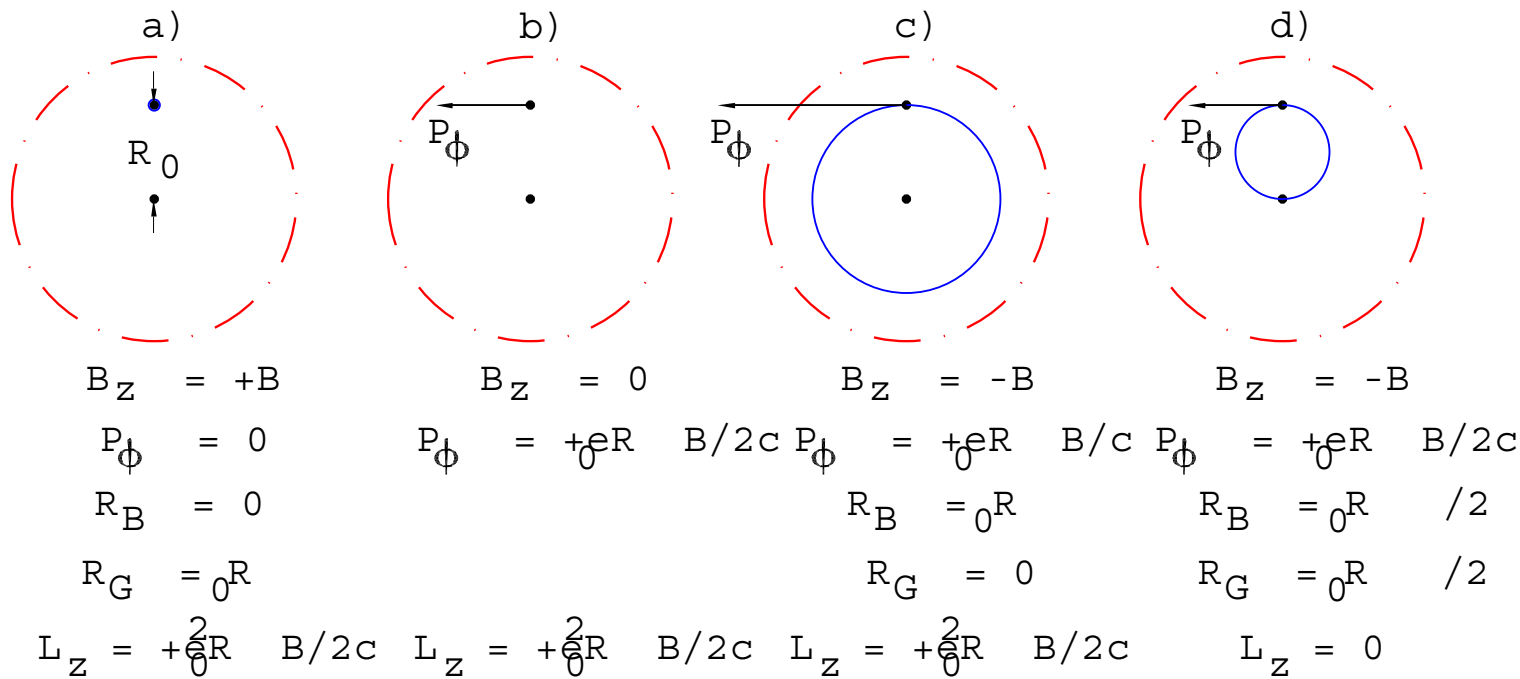
Solution: Alternating Solenoids

Suppose after leaving field B_z , the beam enters field $-B_z$. Then,

$$er^2 B_{z,\text{in}}/2c = L_{z,1} = L_{z,2} = rP_{\phi,2} - er^2 B_z/2c,$$

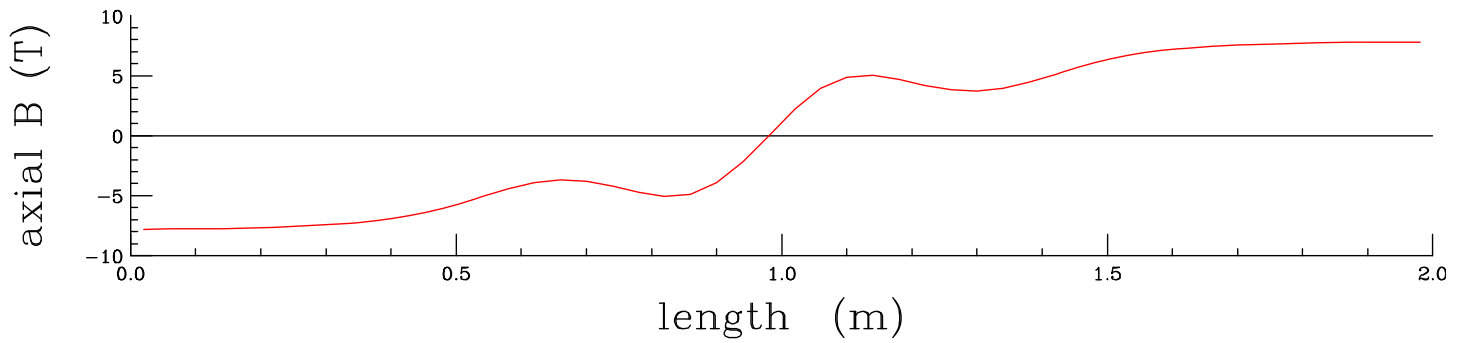
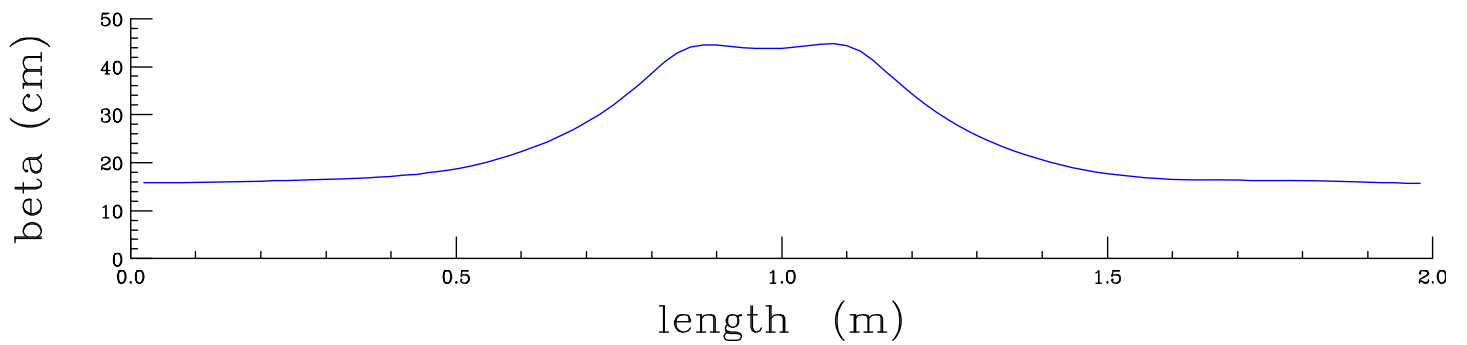
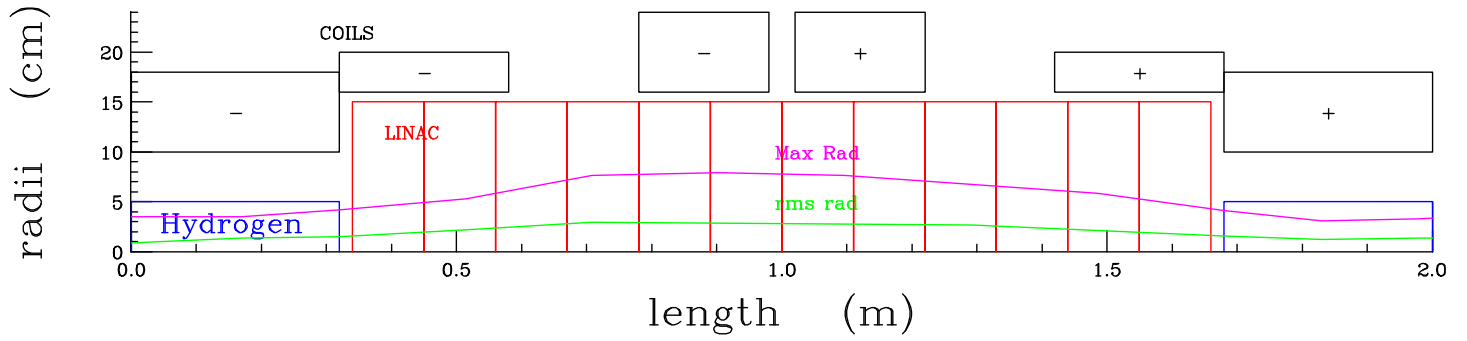
$$\Rightarrow P_{\phi,2} = erB_z/c.$$

Now, if cool in region 2 until $P'_{\phi,2} = erB_z/2c$, and exit, the particle will end up with $P_{\phi} = 0$.



In practice, alternate the fields many times, keeping the canonical momentum always near zero, while the mechanical momentum undergoes damped oscillations.

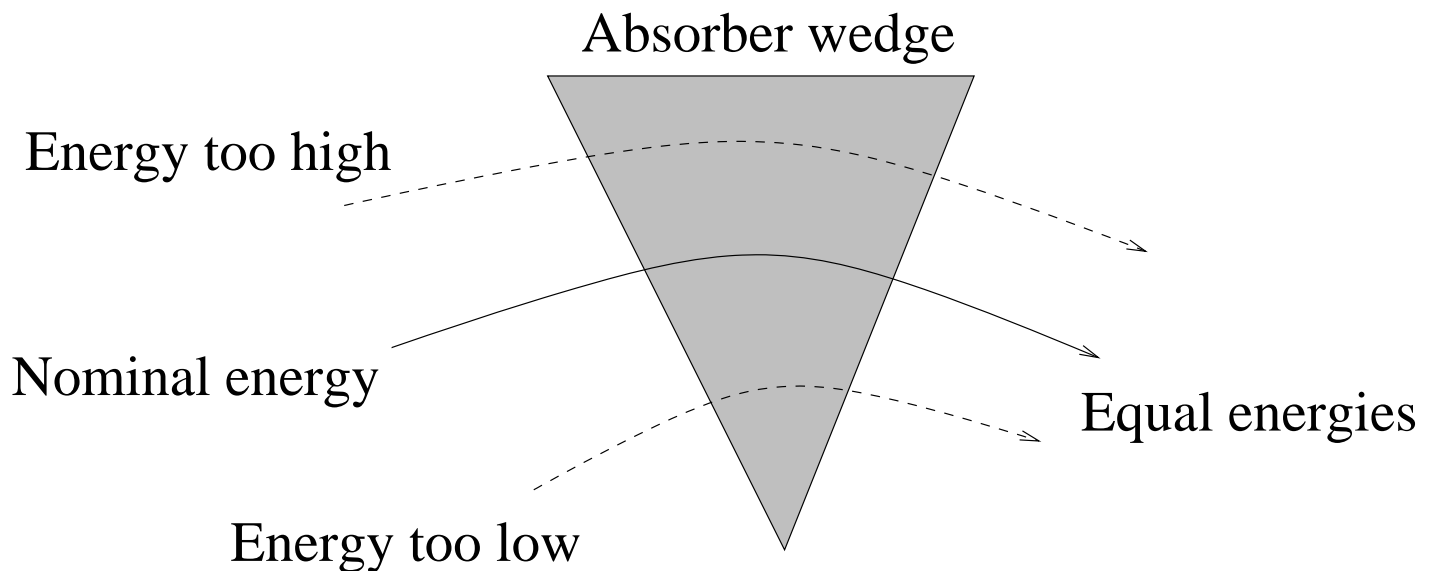
Cooling in a Channel of Alternating Solenoids



But the Energy Spread Rises due to “Straggling”

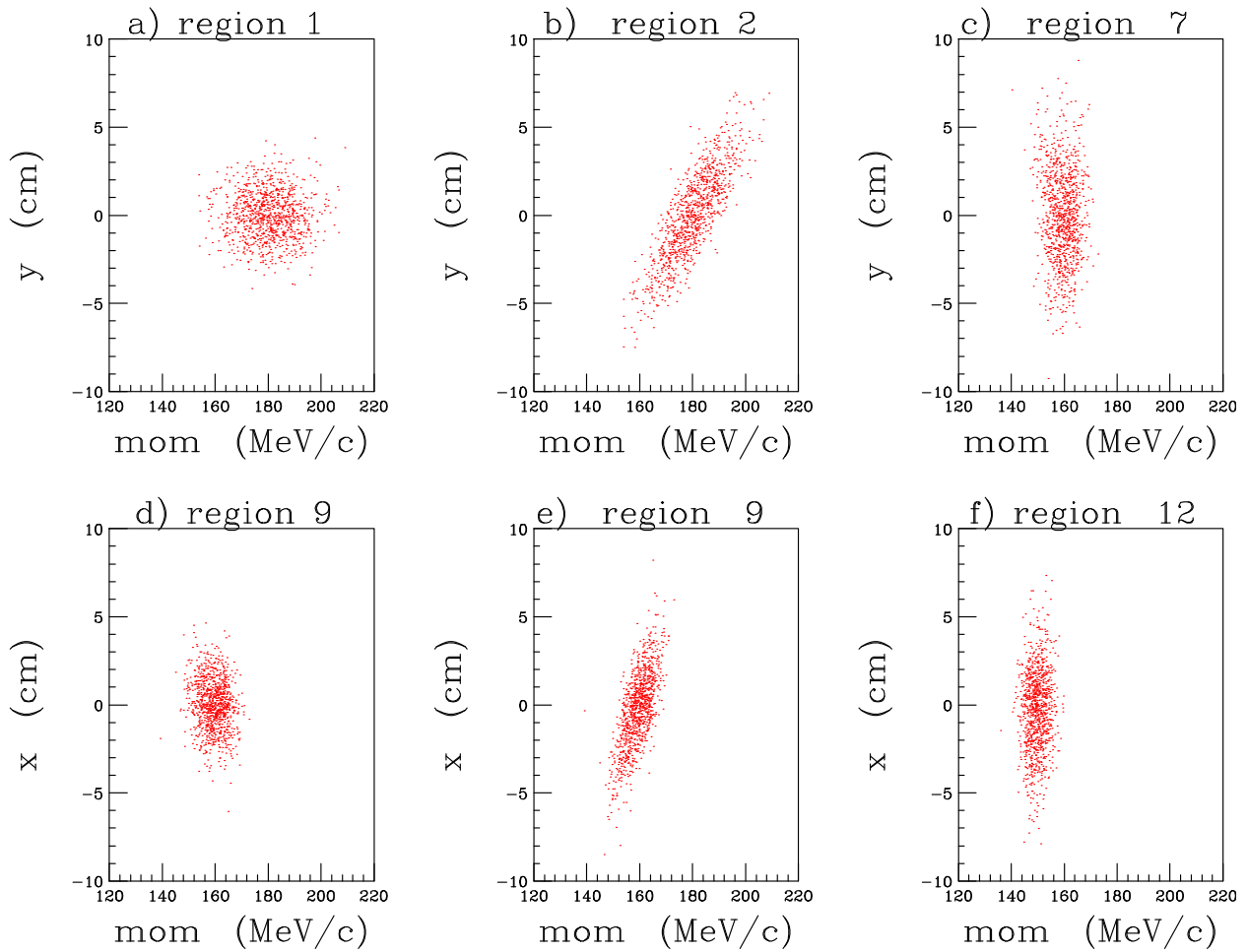
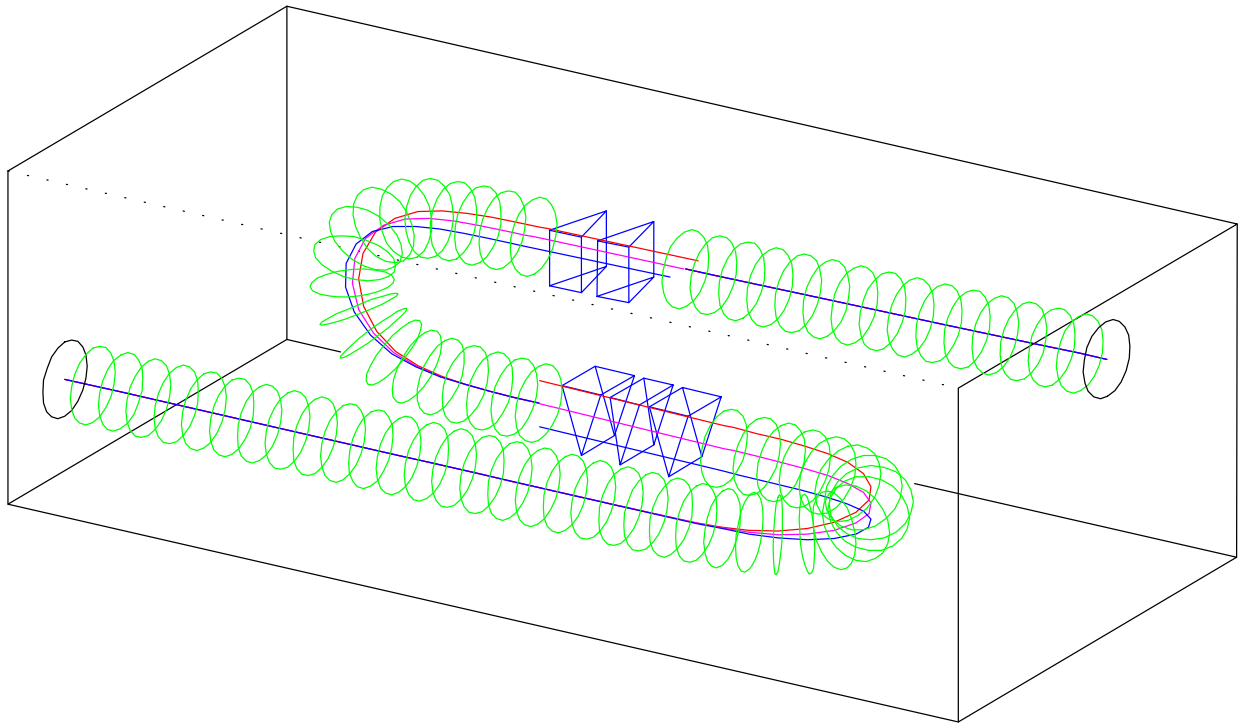
$$\frac{d(\Delta E_\mu)^2}{ds} = -2 \frac{d\left(\frac{dE_\mu}{ds}\right)}{dE_\mu} (\Delta E_\mu)^2 + \frac{d(\Delta E_\mu)_{\text{straggling}}^2}{ds}.$$

- Both terms are positive if operate below minimum of dE_μ/ds curve.
- \Rightarrow Must exchange longitudinal and transverse emittance frequently to avoid beam loss due to bunch spreading.
- Can reduce energy spread by a wedge absorber at a momentum dispersion point:

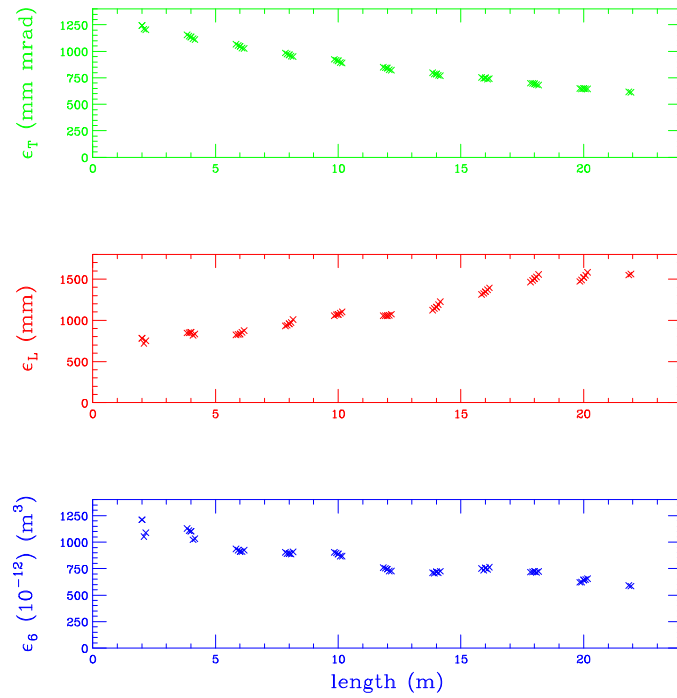


[6-D emittance constant (at best) in this process.]

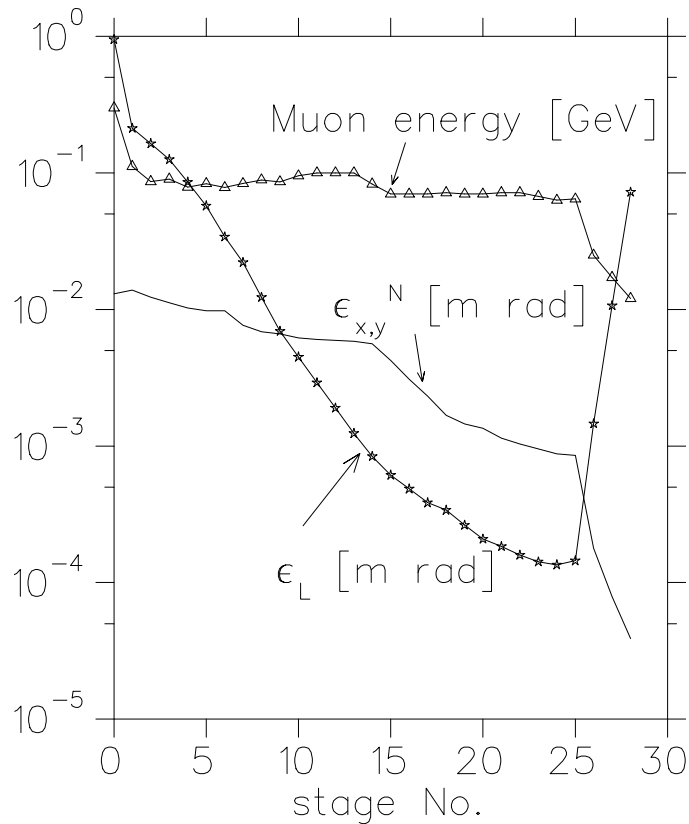
Emittance Exchange Via Wedges + Bent Solenoids



Simulated Cooling Performance



Factor of 2 reduction in 6-d emittance in a 20-m stage.



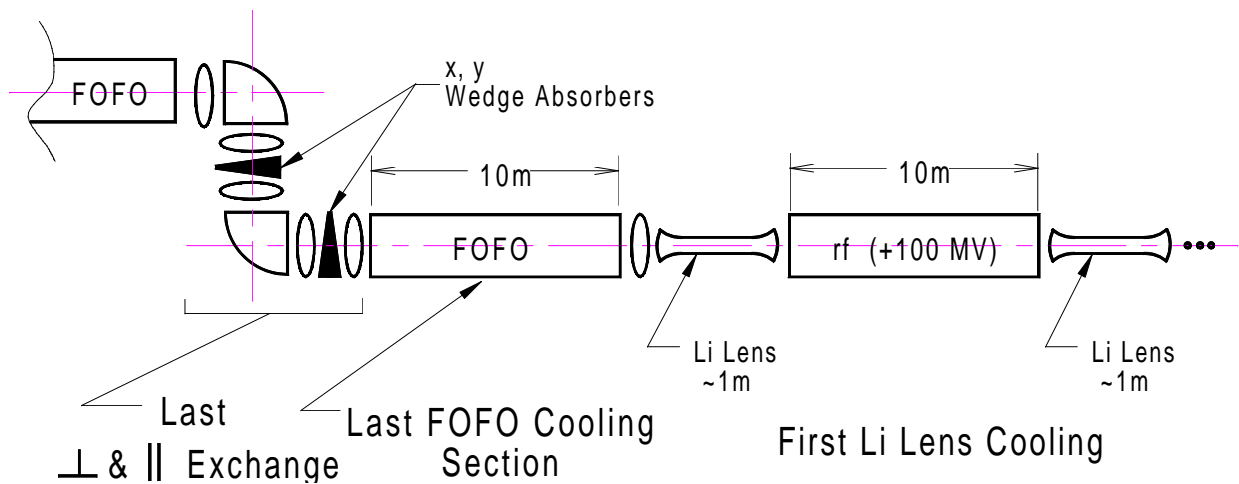
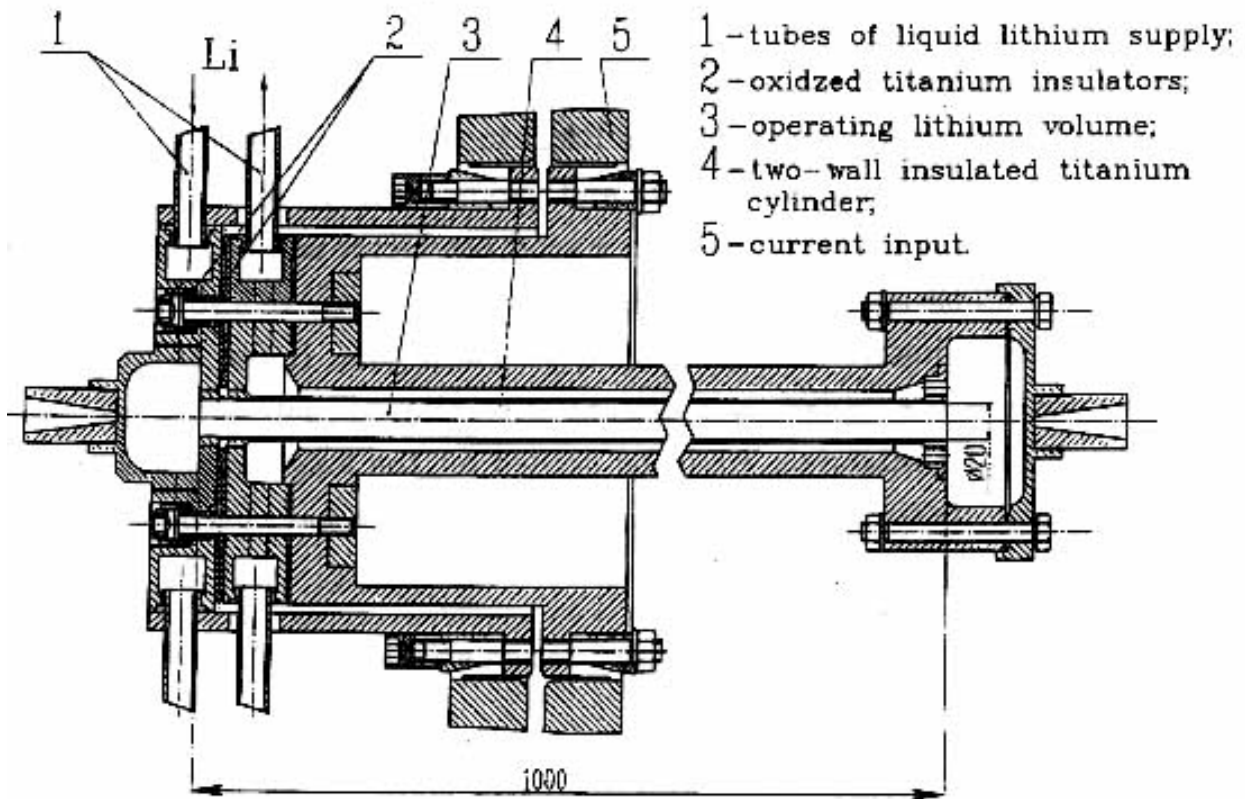
Factor of 10^{-5} reduction in 30 stages.

Cooling in Lithium Lenses

Alternating-solenoid scheme becomes difficult after ≈ 25 stages.

But more cooling is desirable \Rightarrow use lithium lenses.

LITHIUM CURRENT CARRYING COOLING ROD



Cooling, Cooling, Cooling!

Ionization cooling, in which unwanted beam momentum is transferred to atomic electrons, was “invented” by G.K. O’Neill in 1956 when he first proposed storage rings for colliding beams; Phys. Rev. **102**, 1418 (1956).

It was quickly realized that nuclear interactions made ionization cooling impractical for beams of electrons and protons.

In the late 1950’s, Lyman Spitzer noted that a beam of protons could be “cooled” by a co-propagating beam of electrons, particularly if the velocity of the two beam were equal.

This process is now called “electron cooling”, and was developed in Russia by G.I. Budker. It is a variant of ionization cooling in that electrons again take up the unwanted beam momentum.

A practical competitor to electron cooling is “stochastic cooling”, invented by S. Van Der Meer. A variant, “optical stochastic cooling”, may be eventually used at a muon collider.