(Some)
Accelerator Physics of a Muon Collider

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October 20, 1998

Seminar at the Princeton Plasma Physics Laboratory

Muon Collider main page:

http://www.cap.bnl.gov/mumu/mu_home_page.html

Muon Collider R&D Status Report:

http://www.cap.bnl.gov/mumu/status_report.html

Princeton Muon Collider page:

http://puhep1.princeton.edu/mumu/

What is a Muon Collider?

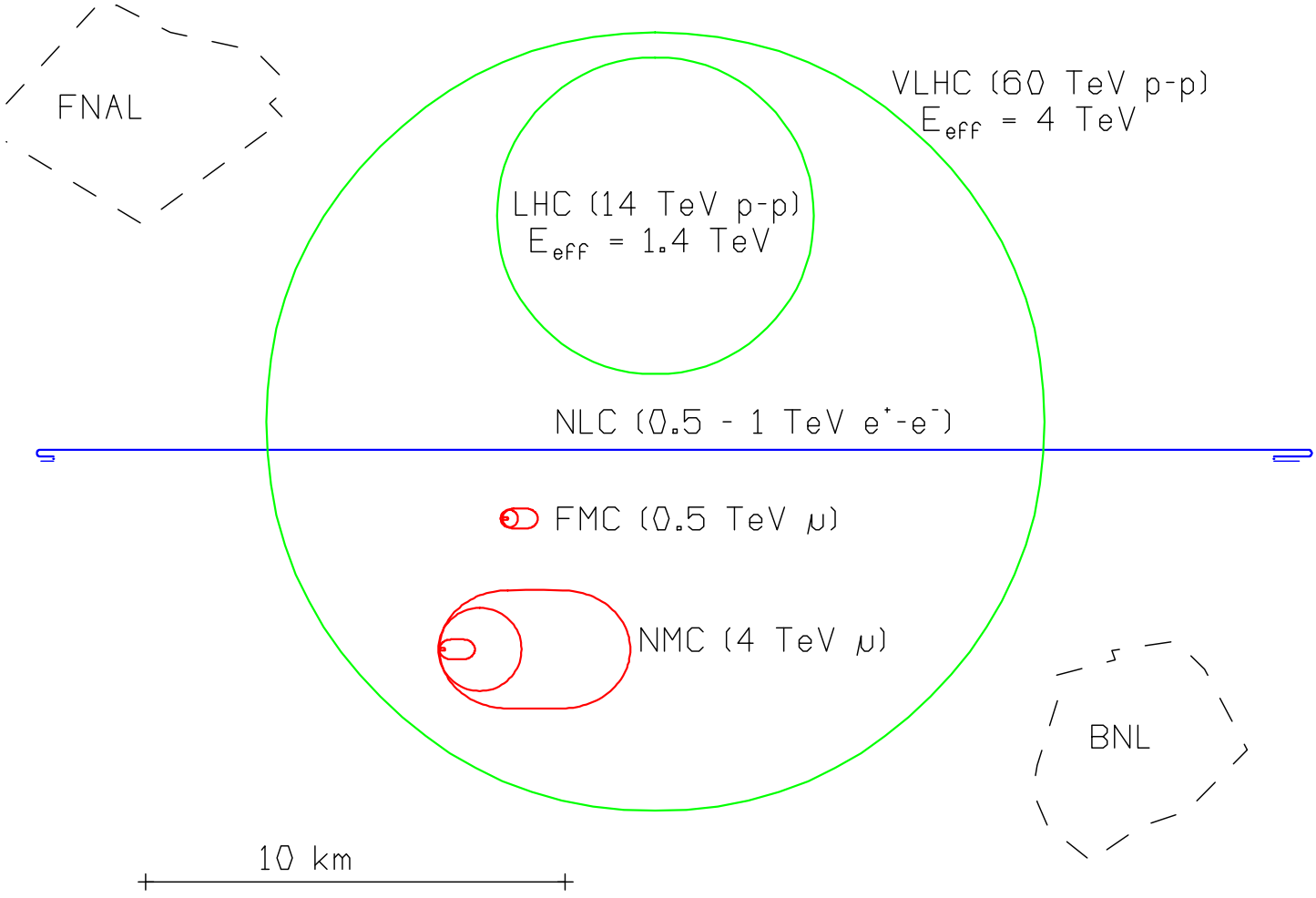
An accelerator complex in which

- Muons (both μ^+ and μ^-) are collected from pion decay following a pN interaction.
- Muon phase volume is reduced by 10^6 by ionization cooling.
- The cooled muons are accelerated and then stored in a ring.
- $\mu^+\mu^-$ collisions are observed over the useful muon life of ≈ 1000 turns at any energy.
- Intense neutrino beams and spallation neutron beams are available as byproducts.

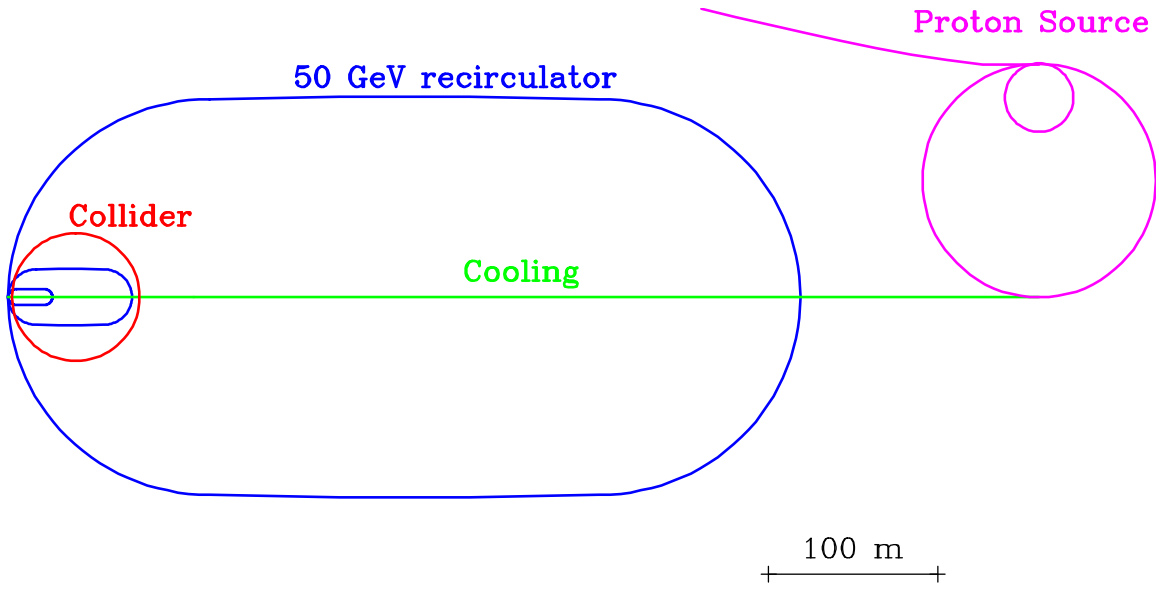
Muons decay: $\mu \rightarrow e\nu \qquad \Rightarrow$

- Must cool muons quickly (stochastic cooling won't do).
- Detector backgrounds at LHC level.
- Potential personnel hazard from ν interactions.

Footprints

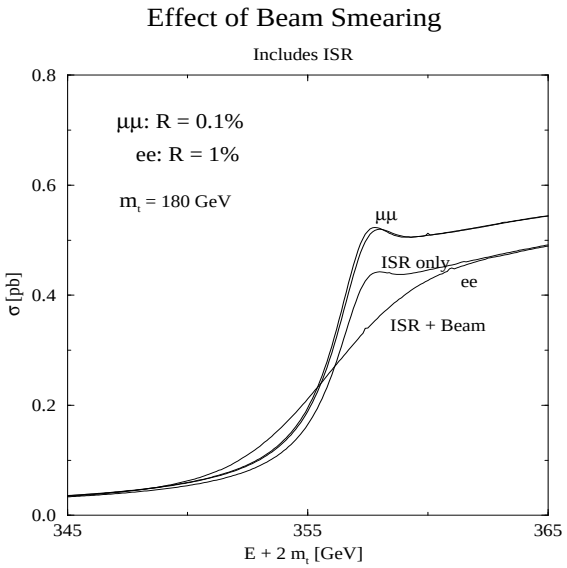


A First Muon Collider to study light-Higgs production:



The Case for a Muon Collider

- More affordable than an e^+e^- collider at the TeV (LHC) scale.
- More affordable than either a hadron or an e^+e^- collider for (effective) energies beyond the LHC.
- Precision initial state superior even to e^+e^-.



- Initial machine could produce light Higgs via s-channel:

  Higgs coupling to μ is $(m_\mu/m_e)^2 \approx 40,000\times$ that to e.

  Beam energy resolution at a muon collider $< 10^{-5}$,

  $\Rightarrow$ Measure Higgs width.

  Add rings to 3 TeV later.

- Neutrino beams from μ decay about 10^4 hotter than present.

Future Frontier Facilities

(Will the U.S. have one?)

- Hadron collider (LHC, SSC): $\approx$ \$100k/m [magnets].

  $\approx$ 2 km per TeV of CM energy.

  Ex: LHC has 14-TeV CM energy, 27 km ring, $\approx$ \$3B.

- Linear e^+e^- collider (SLAC, NLC(?)): $\approx$ \$200k/m [rf].

  $\approx$ 20 km per TeV of CM energy;

  But a lepton collider needs only $\approx 1/10$ the CM energy

  to have equivalent physics reach to a hadron collider.

  Ex: NLC, 1.5-TeV CM energy, 30 km long, $\approx$ \$6B (?).

- Muon collider: $\approx$ \$1B for source/cooler + \$100k/m for rings

  Well-defined leptonic initial state.

  $m_\mu/m_e \approx 200 \Rightarrow$ Little beam radiation.

  $\Rightarrow$ Can use storage rings.

  $\Rightarrow$ Smaller footprint.

  Technology: closer to hadron colliders.

  $\approx$ 6 km of ring per TeV of CM energy.

  Ex: 3-TeV muon collider $\approx$ \$3B (?).

The Muon Collider Collaboration

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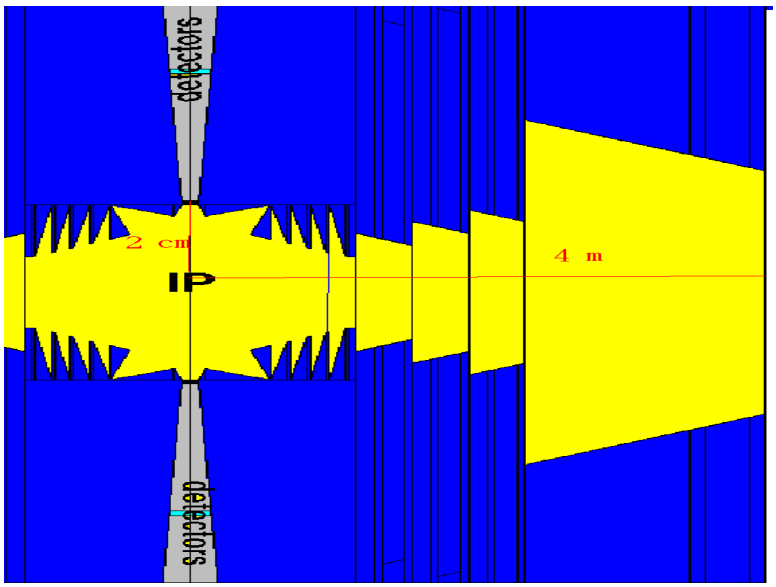
Spokesperson: R.B. Palmer

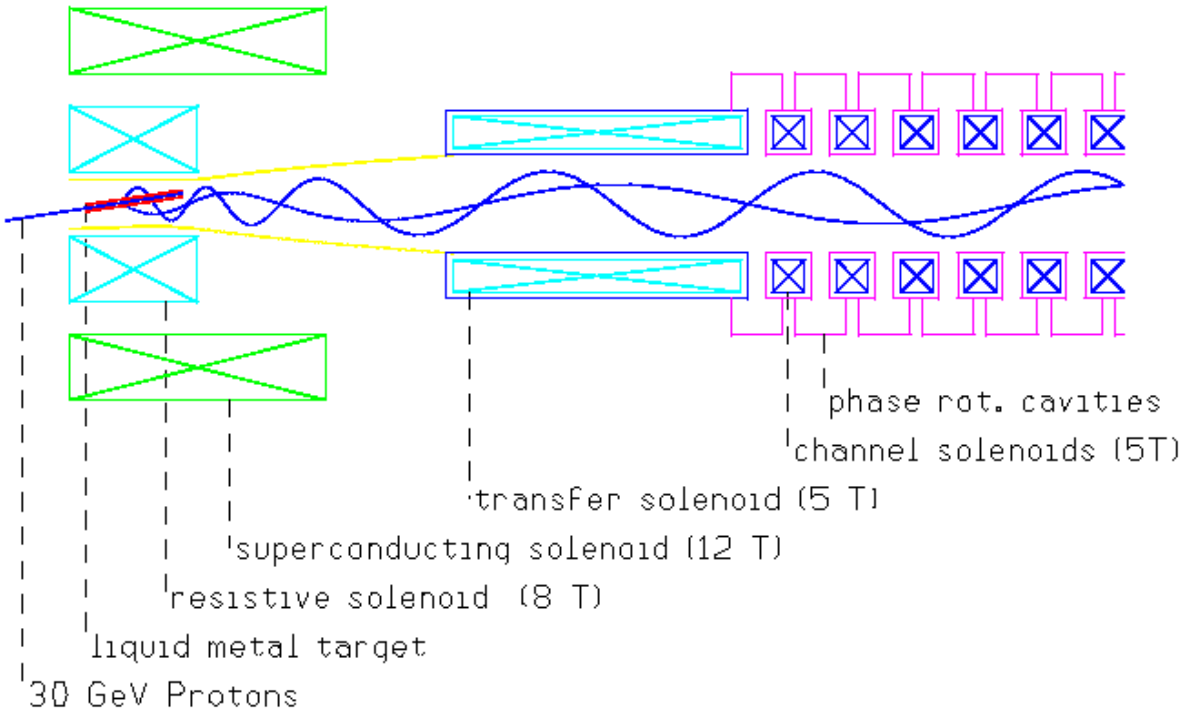
Technical Challenges

- 16-GeV proton driver, 15 Hz, 4-MW beam power, 1-ns bunch length.
- **Targetry and Capture**
- **Muon Cooling**
- Acceleration
- Storage rings
- Interaction region and detector design

A muon's view of the interaction region:



Overview of Targetry for a Muon Collider



- 1.2×10^{14} $\mu^{\pm}$/s via π-decay from a 4-MW proton beam.
- Cooling jacket around stationary target would absorb too many pions.
- Liquid-metal jet target: Ga, Hg, or solder (Bi/In/Pb/Sn).

- 20-T capture solenoid followed by a 1.25-T π-decay channel with phase-rotation via rf (to compress energy of the muon bunch).

Magnetic Bottle Around the Target

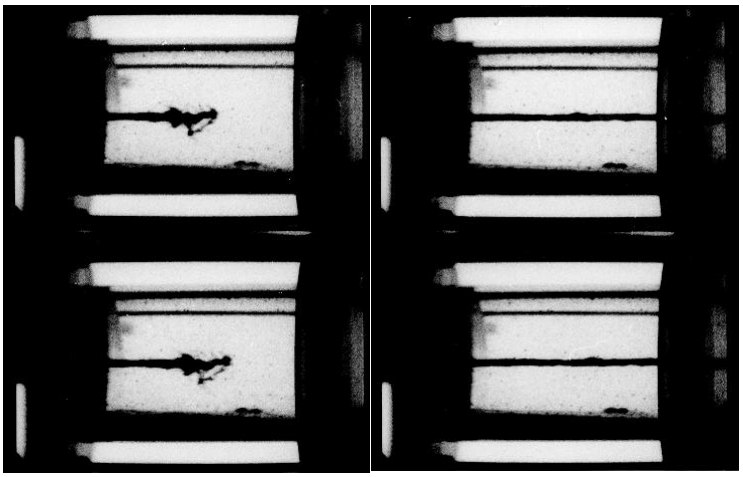
$$p + A \to \pi^{\pm} + X, \qquad\qquad \pi^{\pm} \to \mu^{\pm}\nu.$$

- In high-energy interactions, pions are produced with average transverse momentum $P_{\perp} \approx 200$ MeV/c.
- Goal: Capture pions with $P_{\perp} < 225$ MeV/c.
- Solution: Surround the target with a 20-T solenoid magnet, whose field drops to 1.25 T in the pion-decay channel.
- Peak field at upstream end of target $\Rightarrow$ (some) backwards pions reflected off the high-field region of the **magnetic bottle**.
- As pions advance into the weak-field region, their $P_{\perp}$ drops, $\Rightarrow$ Confined to smaller radii than if produced in a weak-field.
- Adiabatic invariant: $\Phi = \pi r^2 B$ as B drops from 20 to 1.25 T.
- $r = P_{\perp}/eB =$ radius of helix.

$$\Rightarrow \qquad \frac{P_{\perp,f}}{P_{\perp,i}} = \sqrt{\frac{B_f}{B_i}} = 0.25, \qquad [\text{and} \qquad P_{\parallel,f} > P_{\parallel,i}].$$

Targetry Issues

- 1-ns beam pulse ⇒ shock heating of target.
  - Resulting pressure wave may disperse liquid (or crack solid).
  - Damage to target chamber walls?
  - Magnetic field will damp effects of pressure wave.
- Eddy currents arise as metal jet enters the capture magnet.
  - Jet is retarded and distorted, possibly dispersed.
  - Hg jet studied at CERN, but not in beam or magnetic field:



High-speed photographs of mercury jet target for CERN-PS-AA (laboratory tests)

4,000 frames per second, Jet speed: 20 ms-1, diameter: 3 mm, Reynold's Number:>100,000

A. Poncet

- Targetry area also contains beam dump.
  - Need 4 MW of cooling.
  - Harsh radiation environment for magnets and rf.

Effect of a Short Beam Pulse on a Liquid?

Will shock heating disperse the target violently?

Simple model to estimate magnitude of shock pressure wave:

Beam energy heats liquid (no heat flow);

Liquid expands causing strain (shock wave);

Liquid 'tears' if pressure exceeds tensile strength.

Fact: tensile strength (T_S) is about $0.002E$ (Young's modulus) in most metals.

$$\Delta U[\text{J/gm}] = C\Delta T = \frac{C}{\alpha}\frac{\Delta l}{l} = \frac{C}{\alpha}\frac{P}{E} \approx 0.002\frac{C}{\alpha},$$

when $P = T_S$:

Ex: Gallium: $\alpha \approx 2 \times 10^{-5}$/K; $C_P \approx 0.3$ J/gm-K, tears when

$$\Delta U \approx (0.002)(0.3)/(2 \times 10^{-5}) \approx 30 \text{ J/gm}.$$

This is roughly the nominal energy deposition in the target!

Magnetohydrodynamics

Field $\mathbf{E}'$ inside a conductor with velocity $v \ll c$ in field $\mathbf{B}$:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \qquad \text{(MKSA)}.$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{j}, \qquad \mathbf{j} = \sigma \mathbf{E}' = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}),$$

$$\Rightarrow \qquad \frac{\partial \mathbf{B}}{\partial t} = \frac{\nabla^2 \mathbf{B}}{\mu_0 \sigma} + \nabla \times (\mathbf{v} \times \mathbf{B}).$$

$\Rightarrow$ Field diffusion time into long cylinder: $\tau = \mu_0 \sigma r^2$.

Ex: $\sigma_{\text{Hg}} = \sigma_{\text{copper}}/50$, $r = 1$ cm,

$\Rightarrow \tau \approx 4\pi \times 10^{-7} \cdot 10^6 \cdot (10^{-2})^2 \approx 10^{-4}$ s.

Magnetic Reynolds number : $\mathcal{R} = \dfrac{\tau v}{D} \approx \dfrac{10^{-4}\text{s} \cdot 10\text{m/s}}{0.3\text{m}} = 0.003,$

for motion through a solenoid of diameter $D = 0.3$ m.

$\Rightarrow$ The liquid is a "poor" conductor, and the field penetrates quickly.

Eddy Current Effects on Conducting Liquid Jets

- In frame of jet, changing magnetic field induces eddy currents.
- Lenz: Forces on eddy current oppose motion of jet.
- Longitudinal drag force $\Rightarrow$ won't penetrate magnet unless jet has a minimum velocity: $\sigma = \sigma_{\rm Cu}/60,\ \rho = 10\ {\rm g/cm}^3,\ \Rightarrow$

$$v_{z,\rm min} \approx \frac{\sigma r^2 B_0^2}{6\rho D} \approx 60\ {\rm m/s} \left[\frac{r}{1\ {\rm cm}}\right] \left[\frac{r}{D}\right] \left[\frac{B_0}{20\ {\rm T}}\right]^2 .$$

  Ex: $B_0 = 20$ T, $r = 1$ cm, $D = 20$ cm, $\Rightarrow v_{\rm min} = 3$m/s.

- Drag force is larger at larger radius $\Rightarrow$ planes deform into cones:

$$\frac{\Delta z(r)}{r} \approx \frac{\sigma r^2 B_0^2 \alpha}{12\rho v_z} \approx -3\alpha \left[\frac{r}{1\ {\rm cm}}\right] \left[\frac{B_0}{20\ {\rm T}}\right]^2 \left[\frac{10\ {\rm m/s}}{v}\right] .$$

  Ex: $\alpha = L/D = 2,\ r = 1$ cm, $v = 10$ m/s $\Rightarrow \Delta z = 6$ cm.

- Radial pressure: compression as jet enters magnet, expansion as it leaves:

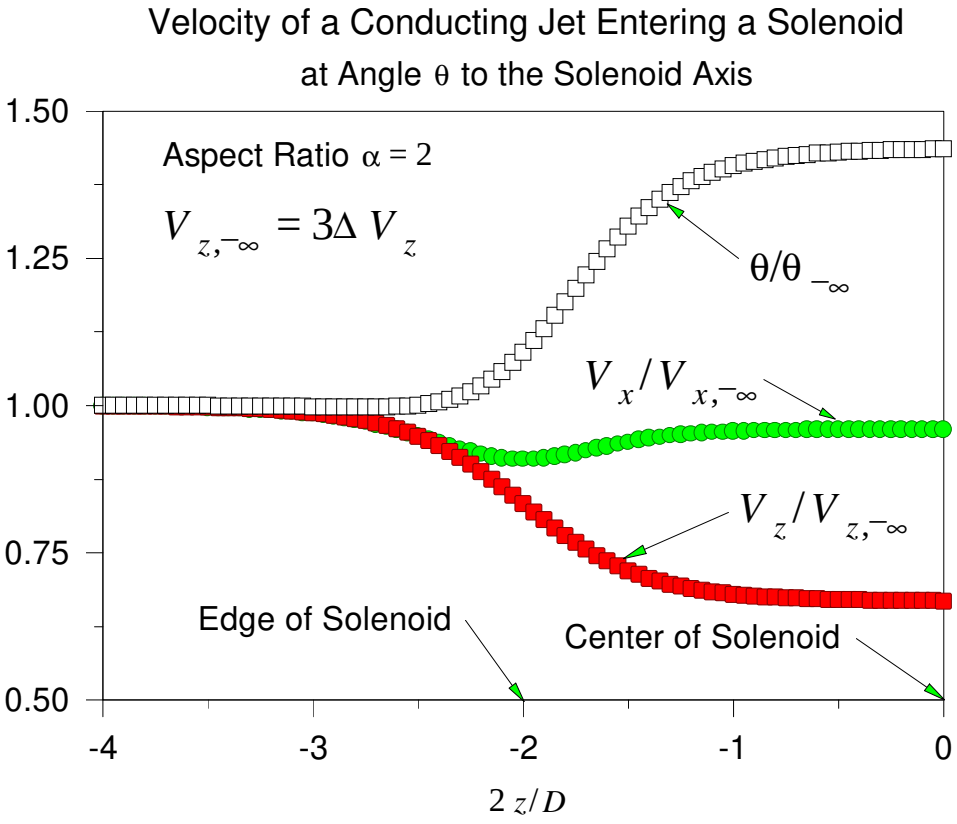
$$P_r \approx \frac{\sigma r^2 B_0^2 v_z}{8D} \approx 50\ {\rm atm.} \left[\frac{r}{1\ {\rm cm}}\right] \left[\frac{r}{D}\right] \left[\frac{B_0}{20\ {\rm T}}\right]^2 \left[\frac{v}{10\ {\rm m/s}}\right] .$$

  Ex: $P = 2.5$ atm for previous parameters.

- Will the jet break up into droplets?

- Jet at angle θ to magnet axis $\Rightarrow$ transverse drag.

  But, $\Delta v_x = \Delta v_z/8$.



  $\Rightarrow \theta$ increases as jet enters magnet.

  Ex: $\alpha = 2,\ v = 3\Delta v_z \Rightarrow \theta_{\text{in}} = 1.5\theta_{\text{out}}$.

- Drag and shear are smaller for larger initial velocity, but pressure rises with velocity.

- Is there a safe working regime?

- Need both FEA analysis and **lab tests**.

Magnetic Damping of Radial Perturbations

If jet blows apart radially, the flux thru rings of metal changes,

$\Rightarrow$ Eddy current damping.

$$\Rightarrow \qquad \Delta P_{r,\mathrm{damp}} \approx \sigma r v_r B_0^2.$$

Ex: Radial pinch $\Rightarrow v_r \approx \dfrac{\sigma r B_0^2}{4\rho}, \Rightarrow P_{r,\mathrm{damp}} \approx \dfrac{\sigma^2 r^2 B_0^4}{4\rho} \gtrsim P_{r,\mathrm{pinch}}.$

Ex: If beam shock $\Rightarrow v_r \approx 1,000$ m/s,

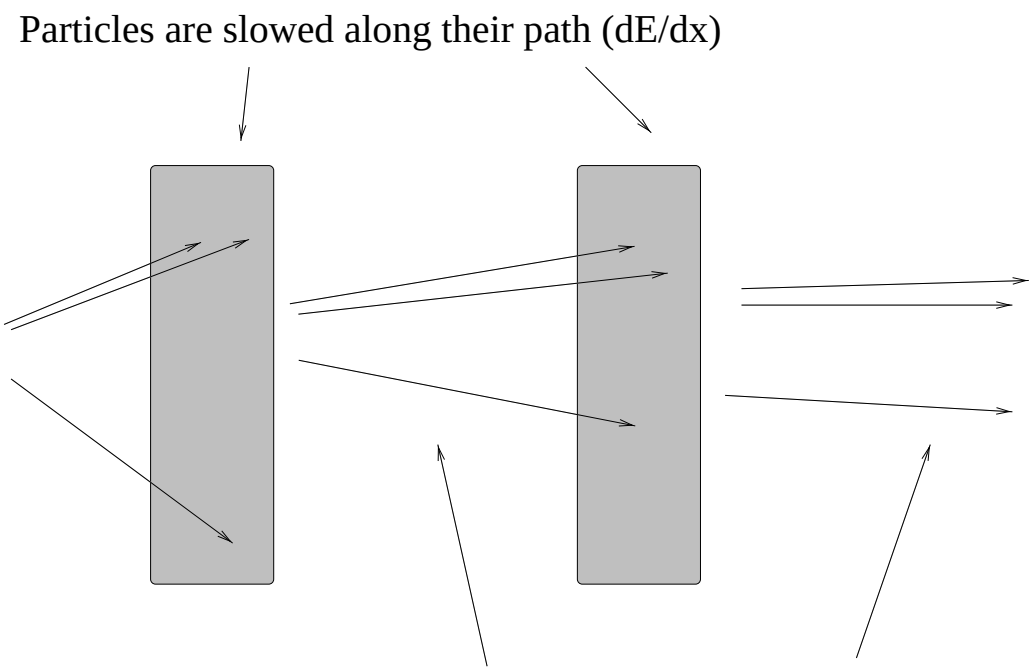
then $P_{r,\mathrm{damp}} \approx 4$ GPa $\approx T_{S,\mathrm{steel}}$.

Also, a strong magnetic field damps the Rayleigh instability (breakup of a jet into droplets due to surface tension) [Chandrasekhar].

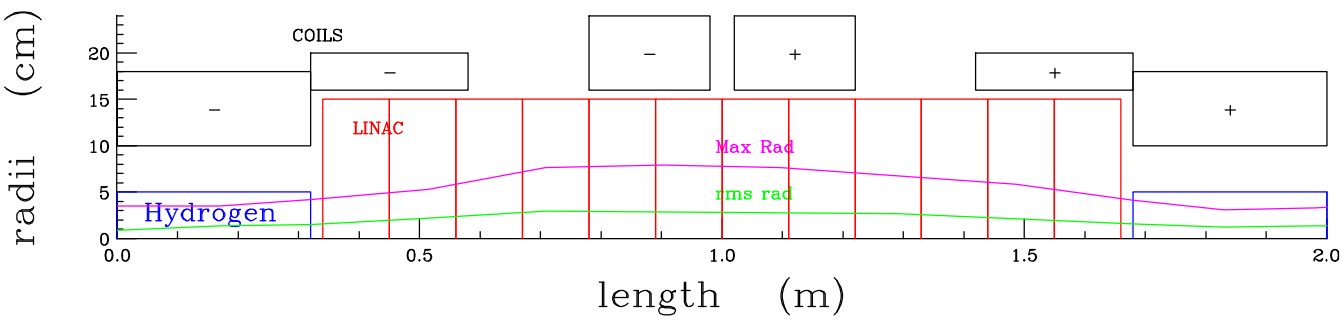
Will test liquid jets in proton beam at Brookhaven National Lab, and in 20-T magnet at National High Magnetic Field Lab.

Ionization Cooling

- Ionization: takes momentum away.
- RF acceleration: puts momentum back along z axis.
- $\Rightarrow$ Transverse "cooling".



- Use channel of LH_2 absorbers, rf cavities and alternating solenoids (to avoid buildup of angular momentum).



Ionization Cooling Theory

Transverse cooling by ionization, heating by multiple scattering:

$$\frac{d\epsilon_{N,\perp}}{ds} = -\frac{1}{\bar{\beta}^2}\frac{dE_\mu}{ds}\frac{\epsilon_{N,\perp}}{E_\mu} + \frac{\beta_\perp (0.014)^2}{2\bar{\beta}^3 E_\mu m_\mu L_R},$$

$\epsilon_{N,\perp} = \sigma_x \sigma_{P_x}/m_\mu c =$ normalized transverse emittance,

$\bar{\beta} = \bar{v}/c$, $\bar{\gamma} = 1/\sqrt{1-\bar{\beta}^2}$

$\beta_\perp = \sigma_x/\sigma_{x'} =$ Betatron function at the absorber,

$$\epsilon_\perp = \frac{\epsilon_{N,\perp}}{\bar{\gamma}\bar{\beta}}, \qquad \sigma_x = \sqrt{\epsilon_\perp \beta_\perp}, \qquad \sigma_{x'} = \frac{\sigma_{P_x}}{\bar{P}} = \sqrt{\frac{\epsilon_\perp}{\beta_\perp}},$$

$L_R =$ Radiation length of absorber.

$$\Rightarrow \text{Equilibrium} \;\; \epsilon_{N,\perp} \propto \frac{\beta_\perp}{\bar{\beta} L_R (dE_\mu/ds)}.$$

$\Rightarrow$ Low-Z absorber (liquid hydrogen is best),

$\Rightarrow$ Put absorber at low-$\beta_\perp$ (beam-waist) where angles are large, so multiple scattering hurts less.

$\Rightarrow$ Need strong focusing (15-T solenoids, Li lens...).

Economics favor $\bar{\beta} < 1$, $\bar{\gamma} \approx 1$, since must restore the beam energy ($\propto \bar{\gamma} - 1$) many times.

However, $\beta(dE_\mu/ds) \propto \beta^{-2/3}$ for low β, so cooling is less effective at smaller β.

Present scenario: Cool at $\bar{\beta} = 0.86$, $P_\mu = 180$ MeV/c,
KE $= 100$ MeV.

The Angular Momentum Problem

A solenoid with field B_z has vector potential $A_\phi = rB_z/2$.

The **canonical momentum**, $\Pi = \mathbf{P} + e\mathbf{A}/c$), is conserved.

The **canonical angular momentum** L is also conserved:

$$\mathbf{L} = \mathbf{r} \times \Pi = \mathbf{r} \times (\mathbf{P} + e\mathbf{A}/c).$$

$$\Rightarrow \qquad L_z = r\Pi_\phi = rP_\phi + er^2B_z/2c.$$

So, if the mechanical transverse momentum, P_ϕ, has been "cooled" to zero inside the solenoid, the charge will emerge with

$$L_{z,\text{out}} = L_{z,\text{in}} = er^2B_{z,\text{in}}/2c.$$

$\Leftrightarrow$ The fringe field of the solenoid imparts an undesirable kick.

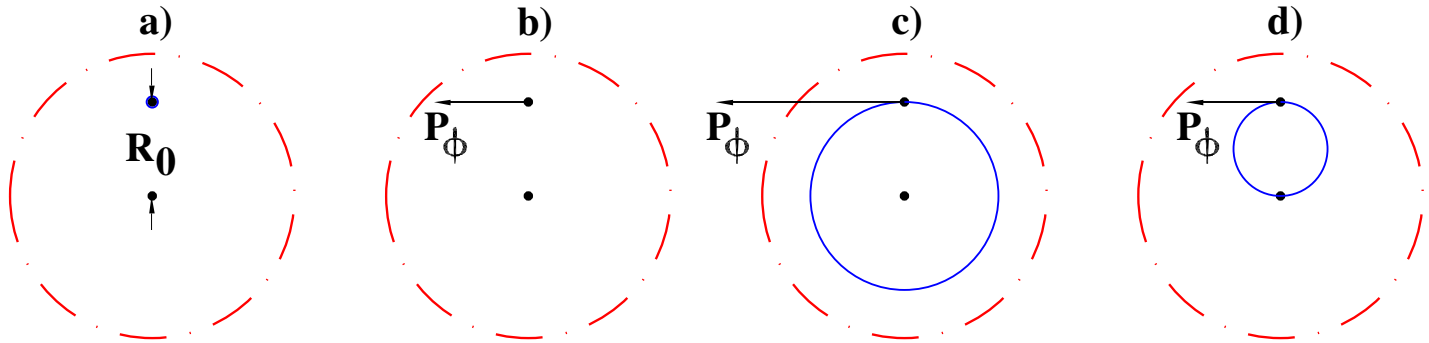
Solution: Alternating Solenoids

Suppose after leaving field B_z, the beam enters field $-B_z$. Then,

$$er^2 B_{z,\mathrm{in}}/2c = L_{z,1} = L_{z,2} = rP_{\phi,2} - er^2 B_z/2c,$$

$$\Rightarrow \qquad P_{\phi,2} = erB_z/c.$$

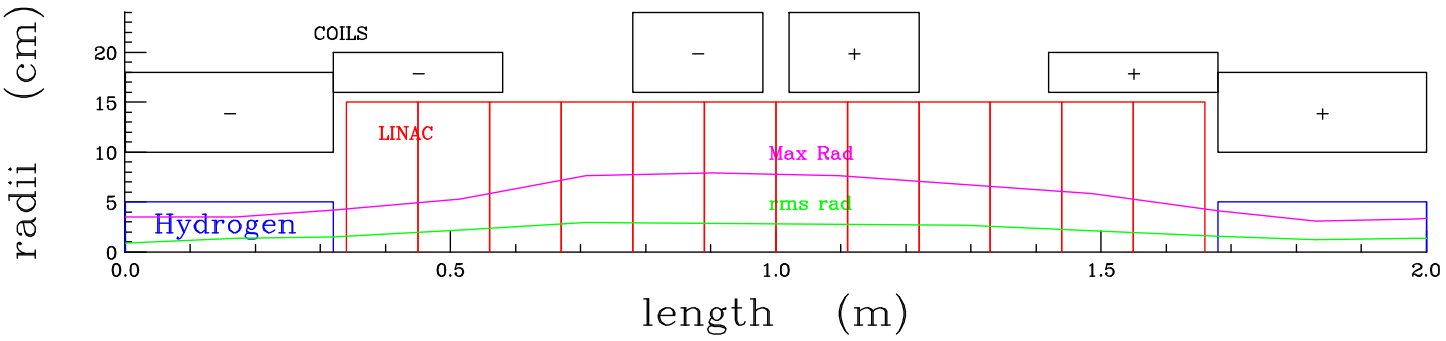
Now, if cool in region 2 until $P'_{\phi,2} = erB_z/2c$, and exit, the particle will end up with $P_\phi = 0$.

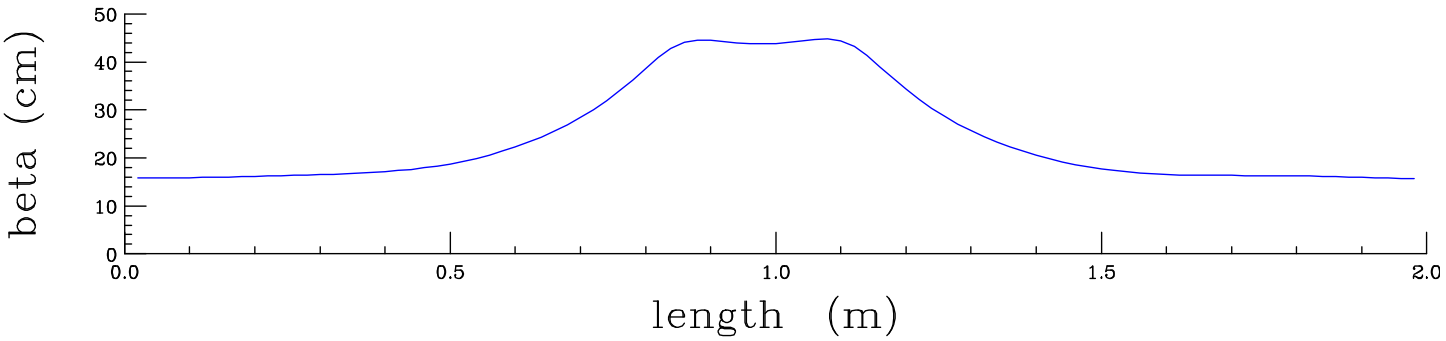


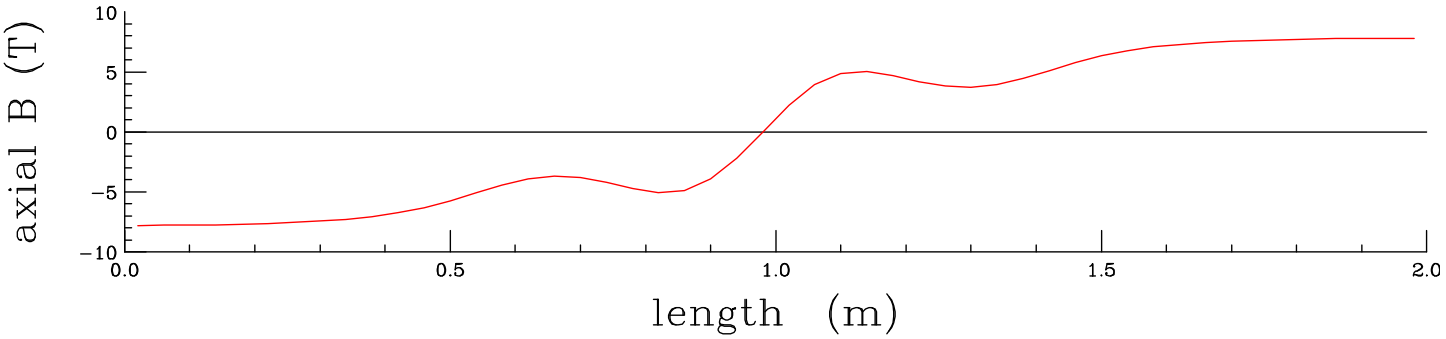
a)	b)	c)	d)
$B_z = +B$	$B_z = 0$	$B_z = -B$	$B_z = -B$
$P_\phi = 0$	$P_\phi = +eR_0B/2c$	$P_\phi = +eR_0B/c$	$P_\phi = +eR_0B/2c$
$R_B = 0$		$R_B = R_0$	$R_B = R_0/2$
$R_G = R_0$		$R_G = 0$	$R_G = R_0/2$
$L_z = +eR_0^2B/2c$	$L_z = +eR_0^2B/2c$	$L_z = +eR_0^2B/2c$	$L_z = 0$

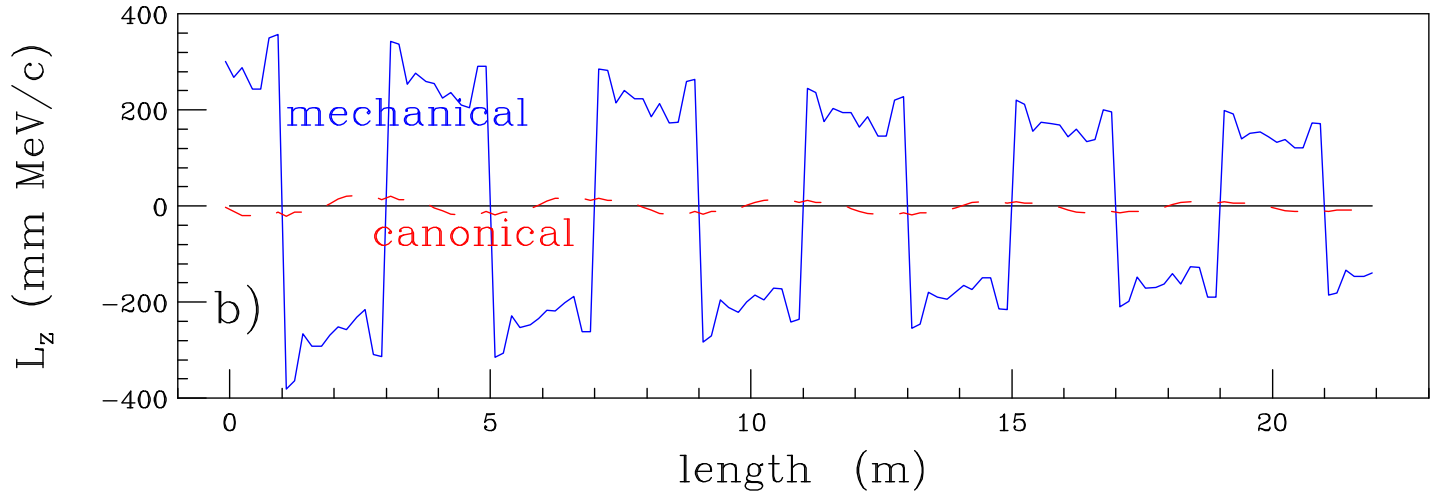
In practice, alternate the fields many times, keeping the canonical momentum always near zero, while the mechanical momentum undergoes damped oscillations.

Cooling in a Channel of Alternating Solenoids





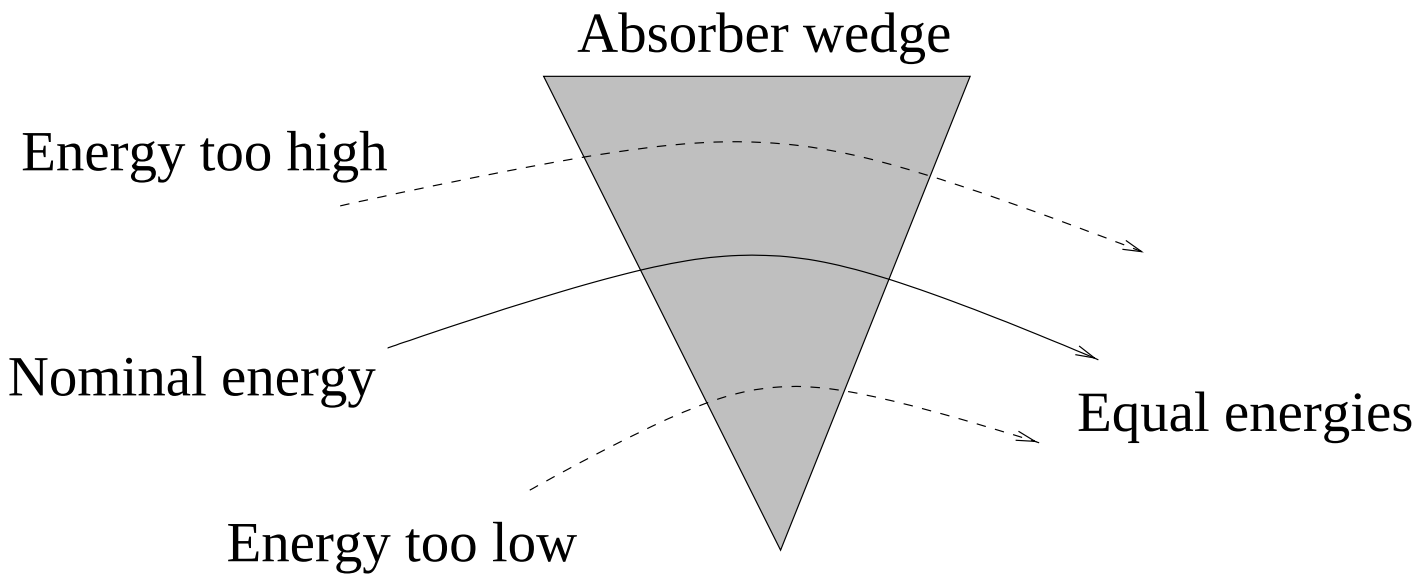




But the Energy Spread Rises due to "Straggling"

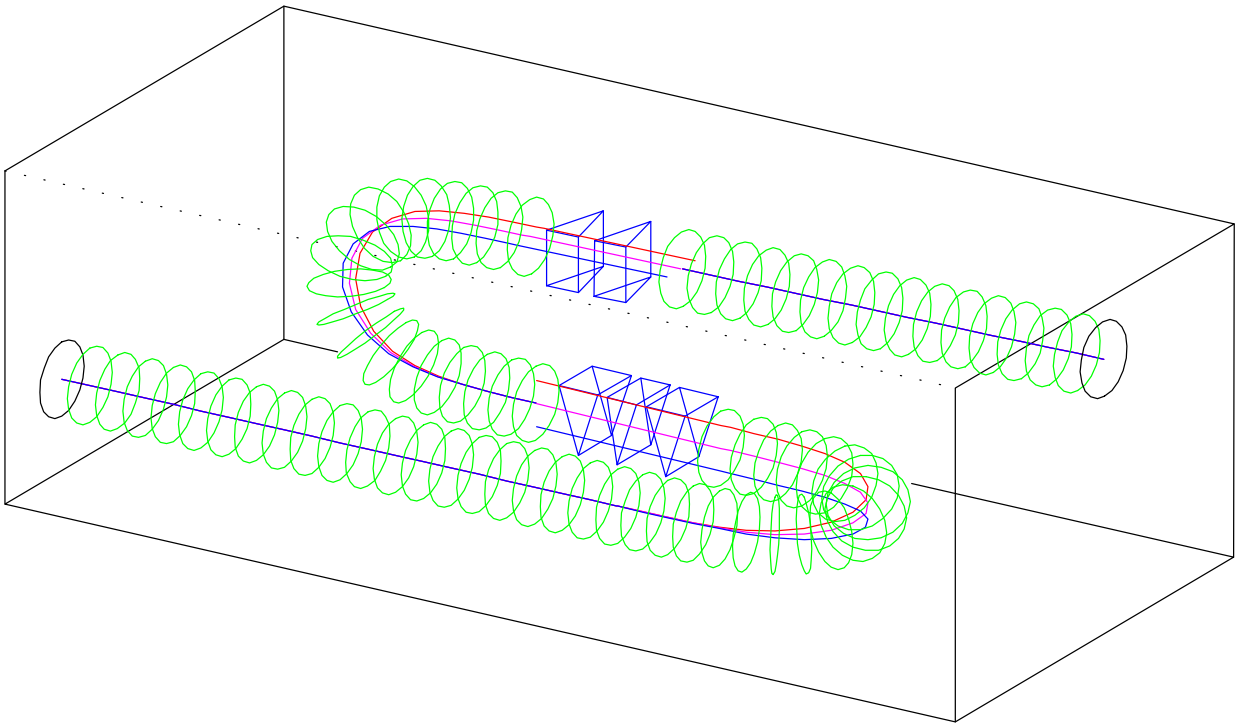
$$\frac{d(\Delta E_\mu)^2}{ds} = -2\,\frac{d\left(\frac{dE_\mu}{ds}\right)}{dE_\mu}(\Delta E_\mu)^2 + \frac{d(\Delta E_\mu)^2_{\text{straggling}}}{ds}.$$

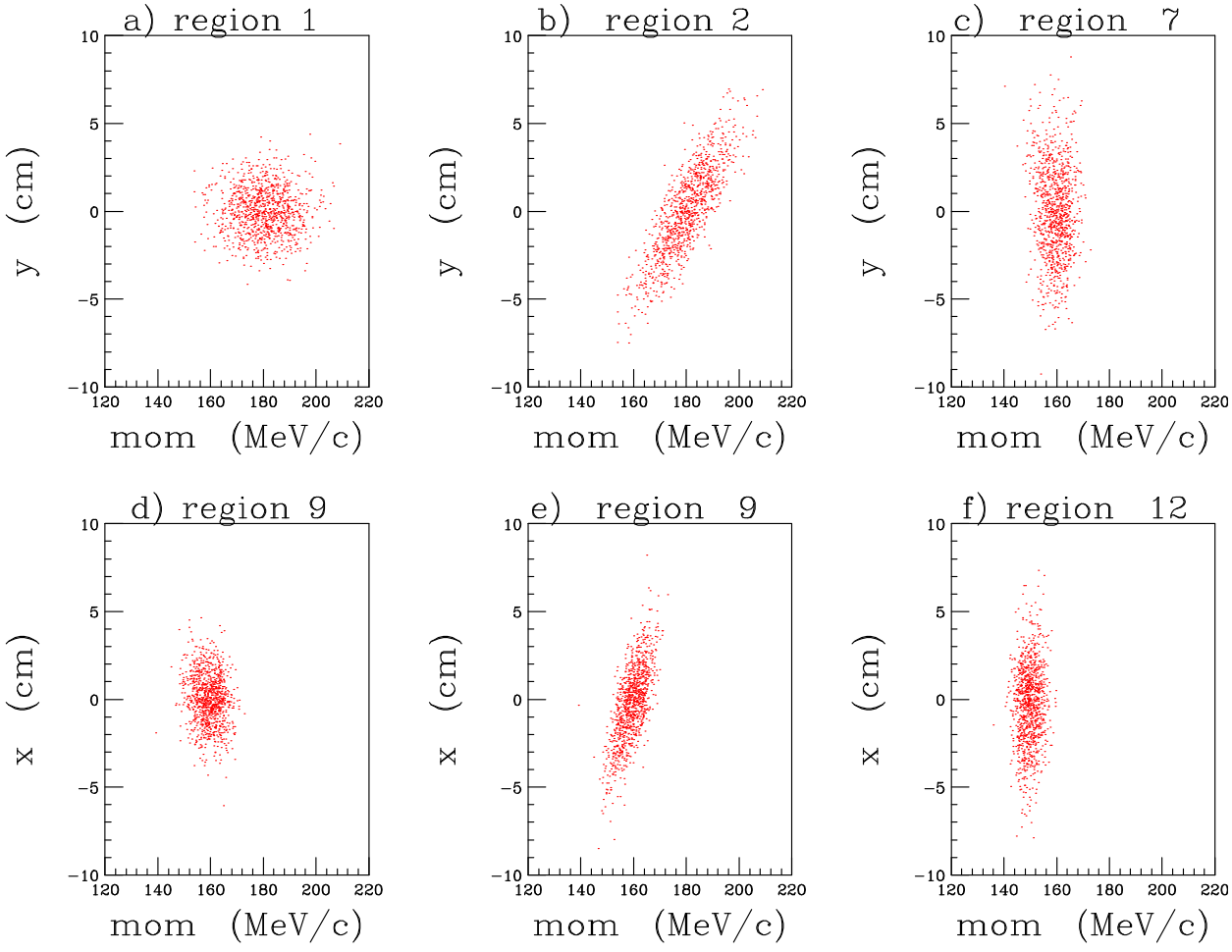
- Both terms are positive if operate below minimum of dE_μ/ds curve.
- $\Rightarrow$ Must exchange longitudinal and transverse emittance frequently to avoid beam loss due to bunch spreading.
- Can reduce energy spread by a wedge absorber at a momentum dispersion point:



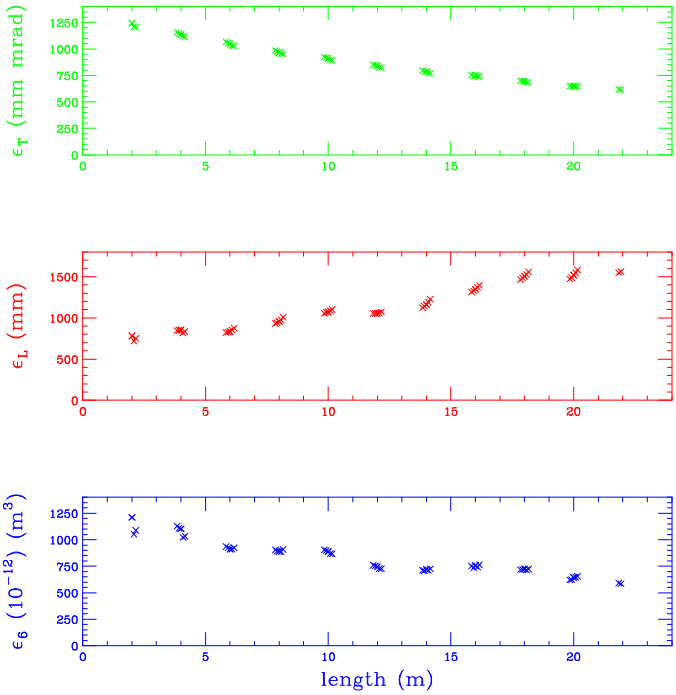
[6-D emittance constant (at best) in this process.]

Emittance Exchange Via Wedges + Bent Solenoids

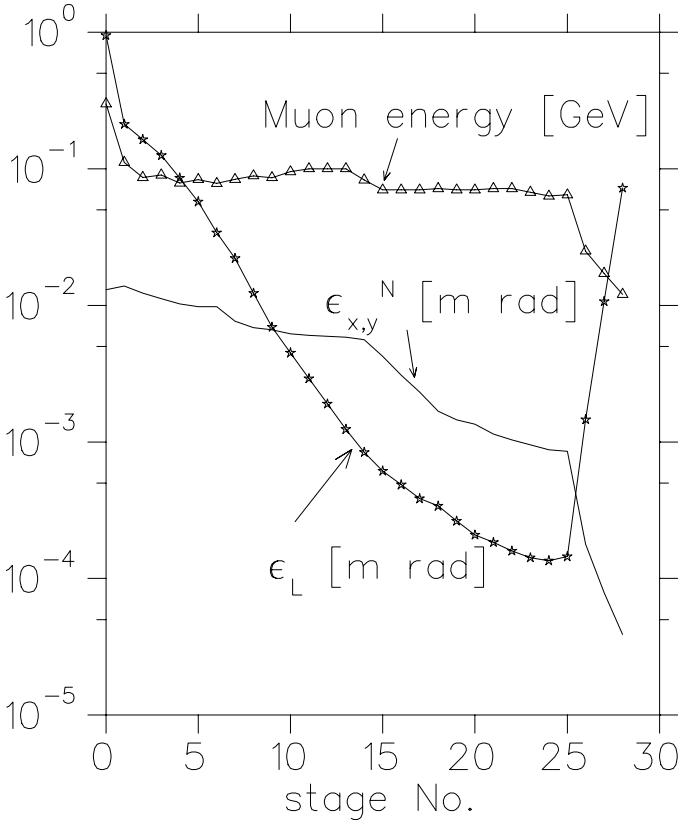




Simulated Cooling Performance

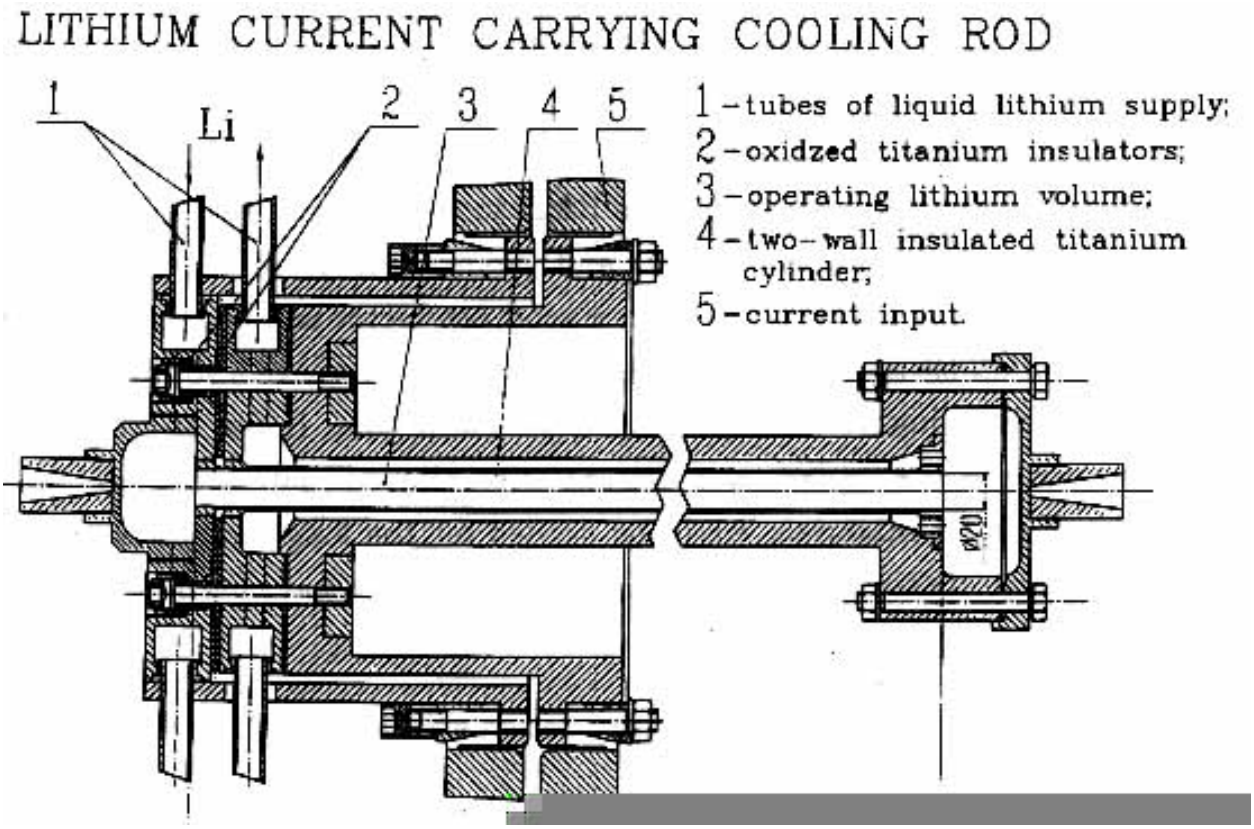


Factor of 2 reduction in 6-d emittance in a 20-m stage.



Factor of 10^{-5} reduction in 30 stages.

Cooling in Lithium Lenses

Alternating-solenoid scheme becomes difficult after ≈ 25 stages.

But more cooling is desirable $\Rightarrow$ use lithium lenses.