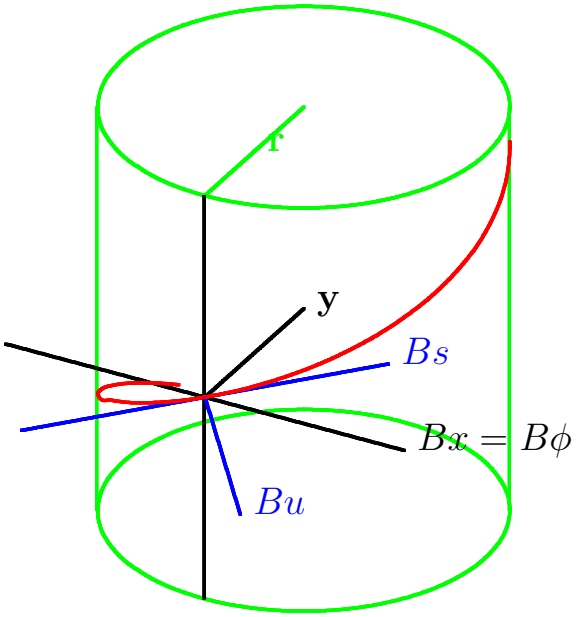
0.1 Momentum Drift



The field aroound a vertical current I

$$B_\phi = I/r$$

Consider a particle travelling on the surface of a cylinder of radius r, spiralling upward at an angle α:

$$B_s = B_\phi \ cos(\alpha)$$

$$B_u = B_\phi \ sin(\alpha)$$

$$\frac{1}{\rho_s} = \frac{d^2y}{ds^2} = \frac{e}{p_s} \ B_u = \frac{e}{p_s} \ B_\phi \ sin(\alpha)$$

and since $d(\phi r) = ds \ cos(\alpha)$

$$\frac{1}{\rho_{cyl}} = \frac{d^2y}{d(\phi r)^2} = \frac{e}{p_s} \ B_\phi \ \frac{sin(\alpha)}{cos^2(\alpha)}$$

If

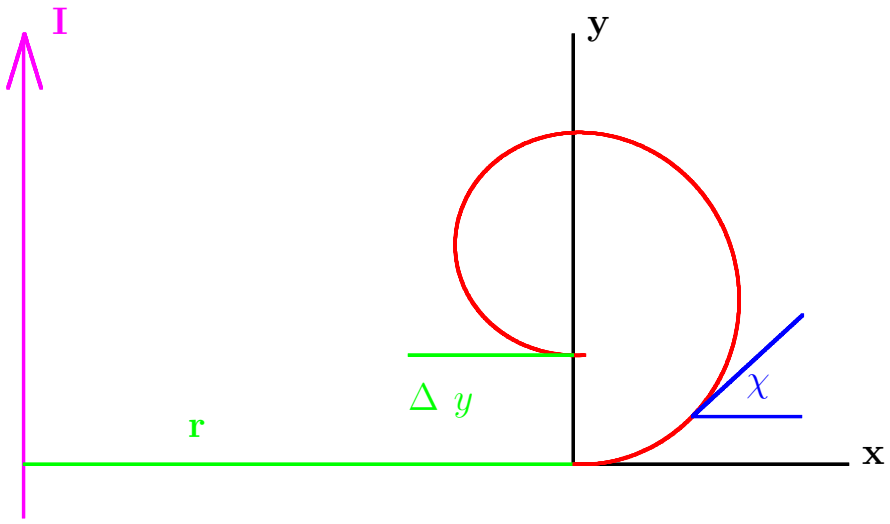
$$\frac{1}{\rho_{cyl}} = \frac{1}{r}$$

then the particle remains on the surface of the cylinder in a smth upward helix, and the upward helix angle for this condition is:

$$\alpha \ \approx \ \frac{sin(\alpha)}{cos^2(\alpha)} \ = \ \frac{p_s}{e \ B_\phi \ r} \qquad (1)$$

Amplitude Drift

Consider now the situation with little or no momentum in the ϕ direction, but a relatively large inital radial momentum. The particle will remain perpendicular to the filed lines and will, to first order, loop in a circle. But to second order, because the B_ϕ is not constant, the particle drifts upwards.



The bending radius ρ in the plane (x,y) including the current $I$is given by:

$$\rho = \frac{p_t}{eB_\phi} \propto \frac{p_t\ r}{I}$$

Defining k

$$\rho = k\ r\ =\ k\ (r_o\ +\ x)$$

Defining the track slope $\chi = tan(dy/dx)$:

$$\frac{ds}{d\chi} = \rho$$

$$dx = ds\ cos(\chi) \qquad dy = ds\ sin(\chi)$$

$$ds = \rho d\chi\ =\ k\ (r_o\ +\ x)\ d\chi$$

if $\rho << r_o$, then to first order :

$$y\ \approx\ \rho_o\ (1 - cos(\chi))$$

$$x\ \approx\ \rho_o\ sin(\chi)$$

where

$$\rho_o = k\ r_o$$

To second order:

$$dy \approx (k(r_o + \rho_o \, sin(\chi) \, d\chi) \, sin(\chi)$$

$\int_0^{2\pi} sin(\chi) = 0$, but $\int_0^{2\pi} sin^2(\chi) = \pi$, so integrating dy over one turn and substituting ρ_o/r_o for k gives a vetical "drift" of:

$$\Delta y_0^{2\pi} \approx \pi \, \frac{\rho_o^2}{r_o} \tag{2}$$

In general, a particles motion will envolve both this amplitude drift and the momentum drift discussed above.