USE OF IONIZATION FRICTION IN THE STORAGE OF HEAVY PARTICLES

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This paper discusses a method for increasing the phase density of a heavy-particle beam based on the use of ionization energy loss. The beam is supposed to pass through a decelerating medium of low nuclear charge which is introduced into the chamber of the storage ring. The damping decrement for betatron and synchrotron oscillations because of "ionization friction" was first determined by Kolomenskii [1]. To evaluate the possibility of making practical use of this phenomenon, a more detailed investigation is carried out in this paper which considers such negative factors as Coulomb scattering, statistical fluctuation of ionization losses, nuclear interactions, and the effects of charge exchange. It was asserted [2] that these processes do not permit the use of ionization friction for compression of phase volume. At energies of several MeV, however, it turns out that conditions can be created where ionization friction predominates over competing processes and one can achieve tens of particle injection pulses into a constant phase volume in a period less than 0.1 sec. A storage ring of this type can find application in nuclear physics.

Oscillation Decrements

We consider a proton storage ring with a chamber filled with a material the density n of which depends on the generalized azimuth θ and the radial coordinate x. The ionization energy loss per unit path length is [3]

$$F=\frac{dE_i}{dS}=4\pi n r_e^2 E_e Z\left(\frac{1}{\beta^2}\ln\frac{2E_e\beta^2\gamma^2}{IZ}-1\right), \tag{1}$$

where $r_e = 2.82 \cdot 10^{-13}$ cm and $E_e = 0.511$ MeV are the classical radius and the rest energy of the electron; β is the ratio of the proton velocity to the light velocity c; $\gamma = (1-\beta)^{-1/2}$; Z is the atomic number; IZ is the mean ionization potential of the atom with $I \simeq 13.5$ eV (for hydrogen, I = 14.9 eV). The density effect is not taken into consideration here because the decelerating medium is considered to be sufficiently rarefied.

The quantity F coincides with the absolute value of the ionization friction force; there the equation for vertical betatron oscillations takes on the form

$$z''+\frac{cF}{\omega\beta E}z'+g_z z=0, \tag{2}$$

where ω is the angular velocity; E is the total energy of the proton, g_z is the usual coefficient of magnetic rigidity; the primes indicate differentiation with respect to θ. It then follows that the oscillation amplitude is damped like $e^{-\Gamma_z t}$ with a decrement

$$\Gamma_z=\left\langle\frac{cF}{2\beta E}\right\rangle. \tag{3}$$

The brackets $\langle\ldots\rangle$ indicate averaging over a revolution.

For the radial oscillations

$$x''+\frac{cF}{\omega\beta E}x'+g_x x=KR_0^2\frac{\varepsilon}{\beta^2}, \tag{4}$$

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where K and R_0 are the local curvature and the mean orbital radius, and the relative deviation of particle energy from the equilibrium value, $\varepsilon = \Delta E/E$, satisfies the equations

$$\varepsilon' = \frac{eV}{2\pi E}(\cos\varphi - \cos\varphi_s) - \frac{1}{E}(E_i' - E_{is}'); \tag{5}$$

$$\varphi' = q\,\frac{\alpha\gamma^2 - 1}{\gamma^2 - 1}\,\varepsilon. \tag{6}$$

Here, V is the amplitude of the accelerating voltage; q is the multiplicity; α is the orbital expansion factor; the subscript s refers to the synchronous particle. The contribution associated with ionization friction can be written in the form

$$E_i' - E_{is}' = F\frac{ds}{d\vartheta} - \left(F\frac{ds}{d\vartheta}\right)_s = R_0(1 + Kx)\,F\,[E(1+\varepsilon),\, x]$$
$$-R_0 F(E,\,0) \simeq FR_0\left[\varepsilon\frac{\partial \ln F}{\partial \ln \gamma} + x\left(K + \frac{\partial \ln F}{\partial x}\right)\right]. \tag{7}$$

For small synchrotron oscillations, the solution of the system (4)-(6) leads to the following expressions for the decrements:

$$\Gamma_\varepsilon = \left\langle \frac{cF}{2\beta E}\left[\beta^2\frac{\partial \ln F}{\partial \ln \gamma} + \psi R_0\left(K + \frac{\partial \ln F}{\partial x}\right)\right]\right\rangle; \tag{8}$$

$$\Gamma_x = \left\langle \frac{cF}{2\beta E}\left[1 - \psi R_0\left(K + \frac{\partial \ln F}{\partial x}\right)\right]\right\rangle, \tag{9}$$

where $R_0\psi(\theta)$ is the closed orbit for a particle with unit deviation of momentum from the equilibrium value.

It is clear from these equations and Eq. (1) that either the radial or the synchrotron oscillations are unstable for $\beta \leqslant 0.7$. Such instability can be suppressed by coupling the radial and vertical motions since, according to (1), (3), (8), and (9), the sum of all decrements is positive:

$$\Gamma = \Gamma_x + \Gamma_z + \Gamma_\varepsilon = \left\langle \frac{cF}{2\beta E}\left(2 + \beta^2\frac{\partial \ln F}{\partial \ln \gamma}\right)\right\rangle = 4\pi\langle n\rangle\frac{r_e^2 cZE_e}{\beta^3 E}\left(1 - \beta^2 + \beta^2\ln\frac{2E_e\beta^2\gamma^2}{IZ}\right), \tag{10}$$

the result being independent of the type of magnetic system, in accordance with the theorem by Jacoby [4]. The quantity Γ characterizes the rate of contraction of the six-dimensional phase volume; this volume is decreased by the factor e^2 in the time Γ^{-1}.

Particle Beam Dimensions

Beam dimensions are determined by Coulomb multiple scattering at small angles and by statistical fluctuations in ionization losses.

Considering oscillations along one of the axes, the time dependence of their amplitudes, including damping, can be represented in the form:

$$a(t) = a_0 e^{-\Gamma_i t} + e^{-\Gamma_i t}\sum_{t_n < t}(\Delta a)_n e^{\Gamma_i t_n}, \tag{11}$$

where $(\Delta a)_n$ is the abrupt change in amplitude because of a collision at the time t_n. This is a random number with zero mean, and $a(t)$ is a random function, the mean value and standard deviation of which tends to the limits when $t \to \infty$

$$\bar{a}_{st} = 0; \quad \overline{|a|^2_{st}} = \frac{\dot{N}\overline{|\Delta a|^2}}{2\Gamma_i}, \tag{12}$$

where $\dot{N}$ is the average number of collisions per unit time, and the bar indicates statistical averaging. If $|\overline{\Delta a}|^2$ depends on the azimuth θ, it is necessary to carry out averaging over a revolution along with the statistical averaging.

We apply these results to vertical betatron oscillations, for which $\Delta a = -i\varphi^* R_0 \Delta\Theta/2$, where $\Delta\Theta$ is the scattering angle and φ is a Floquet function normalized in accordance with [5]. According to Eq. (12)

$$\overline{|a_z|^2_{st}} = \frac{R_0^2}{8\Gamma_z}\langle\dot{\overline{\Theta_z^2}}|\varphi_z|^2\rangle, \tag{13}$$

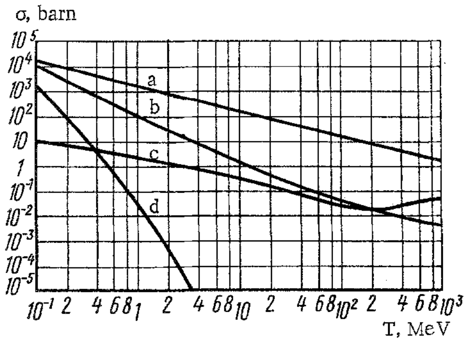

Fig. 1

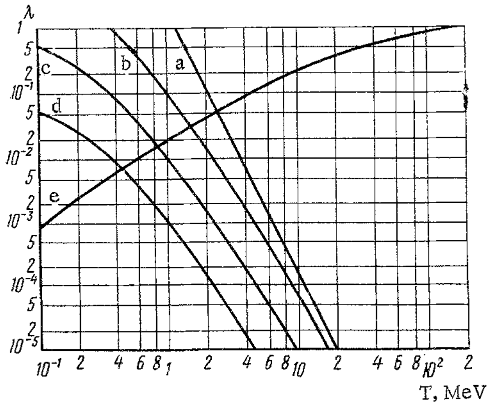

Fig. 2

Fig. 1. Proton interactions with hydrogen: a) ionization cross section $\sigma_{01}\cdot 10^{-4}$; b) "ionization friction cross section" $\sigma_i = \Gamma/\beta c\langle n\rangle$; c) total nuclear interaction cross section σ_n; d) neutralization cross section $\sigma_{10}\cdot 10^{-4}$.

Fig. 2. Proton losses in hydrogen, λ_{10}, because of neutralization: a) $\rho = 0$; b) $\rho = 10^{18}$ atoms/cm^2; c) $\rho = 10^{19}$ atoms/cm^2; d) $\rho = 10^{20}$ atoms/cm^2; e) loss because of nuclear interactions, λ_n.

where $\Theta_t^2 = \dot{N}\,\overline{(\Delta\Theta)^2}$ is the mean square angle for multiple scattering per unit time [3]:

$$\overline{\Theta}_t^2 = 4\pi\beta c r_e^2 n Z(Z+1)\frac{E_e^2}{E^2\beta^4}\ln(183Z^{-1/3}). \tag{14}$$

The mean square amplitude of radial betatron oscillations is calculated in a similar manner. For a collision with a relative energy loss $\Delta\varepsilon$ and a scattering angle $\Delta\Theta$, the derivative x' varies by $R_0\Delta\Theta$ and the orbit is shifted into a position corresponding to the new energy $E(1-\Delta\varepsilon)$, which is equivalent to an additional displacement of the proton by $\Delta x = R_0\psi\Delta\varepsilon/\beta^2$ and $\Delta x' = R_0\psi'\Delta\varepsilon/\beta^2$. Thus the change in amplitude for radial oscillations can be written in the form

$$\Delta a_x = \frac{i}{2}R_0\left[(\psi\varphi_x^{*\prime} - \psi'\varphi_x^*)\frac{\Delta\varepsilon}{\beta^2} - \varphi_x^*\Delta\Theta\right], \tag{15}$$

which leads to the result

$$\overline{|a_x|^2_{st}} = \frac{R_0^2}{8\Gamma_x}\left\langle \overline{\Theta}_t^2|\varphi_x|^2 + \frac{\overline{\varepsilon}_t^2}{\beta^4}\left\{\frac{\psi^2}{|\varphi_x|^2} + \left[\psi^2\left(\frac{|\varphi_x|}{\psi}\right)'\right]^2\right\}\right\rangle, \tag{16}$$

where $\overline{\varepsilon}_t^2 = \dot{N}\,\overline{(\Delta\varepsilon)^2}$ is the mean square fluctuation of the ionization losses per unit time [3]:

$$\overline{\varepsilon}_t^2 = 2\pi\beta c r_e^2 n\gamma^2(2-\beta^2)Z\frac{E_e^2}{E^2}. \tag{17}$$

Analyzing the synchrotron oscillations in the same way, one can show that the momentum spread of the particles reaches a value

$$\overline{\left(\frac{\Delta p}{p}\right)^2_{st}} = \frac{1}{\beta^4}\overline{|\varepsilon_{max}|^2_{st}} = \frac{\overline{\varepsilon}_t^2}{2\Gamma_\varepsilon\beta^4}. \tag{18}$$

These results make sense only when $\beta \gtrsim 0.7$, when all the decrements can be made positive. At lower energies, it is necessary to couple the vertical and radial motions. In the case of strong coupling, which ensures energy exchange between radial and vertical oscillations in a time less than the effective damping time, the decrements and average increases in the squares of the amplitudes for both directions become identical and are, respectively, $(\Gamma_x + \Gamma_z)/2$ and $(|\Delta a_x|^2 + |\Delta a_z|^2)/2$. This leads to the following values for the stationary mean square amplitudes:

$$\overline{|a_x|^2_{st}} = \overline{|a_z|^2_{st}} = \frac{\dot{N}\left(\overline{|\Delta a_x|^2} + \overline{|\Delta a_z|^2}\right)}{2(\Gamma_x+\Gamma_z)} = \frac{R_0^2}{8(\Gamma_x+\Gamma_z)}\left\langle \overline{\Theta}_t^2(|\varphi_x|^2 + |\varphi_z|^2) + \frac{\overline{\varepsilon}_t^2}{\beta^4}\left\{\frac{\psi^2}{|\varphi_x|^2} + \left[\psi^2\left(\frac{|\varphi_x|}{\psi}\right)'\right]^2\right\}\right\rangle. \tag{19}$$

Particle Loss in Single Events

Nuclear interactions, charge exchange, large-angle Coulomb scattering, and collisions with large momentum transfer lead to particle loss in single events.

The dependence of the proton–proton nuclear interaction cross section of kinetic energy T is shown in Fig. 1c [6]. The relative magnitude of the loss during the effective damping time Γ^{-1} is determined by the formula $\lambda_n = \sigma_n/\sigma_i$, where $\sigma_i = \Gamma/\beta c \langle n \rangle$ can be called the ionization loss cross section (see Fig. 1b). The dependence of λ_n on T for hydrogen is shown in Fig. 2e. It is clear that $\lambda_n \geq 1$ for $T \geq 100$ MeV, i.e., practically all previously stored beam is lost in the period between two injection cycles. Consequently, storage is possible only for $T \leqslant 100$ MeV and is most effective at energies of a few MeV.

At such low energies, charge exchange transforming protons into neutral atoms can make a significant contribution to the losses. The cross section for this process, σ_{10}, for hydrogen is shown in Fig. 1d [7], and the corresponding relationship $\lambda_{10}(T)$ is shown in Fig. 2a. It is clear that charge exchange essentially limits the storage possibilities on the low-energy side. However, this effect can be reduced by using reverse ionization of the neutral atoms formed. For this purpose, the decelerating medium must be made up in the form of a thin target set up in a gap in which there is no magnetic field. Then the majority of neutral atoms will be secondarily ionized because the corresponding cross section is very large (see Fig. 1a) [8]. If the surface density of the target is ρ (atoms/cm^2) and the condition $(\sigma_{10} + \sigma_{01})\rho \gg 1$ is satisfied, the neutralization cross section in a thick target is determined by the expression $(\sigma_{10})_{\rm eff} \simeq \sigma_{10}/\rho\sigma_{01}$ and the particle loss in a time Γ^{-1} is $\lambda_{10}\sigma_{10}/\rho\sigma_i\sigma_{01}$ (see Fig. 2b-d).

In Coulomb scattering by nuclei, the main contribution to loss will be made for angles $\Theta \leqslant 1$; one can therefore use the formula for the differential cross section [3]:

$$\frac{d\sigma_c}{d\Theta} = \frac{2\pi}{\Theta^3}\left(\frac{r_e E_e Z}{\beta^2 E}\right)^2. \tag{20}$$

Designating the permissible scattering angle by Θ_0 and using Eq. (10), we find that the Coulomb loss in the nonrelativistic case is

$$\lambda_c = \frac{1}{\sigma_i}\int_{\Theta_0}^{\pi/2} \frac{d\sigma_c}{d\Theta}\, d\Theta \simeq \frac{ZE_e}{4E\Theta_0^2}. \tag{21}$$

However, one ought to take into account only those events with $\lambda_c > \lambda_n$. Actually, when $\lambda_c < \lambda_n$, Coulomb scattering at a large angle would be exprienced by those particles which are lost because of nucleus interactions anyway. On the other hand, when $\lambda_c > \lambda_n$, there is no need to consider nuclear scattering. In cases of practical interest, $\Theta_0^2 \simeq 0.04$–0.1 and therefore $\lambda_c \simeq (2\text{-}3)\cdot 10^{-3}$. This means that for $T \geqslant 0.5$ MeV, nuclear Coulomb scattering makes no contribution to single-event losses.

Scattering by electrons also can produce no losses because the maximum scattering angle for protons $\Theta_{max} \simeq E_e/E \simeq 5\cdot 10^{-4}$ is considerably less than the permissible value.

Finally, we discuss the possibility of loss because of collisions with large momentum transfer. In considering nuclear collisions, it is sufficient to consider those which involve scattering at an angle $\Theta < \Theta_0$ because losses resulting from collision with $\Theta > \Theta_0$ are already taken into account. In the nonrelativistic case, the momentum transferred by a proton to a nucleus has a simple relation to the scattering angle: $(\Delta p/p) \simeq (\Theta^2/2A) < (\Theta_0^2/2A) \simeq (0.02\text{-}0.05)/A$, where A is the atomic weight of the target material. Collisions with electrons lead to a smaller change in momentum: $(\Delta p/p) \simeq (2E_e/E) \simeq 10^{-3}$. In cases of practical interest, such a change does not take the proton out of the region of stability.

Nuclear scattering and charge exchange are therefore the main source of single-event loss. The energy dependence of the total loss $\lambda = \lambda_n + \lambda_{10}$ is shown in Fig. 3. The inverse quantity, λ^{-1}, which is roughly the same as the number of injection pulses which can be stored in the system, is ~100 for a hydrogen target density $\rho \sim 10^{20}$ atoms/cm^2 $\simeq 1.7\cdot 10^{-4}$ g/cm^2.

Illustrative Calculations

We reduce the formulas obtained to a form suitable for calculation in the nonrelativistic case ($T \leqslant 10$ MeV). We introduce the notation $\Gamma_\varepsilon = \zeta\Gamma$, $\Gamma_x + \Gamma_z = (1-\zeta)\Gamma$, where the positive quantity $\zeta < 1$ has the form

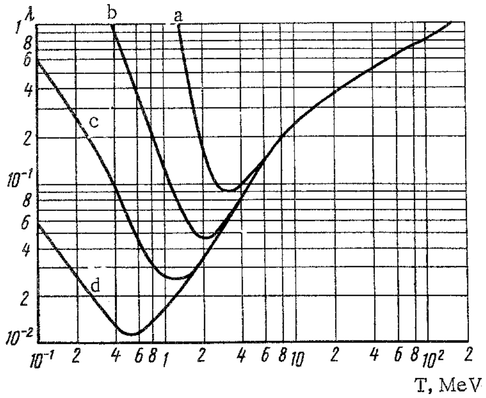

Fig. 3. Total proton loss in hydrogen: a) ρ = 0; b) $\rho = 10^{18}$ atoms/cm^2; c) $\rho = 10^{19}$ atoms/cm^2; d) $\rho = 10^{20}$ atoms/cm^2.

$$\zeta = \frac{\Gamma_\varepsilon}{\Gamma} = \left(2.5 + 0.5 \ln \frac{T}{Z}\right) \left\langle \frac{n}{\langle n \rangle} \psi R_0 \left(K + \frac{\partial \ln n}{\partial x}\right)\right\rangle - 4$$
$$- \ln \frac{T}{Z} \simeq \frac{1}{Q^2}\left(2.5 + 0.5 \ln \frac{T}{Z}\right)\left(1 + R_0 \frac{\partial \ln n}{\partial x}\right) - 4 - \ln \frac{T}{Z} \tag{22}$$

(T is measured in MeV). The second part of the formula is valid for an azimuthally symmetric accelerator with coincident betatron frequencies Q; in the general case, it is suitable for a rough estimate. The stationary mean square amplitude (18) and (19) can then be written as

$$\overline{\left(\frac{\Delta p}{p}\right)^2_{\rm st}} = \frac{2.7 \cdot 10^{-4}}{\zeta}; \tag{23}$$

$$\overline{|a_x|^2_{\rm st}} = \overline{|a_z|^2_{\rm st}} = \frac{0.68 \cdot 10^{-4} R_0^2}{(1-\zeta)\langle n \rangle} \left\{ \left\langle n \left(\frac{\psi^2}{|\varphi_x|^2} + \left[\psi^2 \left(\frac{|\varphi_x|}{\psi}\right)'\right]^2 \right)\right\rangle + (Z+1)\left(5.2 - \frac{1}{3}\ln Z\right)\right.$$
$$\left. \times \langle n (|\varphi_x|^2 + |\varphi_z|^2)\rangle \right\} \simeq \frac{0.68 \cdot 10^{-4} R_0^2}{Q(1-\zeta)}\left[\frac{1}{Q^2} + 2(Z+1)\left(5.2 - \frac{1}{3}\ln Z\right)\right]. \tag{24}$$

It is clear from this equation that the main contribution to the transverse dimensions of the beam is made by the second term, which takes multiple Coulomb scattering into account, and the first term can be neglected. When $Q^2\zeta \gg 0.05\ (1 - \zeta)$, one can also neglect the contribution of orbital spread to beam width. In this approximation, Eqs. (23) and (24) indicate that the squares of the semiaxes of the equilibrium transverse beam cross section are

$$r^2_{x,z} \simeq 1.4 \cdot 10^{-3} \frac{1+Z}{1+\zeta}(1 - 0.064 \ln Z) R_0^2 |\varphi_{x,z}|^2_{\rm max} \left\langle \frac{n}{\langle n \rangle}(|\varphi_x|^2 + |\varphi_z|^2)\right\rangle \tag{25}$$
$$\simeq 2.9 \cdot 10^{-3} \frac{1+Z}{1-\zeta}(1 - 0.064 \ln Z)\frac{R_0^2}{Q^2}.$$

Hydrogen is the best decelerating medium. The dimensions of the beam increase like $\sqrt{Z+1}$ with increasing Z. The dimensions of the storage-ring vacuum chamber are determined by the permissible magnitude of the loss, which is estimated for each direction over the time period Γ^{-1} by means of the equation [5]:

$$\Pi_i \simeq 2\varkappa_i e^{-\varkappa_i} \frac{\Gamma_i}{\Gamma}, \tag{26}$$

where $\varkappa_i$ is the ratio of the square of the permissible amplitude to the square of the stationary amplitude. For example, demanding that the loss not exceed 10^{-2}, we find that $\varkappa \simeq 6$, i.e., the semiaxes of the chamber must be approximately 2.5 times greater than the beam semiaxes. In addition, the dimension of the separatrix must be $\sqrt{\varkappa}$ times greater than the momentum spread of the beam. To accomplish this, the amplitude of the accelerating voltage must not be less than

$$eV \simeq \pi q |1 - \alpha| T \left(\frac{\Delta p}{p}\right)^2_{\rm per} = \pi q \varkappa |1 - \alpha| T \overline{\left(\frac{\Delta p}{p}\right)^2_{\rm st}} \simeq 0.85 \cdot 10^{-3} \frac{\varkappa}{\zeta} q |1 - \alpha| T. \tag{27}$$

On the basis of Eqs. (23)-(26), one can estimate the permissible angle for single scattering and the permissible variation in particle momentum: $\Theta_0 \simeq (rQ\sqrt{\varkappa}/R_0) \simeq 0.2$-$0.3$; $(\Delta p/p)_{\rm per} \simeq 0.06$-$0.1$. These results were used in the preceding section.

The average density of the decelerating medium and the size of the decrements are limited by the capabilities of the accelerating system which compensates for the average ionization energy loss:

$$\Delta E = 7.5 \cdot 10^{-21} \frac{\langle n \rangle R_0 Z}{T} \left(1 + 0.2 \ln \frac{T}{Z}\right); \tag{28}$$

$$\Gamma = 1.65 \cdot 10^{-13} \frac{\langle n \rangle Z}{T^{3/2}}, \tag{29}$$

where ΔE and T are measured in MeV, R_0 is in centimeters, n is in atoms/cm^3, and Γ is in sec^{-1}.

As an illustration, we consider a 1.5-MeV storage ring with an average radius of 50 cm and betatron frequencies in the range 3-4. If a hydrogen target is used for deceleration, estimates based on Eq. (25) give: $r_x \simeq r_z \simeq (1\text{-}1.3)/\sqrt{1-\xi}$ cm. Setting $\xi = 0.3$, we obtain $r_x \simeq r_z \simeq 1.2\text{-}1.5$ cm. For permissible losses because of multiple scattering ~0.01, the chamber radius must be 3-3.5 cm. To avoid particle escape from the separatrix, it is necessary to have an accelerating voltage of about 30 kV. Assuming a permissible energy loss per turn of ~9 kV ($\cos \varphi_s \simeq 0.3$) and using Eq. (22), we find that the surface density of the target is ~10^{19} atoms/cm^2 $\simeq 1.7 \cdot 10^{-5}$ g/cm^2. Furthermore, the oscillations will be damped with a decrement $\Gamma \simeq 3$ msec^{-1}.

Figure 2 indicates that the losses because of nuclear interactions and charge exchange are respectively 0.028 and 0.003. With the chamber dimensions specified, Coulomb scattering need not be considered because its effective cross section is 10-15 times less than the nuclear cross section. Collisions involving large momentum transfer make no contribution in this case either because the total losses (including multiple losses) are approximately 0.04, i.e., storage of ~25 injection pulses is possible in such a system requiring a period of approximately 10 msec. The maximum number of particles, which is determined by space charge forces, is approximately $2 \cdot 10^{12}$ protons in such a storage ring.

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