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Internal

SOME ESTIMATES OF PROPERTIES OF INTERSECTING BEAM ACCELERATORS

D. W. Kerst

University of Illinois and Midwestern Universities Research Association*

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Several calculations not given in MURA DWK/12 but discussed in MURA meetings will be described here along with some other considerations which need to be examined more in the study of a system for intersecting beams.

1. Multiple coulomb scattering of the beam.

If we assume the vacuum chamber to contain nitrogen at 10^{-5} mm pressure, what is the angle containing half the beam after scattering has continued for 10^3 seconds?

$$\bar{\theta}_{\frac{1}{2}} = \frac{Z\sqrt{N}}{2\beta^2(T+Mc^2)}$$ where N is the number of moles of gas of atomic number Z per square centemeter. $T + MC^2$ in Mev.

$$\bar{\theta}_{\frac{1}{2}} = \frac{7\sqrt{3 \times 10^{10} \times 10^3 \text{ sec} \times 10^{-5}/(760 \times 22{,}400)}}{2000 \times \text{Bev}}$$
$$= \frac{.015}{\text{Bev}} \text{ radians}$$

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At 10 Bev, $\theta_{\frac{1}{2}}$ = .0015 radians

At 3 Bev, $\theta_{\frac{1}{2}}$ = .005 radians

Thus at a conservative pressure of 10^{-5} mm and at 10 Bev, the scattering is less than 1.5 milliradians for half the beam after 1000 seconds. This gives approximately a one centimeter spread of the beam.

2. Beam Life.

Nuclear reactions or collisions with the residual gas will consume the beam eventually.

At 10^{-5} mm of pressure we have 10^{13} nucleons per cubic centimeter. These nucleons are in clusters forming nuclei which provide a certain amount of shielding of each other cutting down the cross section per nucleon say to 50%. The rate of reaction is then $n v \sigma = 50\% \times 10^{13} \times 3 \times 10^{10}$ cm/sec $\times 5 \times 10^{-26}$ cm^2 = 7.5×10^{-3} per sec. So the mean life of the beam is $(1/7.5) \times 10^{+3}$ = 130 seconds at 10^{-5} mm.

3. Background

These disintegrations and scatterings estimated in section 2 cause a background with which the experimenter must contend. If we have 5×10^{12} effective gas nucleons/cc. and 5×10^{14} moving protons in a machine or $\dfrac{5 \times 10^{14}}{\pi \times 10^{4}\ \text{cm}} = 1.6 \times 10^{10}$ protons/cc or 300 is the ratio of effective background nucleons to target protons in the target section. But the target protons are moving and hence twice as much volume is effective for producing desired reactions. Thus in the target the background at 10^{-5} mm is 150 times the desired count. In a one meter long target section 1/3 curie of background would exist. Perhaps we should strive for 10^{-7} mm pressure.

There are two important characteristics of this background: First, it is expected to be largely confined to the orbital plane and to travel forward, while the reactions of the intersecting beams would produce products traveling in all directions if the center of mass is at rest in the laboratory. Second, troublesome background may originate from the up-stream portions of the beam in each machine. To eliminate this a specific type of shielding would be needed. The magnets themsleves would ehlp shield and using particles which leave the orbital plane would help to avoid the background.

Exactly what would happen to the ionized residual gas in the vacuum chamber is not clear. There is an electric field of the order of 5000 volts/cm on the space surrounding the beam due to the charge in the beam, but the plasma of ions plus electrons would just polarize by .03 mm to neutralize this so the positive gas ions would not be sent far out of the beam. The beam would tend to become neutral by eventually collecting electrons. If an electric field parallel to the magnetic field were put on the space to pull ions and electrons to the opposite walls and if some mechanism could be used to collect them and to keep neutralized ions from going back as gas, we would have an usefull ion pump effect. Such things have been done (R.G. Herb's pump). Even a small electric field can hold back the ion gas pressure if ions can be screened from the polarization field produced by their separation from electrons. This field is $E^2/8\pi = P =$ pressure in dynes/cm^2. If $P = 10^{-5}$ mm of mercury ion pressure,

$$E = \sqrt{8\pi \ 1{,}300 \text{ dynes/cm}^2/\text{mm/cm}^2 \ P \text{ mm/cm}^2} = .55 \text{ ESU/cm}$$
$$= 170 \text{ volts/cm.}$$

It would be helpfull to have considerations of the shielding problem for typical experimental examples. (See R.W. Williams CAP-6 "Shielding problems of the 6 Bev Electron Synchrotron" and Citron's CERN Reports).

4. Phase space requirements.

If all time and space changes experience by the particles are adiabatic with regard to betatron oscillations and synchrotron oscillations, then the phase space occupied by a high energy beam composed of N rings of charge each successively carried from the injection radius and deposited at the high energy radius is N times as big as the phase space required for each injected ring. The importance of examining this problem was pointed out by Prof. E.P. Wigner. Some of the processes occuring during R. F. acceleration may not be adiabatic. The estimate given here assumes phase space is conserved. The results do not guarantee that one can be clever enough to accomplish building up a big beam, but they show what Liouville's theorem allows.

$\Phi = N \Delta r_i \Delta p_{ri} \Delta z_i \Delta p_{zi} \Delta \theta_i \Delta p_{\theta i}$ = constant where the subscript i refers to injection. The quantities are cannonical variables. We would like to see what p_θ is. If we assume that our case is like an example with a vector potential which has just a θ component, we can make a simple calculation.

From the Lagrangian we have

$p_\theta = (p + e/c \quad A_\theta) R$ where p is the total momentum. But $A_\theta = - \phi/2\pi R$ where ϕ is the flux included within the radius R the negative sign is in conformity with Lenz' Law.

Thus

$$p_\theta = p R - \frac{e}{c} \frac{\phi}{2\pi} \quad \text{and}$$

$$\Delta p_\theta = p \Delta R + R \Delta p - \frac{e}{c} \bar{B}_z R \Delta R$$

where $\bar{B}_z$ is the average bending field for an orbit of mean radius R.

But $p = \frac{e}{c} \overline{B_z} R$ so

$$\Delta p_\theta = R \Delta p$$

Thus we have

$\Delta p_\theta \, \Delta \theta = \Delta p \, \Delta s$ where Δs is the spread in arc around the orbit measured away from the center of the ensemble. So

$$\Phi = N \, \Delta r_i \, \Delta p_{ri} \, \Delta z_i \, \Delta p_{zi} \, \Delta s_i \, \Delta p_{si}$$

If we assume no coupling between betatron oscillations and phase oscillations, then betatron oscillation phase space and phase oscillation phase space are separately conserved. This is the case without a tipped R.F. cavity edge or without a radial or axial variation of R.F. voltage, for examples.

Then for final conditions

$$\Delta z \, \Delta p_z \, \Delta r \, \Delta p_r = \Delta r_i \, \Delta p_i \, \Delta z_i \, \Delta p_{zi}$$

We know that

$$\left.\begin{matrix} \Delta r \\ \Delta z \end{matrix}\right\} p^{-\frac{1}{2}}$$ due to adiabatic compression of the beam during acceleration if σ_x, σ_z are constant. Also under these

[illegible] p_θ [illegible] $\frac{\ }{c} \; \frac{\ }{2\pi}$ [illegible]

$$\left.\begin{matrix} \Delta p_r \\ \Delta p_z \end{matrix}\right\} p^{+\frac{1}{2}} \quad \text{keeping } \Delta r \, \Delta p_r \, \Delta p_z \cong \text{constant.}$$

If we increase the momentum, p, by $\sim$ 100 during acceleration, then $\Delta r / \Delta r_i = \Delta z / \Delta z_i = 1/10$ so a one centimeter diameter final beam would result from a 10 cm. diameter initial beam. Not only could we use an injected beam of 10 cm. diam., but we could use a Δp_r or Δp_z in the injected beam so that the resultant betatron oscillations has an amplitude of 10 cm/2. That is:

$$5 \text{ cm} \cong \frac{\Delta p_{ri}}{p_i} \lambda\!\!\!\!- = \frac{\Delta p_{ri}}{p_i}\left(\frac{R}{N\sigma_r}\right) = \frac{\Delta p_{zi}}{p_i}\left(\frac{R}{N\sigma_z}\right)$$

where N is the number of sectors and we are ignoring the orbital scallops due to A.G. If the angular spread of the beam is equal to $\Delta p_{ri}/p_i$ as calculated from the formula above, the injection system is matched to the accelerator and we can fill the orbital oscillation phase space with one turm. If the injector we have cannot fill this space, then in principle we can shoot in several turns either swinging $\Delta p_{ri}/p$ around by directing the gun or by moving the position of the gun by Δr_i or by both until we have built up the Δr_i and the Δp_{ri} allowed by the accelerator aperture. The phase space in this example is $\Delta r_i \Delta p_{ri} = \Delta r_i^2 \; p_i/\lambda\!\!\!\!-$. If there are ~ 12 waves around the machine, $R/\lambda\!\!\!\!- = \nu_r = 12$, and if $R \cong 10^4$ cm.,

$$\Delta r_i \frac{\Delta p_{ri}}{p_i} = \frac{\nu_r \Delta r^2}{R} = \frac{\Delta r^2\, 12}{10^4} = (5)^2 \times 10^{-3} =$$

.025 available space, but Van De Graaf electro static accelerators and linacs produce 1 cm^2 beam with .001 radians spread $= \Delta p_{rI}/p_I$ or $\Delta r_I \; \Delta p_{rI}/p_I = .0005$ where Δr_I and Δp_{rI} are from a specific injector. Consequently these injectors would have to spray in ~ 50 turns = q to fill the aperture.

In general we should have

$$\Phi = \Phi_\beta \Phi_s$$

$$\Phi_\beta = q \nu_r \nu_z \Delta r_I^2 \Delta z_I^2 p_i^2/R^2 = \nu_r \nu_z \Delta r_i^2 \Delta z_i^2 p_i^2/R$$

where q is the number of spirals put in at injection time.

And for synchrotron oscillations:

$$\Phi_s = n \Delta s_i \Delta p_i = n \Delta s_i (\Delta p_i/p_i) p_i$$
$$= n \Delta s_i \frac{1}{2} \frac{\Delta KE_I}{KE_i} p_i$$

if injection is at classical energy.

or

$$\Phi = n q \nu_r \nu_z \Delta r_I^2 \Delta z_I^2 \Delta s_i \frac{\Delta KE_i}{KE_i} p_i^3/2R^2 \quad \text{at injection}$$

and at high energy if R does not change much:

$$\Delta p = \frac{e}{c} \Delta(H\rho) = \frac{e}{c}(k+1)\frac{H}{R}\rho\,\delta r = p(k+1)\,\delta r/R$$

$$\Phi = \nu_r \nu_z \Delta r^2 \Delta z^2 \frac{p^2}{R^2} \Delta s (k+1) p \frac{\delta r}{R}$$

where δr is the shift of the orbit associated with Δ p.

Equating:

$$n = \frac{\Delta r^2 \Delta z^2 \Delta s\, \delta r (k+1)\, 2\, p^3}{q \Delta r_I^2 \Delta z_I^2 \Delta s_i (\Delta KE_I/KE_i) p_i^3 R} = \frac{\text{Final phase space}}{\text{Initial phase space per F.M. cycle}}$$

If Φ_β is separately conserved i.e. no coupling with phase oscillations,

$$\Delta r^2 \Delta z^2 p^2 \nu_r \nu_z/R^2 = q \Delta r_I^2 \Delta z_I^2 p_i \nu_r \nu_z/R^2$$

and $$\eta = \frac{\Delta s\; 2(k+1)\, p\, \delta r}{\Delta s_i (\Delta KE_f / KE_i)\, p_i\, R}$$

Suppose $\Delta s = \Delta s_i (= 2\pi R)$ (no bunching)

$k \sim 100$

$p/p_i \sim 100$

$\Delta KE_f/KE_i = .001$ such as a Van de Graaf

$R \sim 10^4$ cm

$N = 2000.\, \delta r$. is the number of rings which can be separately frequence modulated up to full energy. Thus if we allow the energy spread of the beam to produce $\delta r = \frac{1}{2}$ cm, $N = 1000$ F.M. cycles. We noticed earlier that q could be 50 for an electrostatic accelerator (to take that as an example for which we have information). Thus: $Nq = 50{,}000$ spirals from the injector would reach ~ 1 cm^2 beam at full energy. This must be done in about 100 seconds because of the life time of the beam discussed earlier so we would have to F.M. at the rate of about 10 per second.

If we put 50,000 injected spirals together at $\beta = 1/10$ and at 2 milliampere out of the injector, we would have 50 amperes becoming 500 amperes after acceleration. This beam would contain

$$\frac{500}{10} \text{ emu} \quad \times \frac{c}{4.8 \times 10^{-10}} \quad 2\pi R = 6.6 \times 10^{15} \text{ particles}$$

per machine. Such a beam current if not neutralized by the plasma of ionized gas in the vacuum vessel would not have any focussing effect on itself at relativisitic speeds because the space charge repulsion in the beam is just canceled by the space current attraction at the velocity of light. However, if electrons accumulated in the beam and removed the space charge repulsion, the magnetic field of 200 gauss at the edge of the beam due to its own current would give an added $\Delta k = +200$ within the beam if the current density in the beam were uniform, thus providing much more radial and axial focussing. Outside the beam Δk would reverse sign and become $\Delta k = -200$ at the outside edge, but it still provides a beam focussing effect because parallel currents attract each other. A strong nonlinearity occurs in the restoring force. If a single particle were oscillating in a parabolic smooth approximation potential well shown by the dotted line, then the space current would distort the potential well as shown by the solid line.

It seems that a better injector for a machine with $\nu \sim 12$ and Δr_i 5 cm might be one which needed more phase space; that is one which gave a fat beam about 10 cm in diameter of 50 millamps and with $\Delta p_I / p_i = .01$ radians. Then only one injection turn would be required, $q = 1$.

Since Δ ps/ps $\cong$ .05/($\sqrt{h}$) for low energy injection (LJL MAC-3)

Δ KEi/Kei = 1/(10$\sqrt{h}$) where h is the harmonic number. It would be good to be able to inject with a large enough energy spread to fill the Φ_s space so that filamentation of this space occupied during phase oscillation by the ensemble does not affect the eventual yield. What would be best for this is an injector with an R.F. bunched beam injecting bunches which land in the pearls of synchrotron oscillation phase space. It may well be that if a full Δ ps/p emerges from the injector with no bunching, then some particles lost on the first F.M. cycle would coast around and be caught on the second or on some subsequent cycle.

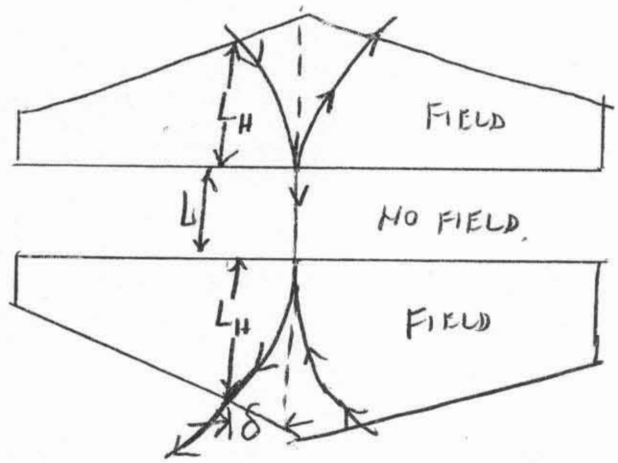
If coupling between synchrotron and betatron oscillations could be controlled by changing the slant of the R.F. cavity with radius it may be possible to permanently exchange synchrotron phase space for betatron phase space and to thereby do better with likely apertures and injectors.

The example given here has an order of magnitude more final current than the examples treated in earlier sections, but a larger Δ KEi/Kei, say.005, and a higher p_i would bring this current down proportionally.

It is very interesting to know that L. Alvarez and F.S. Crawford found that they could pile up four rings of particles in the 184" Berkely cyclotron by successive injection and frequency modulation with an interrupted oscillator. No more current could be brought up however, although the coasting beam lifetime is about a minute.

5. Beam Separation Geometry.

How long must L_H be to separate the beams 10 cm? We want $\delta = 5$ cm if $\rho = 2500$ cm at 10 Bev and 14 kilogauss



$L_H = \sqrt{2\rho\delta} = 160$ cm. Thus if L = 100 cm. The total straight section is 2 x 160 + 100 = 4.2 meters. This is not too long to be without focussing magnets in a radial sector Mark I; but since the beams are 10 cm apart, focussing with finite gradients could probably take place within the bending field even with a gap of about 10 cm width.

6. Kinematics.

If two particles have momentum p_1' and p_2' in the laboratory system, the energy in the center of mass system is (Jauch MURA-JMJ-3)

$$E_1 + E_2 = \sqrt{(M_1 c^2)^2 + (M_2 c^2)^2 + 2(E_1' E_2' - c^2 p_2' p_1')}$$

E is the kinetic plus rest energy of a particle. If the particles are identical and are going in opposite directions in the labsystem, this becomes

$$E^2 = 2 + 2\left(E_2' E_1' + \sqrt{\left[(E_2')^2 - 1\right]\left[(E_1')^2 - 1\right]}\right)$$

using MC^2 as the unit of energy.

If $(E_2')^2 \gg 1$ and $(E_1')^2 \gg 1$, then

$$E_{CM}^2 = 2 + 4E_1' E_2' \cong 4E_1' E_2'$$

or

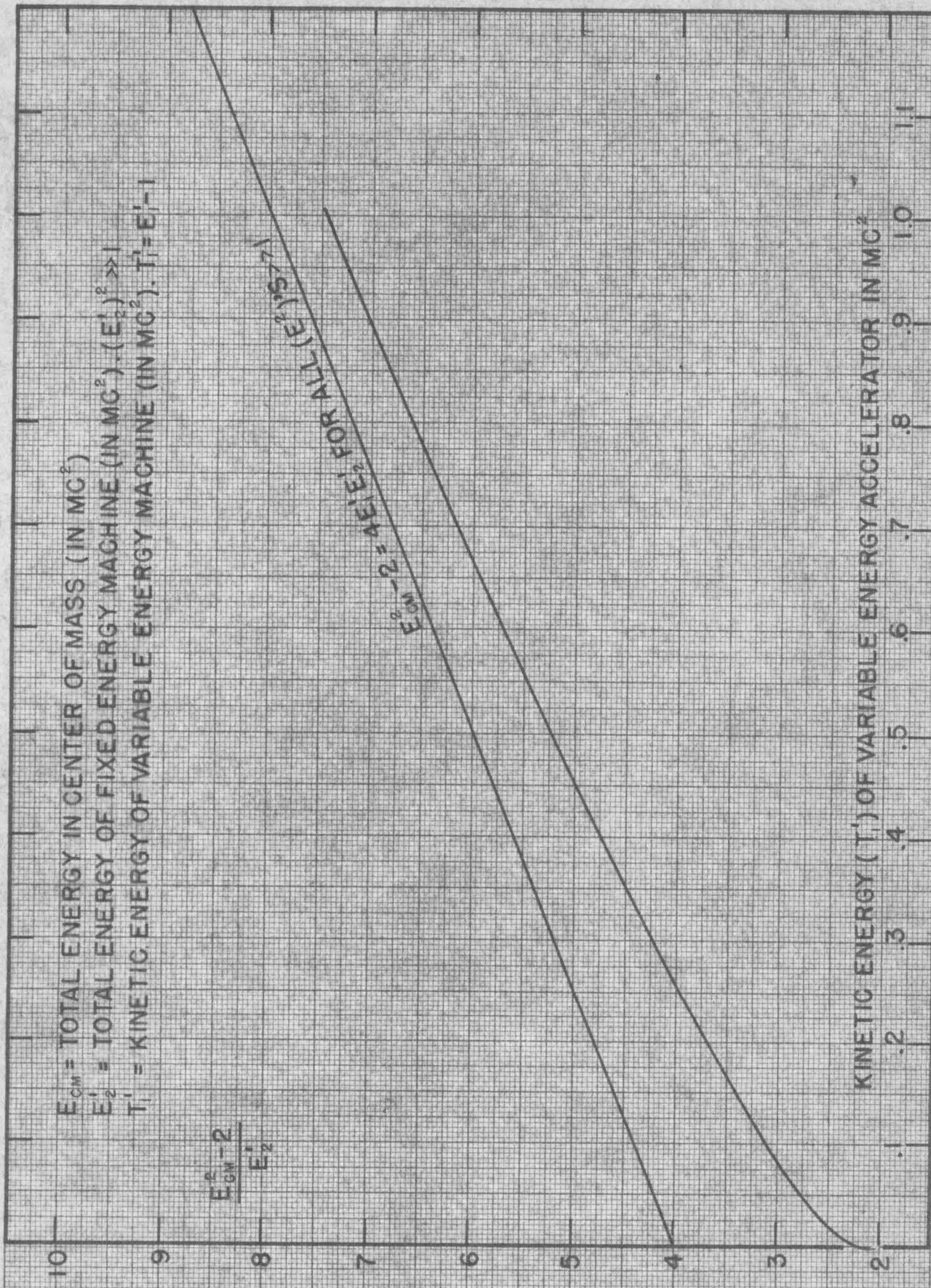
$$E = 2\sqrt{E_1' E_2'} \quad \text{when } E^2 \gg 2$$

If we want a certain E in the center of mass system and if we shoot two machines of radii $R_1 \propto E_1'$ and $R_2 \propto E_2'$ at one another we have $R_1 R_2 = \text{constant} = R_o^2$. The cost of a machine varies as R^3 so

$$\$ = \left(\frac{R_1}{R_o}\right)^3 + \left(\frac{R_2}{R_o}\right)^3 = \left(\frac{R_1}{R_o}\right)^3 + \left(\frac{R_o}{R_1}\right)^3$$

. This cost has its minimum at $R_1 = R_2 = R_o$ so it is most economical to have both accelerators of the same energy. The cost doubles if $R/R_o = 1.5$ for one of the machines and $1/1.5$ for the other machine. That is if one machine is $2\frac{1}{4}$ times bigger than the other, the cost is twice minimum.

The graph shows that for cases where $(E_2')^2 \gg 1$ the formula $E^2 - 2 = 2E_1' E_2'$ gives E with less than $3\frac{1}{2}\%$ overestimate as soon as $T' \geq MC^2$.



E_{CM} = TOTAL ENERGY IN CENTER OF MASS (IN MC^2)
E_2' = TOTAL ENERGY OF FIXED ENERGY MACHINE (IN MC^2). $(E_2')^2 >> 1$
T_1' = KINETIC ENERGY OF VARIABLE ENERGY MACHINE (IN MC^2). $T_1' = E_1' - 1$
$E^2_{CM} - 2 = 4E_1'E_2'$ FOR ALL $(E_2')^2 >> 1$
$\frac{E^2_{CM} - 2}{E_2'}$
KINETIC ENERGY (T_1') OF VARIABLE ENERGY ACCELERATOR IN MC^2
2
3
4
5
6
7
8
9
10
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1

If only one machine is used bombarding a fixed target then $T_1' = 0$ and

$$E^2 = 2 + 2E_2' \cong 2E'$$
$$E = \sqrt{2E'}$$

If we aks what single machine with energy E' gives the same energy as two machines with energies E_1' and E_2' bombarding each other we have.

$$\sqrt{2E'} = 2\sqrt{E_1' E_2'} \qquad \text{or}$$

$E' = 2E_1' E_2'$ provided $(E_2')^2 \gg 1$ and $E_1' \geqq 2$ that is $T_1' \gg 1$ to the approximation that a rest mass is one Bev, this means that if #1 machine gives 1 Bev ($E_1' = 2$) and #2 machine gives 9 Bev ($E_2' = 10$) the equivalent machine would have to be 40 Bev = E'. For $T_1' < MC^2$ the graph should be used. If #1 were 3 Bev = T_1', we would have 80 Bev = E'. While if $E_1' = E_2' = 10$ Bev, we have 200 Bev.

Calculating accurately two 21.6 Bev accelerators would be equivalent to one trillion electron volt (Tev) accelerator, but of course they would give only 43.2 Bev plus two rest masses in the center of mass system.

It would probably be important to be able to vary the field strengths of both magnets together so that the energy of the reaction products could be slowed down to an energy easily manageable with detecting equipment, provided new thresholds are reached at these high energies. On the other hand, any very short lived particles which do not live long enough to leave the target area near threshold energy might have their lifetimes and consequently their path lengths increased an order of magnitude by high energy bombardment so that they could escape far enough to be detected.

If $E_1' \neq E_2'$ the center of mass is not at rest in the laboratory and the spacial distribution of the reaction products is peaked in the direction the higher energy particle goes. This provides a way to get slow particles, although the energy may be far above threshold energy, by using reaction products emitted in a direction opposite to that of the center of mass.

These considerations have been rough and inconsistent in some chosen numbers, but they show some of the questions which must be studied if we are to make a judgement of the feasibility of making such a beam system now or of allowing for the addition of a second machine later.