ON THE OSCILLATION DECREMENTS IN ACCELERATORS IN THE PRESENCE OF ARBITRARY ENERGY LOSSES

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In accelerators, particles experience energy losses caused by various effects (ionization and bremsstrahlung on residual gases, magnetic bremsstrahlung, etc.). These losses and their compensation by the high-frequency accelerating field lead to the following consequences: 1) oscillations (betatron and synchrotron) are excited as a result of the discrete nature of the losses, while the excitation mechanism recalls the excitation of oscillations in an oscillator under the action of noise; 2) the energy losses, on the average, produce a force which may play the role of positive or negative friction acting on the oscillations. Until now, this problem was considered only for one particular case – the loss due to magnetic bremsstrahlung in the relativistic motion of electrons (see, for instance, [1], Ch. 5).

The present article provides the expressions for the decrements of betatron and synchrotron oscillations in the presence of arbitrary energy losses. These expressions, which hold for any particle energy, make it possible to estimate the magnitude and the sign of the friction connected with the losses and also the efficiency of any method of artificial oscillation damping.

We shall abstract ourselves from the above-mentioned oscillation excitation connected with loss fluctuations. This excitation basically depends on the actual conditions, in particular, the fluctuation distribution function.

Let the instantaneous energy loss P depend on the total particle energy E, the magnetic field B at the point in question, the radial coordinate x (along the normal to the equilibrium orbit), and the generalized azimuth θ:

$$P = P(E, B, x, \theta).$$

For determining the oscillation decrements, we shall consider the equation of motion of particles:

$$\frac{d}{dt}(m\mathbf{v}) = \frac{e}{c}[\mathbf{vB}] + e\mathscr{E} - \frac{P}{v^2}v, \tag{1}$$

where the last term represents the damping force connected with the energy losses, $\mathbf{v}$ and m are the velocity and the total particle mass, respectively, and $\mathscr{E}$ is the electric field. Equation (1) holds if the direction of the dissipative force is in opposition to the velocity direction.

From Eq. (1), we can obtain linearized equations describing the motion in the symmetry plane of a cyclic accelerator in the vicinity of the equilibrium orbit (see [1], Ch. 5, paragraph 2):

$$\frac{d^2x}{d\theta^2} + \left(\Gamma_1 + \frac{\Gamma}{\beta^2}\right)\frac{dx}{d\theta} + \frac{K^2}{K_0^2}(1-n)\,x = \frac{K}{K_0^2}\cdot\frac{\varepsilon}{E_s}\,; \tag{2}$$

$$\begin{aligned}\frac{1}{E}\cdot\frac{d\varepsilon}{d\theta} = &-\frac{eV\sin\varphi_s}{2\pi E}\eta - \\ &-\Gamma(\theta)\left[\left(\frac{\partial\ln P}{\partial\ln E} - \frac{1-\beta^2}{\beta^2}\right)\frac{\varepsilon}{E} + \right. \\ &\left. + \left(1 - n\frac{\partial\ln P}{\partial\ln B} + \frac{1}{K}\cdot\frac{\partial\ln P}{\partial x}\right)Kx\right],\end{aligned} \tag{3}$$

where $K = K(\theta)$ is the curvature of the orbit, $2\pi/K_0$ is the length of the orbit, $n = (1/KB) \cdot (\partial B/\partial x)$ is the field index, $\beta = (v/c)$; $\varepsilon = E - E_s$; $\eta = \varphi - \varphi_s$, where φ_s and E_s are the equilibrium values of the phase and the energy, respectively, q is the multiplicity of acceleration, ω is the angular frequency, V is the amplitude of the accelerating voltage, and

$$\frac{d\eta}{d\theta} = q\left(Kx - \frac{\Delta\beta}{\beta}\right); \quad \Gamma = \frac{P}{\omega E}; \; \Gamma_1 = \frac{1}{E} \cdot \frac{dE_s}{d\theta}. \tag{4}$$

The closed perturbed orbit $x_0(\theta)$ is described by the expression

$$x_0(\theta) = \frac{\psi(0)}{K_0} \cdot \frac{\varepsilon}{E}, \tag{5}$$

where $\psi(\theta)$ is the known periodic function of the orbit.

By means of Eqs. (2)-(5), using the methods applied in [1] (see Ch. 5, paragraph 3, or Appendix D), we can find the expressions for the decrement $\langle\zeta_x\rangle$ of radial betatron oscillations and the decrement $\langle\zeta_s\rangle$ of synchrotron oscillations, connected with energy losses:

$$\langle\zeta_x\rangle = \frac{1}{2}\left\langle \Gamma\left(\frac{1}{\beta^2} - F\right)\right\rangle; \tag{6}$$

$$\langle\zeta_s\rangle = \frac{1}{2}\left\langle \Gamma\left(\frac{\partial \ln P}{\partial \ln E} - \frac{1-\beta^2}{\beta^2} + F\right)\right\rangle, \tag{7}$$

where $\langle\;\rangle$ denotes averaging with respect to θ;

$$F = \frac{K\psi}{K_0}\left[\left(1 - n\frac{\partial \ln P}{\partial \ln B}\right) + \frac{1}{K} \cdot \frac{\partial \ln P}{\partial x}\right]. \tag{8}$$

From Eqs. (6) and (7), we find the sum of decrements

$$\sigma = \langle\zeta_x\rangle + \langle\zeta_s\rangle = \frac{1}{2}\left\langle \Gamma\left(1 + \frac{\partial \ln P}{\partial \ln E}\right)\right\rangle, \tag{9}$$

which characterizes the rate of change in the over-all phase volume of oscillations during the process of motion. The found Eq. (9) indicates that this rate is determined only by the dependence of losses on the energy E. If the lost power P decreases with an increase in D faster than E^{-1}, then, $\sigma < 0$, which corresponds to an increase in the total phase volume, for which simultaneous damping of betatron and synchrotron oscillations cannot be secured. This can be ensured if P decreases more slowly than E^{-1}, and even more so if P increases with E.

As an example, we shall provide approximate estimates of the σ value for certain particular cases.

1. Ionization losses lead to the buildup of oscillations for $\beta < (1/\sqrt{2})$:

$$P \backsim \frac{1}{\beta}, \; \sigma \approx \frac{1}{2}\left\langle \Gamma \frac{2\beta^2 - 1}{\beta^2}\right\rangle. \tag{10}$$

2. Bremsstrahlung losses lead to the buildup of oscillations for $\beta < (1/2)$:

$$P \backsim \beta E^2, \; \sigma \approx \frac{1}{2}\left\langle \Gamma \frac{4\beta^2 - 1}{\beta^2}\right\rangle. \tag{11}$$

3. The loss due to magnetic relativistic bremsstrahlung ($\beta = 1$)

$$P \backsim E^2, \; \sigma = \frac{3}{2}\langle\Gamma\rangle \tag{12}$$

leads, as is known, to the damping of oscillations, which is of great importance for the operation of electron accelerators and accumulators.

The calculation of the root-mean-square amplitudes of the oscillations excited as a result of losses due to ionization, bremsstrahlung, etc. with an allowance for the friction described by Eqs. (6) and (7) will be presented in another paper.

LITERATURE CITED

1. A. A. Kolomenskii and A. N. Lebedev, Theory of Cyclic Accelerators [in Russian], Moscow, Fizmatgiz (1962).