IONIZATION COOLING: PHYSICS AND APPLICATIONS

V.V.Parkhomchuk and A.N.Skrinsky

Institute of Nuclear Physics,
630090, Novosibirsk 90, USSR

Ionization cooling is based on use of the ionization energy losses of charged particles, which travel in matter, with the external source compensation for the average energy losses of equilibrium particles /1-3/. When the motion in matter occurs at the average electron density N_e, the power of losses $\mathcal{P}_{fr}$ and the frictional force F_{fr} are, correspondingly, equal to

$$\mathcal{P}_{fr} = 4\pi \frac{N_e e^4 L}{m_e v} \; ; \quad \vec{F}_{fr} = -\frac{\mathcal{P}_{fr}}{v^2}\vec{v} \qquad (1)$$

where e and m_e are the charge and mass of an electron, $L_i = \ln\left(\frac{2m_e\gamma^2 v^2}{I}\right) \approx 8 \div 12$, Z_i is the atomic number of matter, I is the effective ionization potential, and $\vec{v}$ is the velocity of particle motion.

1. For simplicity, let us first consider the case of a straight-line motion of the beam under cooling in matter with ionization losses. The specific features of the ionization cooling manifest themselves in this case as well, but the sum of the decrements is completely in dependent of the specific features of the particle motion, including the presence of focusing. Assuming that the deviations of the longitudinal momentum and the transverse momentum (with respect to the direction of the force which recovers the energy losses) are small, $|\Delta P_\parallel|$, $|\Delta P_x|$, $|\Delta P_z| \ll P$, we immediately obtain, in a linear approximation, the damping decrements in the form

$$\delta_{x,z} = \frac{F_{fr}}{P} = \frac{\partial F_{fr}^{x,z}}{\partial P_{x,z}} \; ; \quad \delta_\parallel = \frac{\partial F_{fr}}{\partial P_\parallel} \qquad (2)$$

and an expression for the sum of the decrements:

$$\sum\delta = \delta_x + \delta_z + \delta_\parallel = \mathrm{div}_p \vec{F}_{fr} = \frac{2\mathcal{P}_{fr}}{Pv}\left(1 - \frac{P_\parallel}{2v}\frac{\partial v_\parallel}{\partial P_\parallel}\right) + \frac{1}{v}\frac{\partial \mathcal{P}_{fr}}{\partial P_\parallel} =$$

$$= \begin{cases} \dfrac{\mathcal{P}_{fr}}{2E_k} + \dfrac{\partial \mathcal{P}_{fr}}{\partial E_k}, & v \ll c \\ \dfrac{2\mathcal{P}_{fr}}{E} + \dfrac{\partial \mathcal{P}_{fr}}{\partial E}, & v \to c \end{cases} \qquad (3)$$

In the non-relativistic case, a rapid fall in the frictional force with increasing of the kinetic energy E_k leads to a negative value of the longitudinal decrement and to a substantial decrease of the sum of the decrements compared with $\delta_x + \delta_z$:

$$\sum\delta = \frac{\mathcal{P}_{fr}}{E_k}\cdot\frac{2}{L_i} = 16\pi r_e^2 c \frac{m_e}{M} N_e \left(\frac{v}{c}\right)^{-3}, \; v \ll c. \qquad (3a)$$

This is 3÷5 times less than the sum of the transverse decrements only.

In the relativistic region, where the ionization losses become, in practice, energy-independent, the sum of the decrement is equal to

$$\sum\delta = \frac{2\mathcal{P}_{fr}}{E} = 8\pi r_e^2 c L_i N_e \gamma^{-1}, \; v \to c. \qquad (3b)$$

The magnetic structure of a cyclic ionization cooler and the shape of a target should be such that the decrements from transverse degrees of freedom are 'transferred' to a longitudinal one. To ensure the energy spread damping, it is sufficient to place the targets on a section where the position of the closed orbit of a particle depends on its energy and to make the target thickness variable in order that the larger thickness t of the target correspond to the higher energy:

$$\frac{1}{t}\frac{dt}{dx} = \left(\frac{\delta_\parallel}{\delta_x}\psi\right)^{-1}, \qquad (4)$$

where ψ is the dispersion function in the target region ($\psi = P\,dx/dP$). If $v \ll c$ it is necessary to use the coupling with both transversal degrees simultaneously.

2. The multiple scattering on the nuclei and electrons of the target itself and the fluctuations of ionization losses are referred to the main 'heating' factors determining the equilibrium of the momentum spreads. To a sufficient accuracy, the direct diffusion of transverse and longitudinal momenta can be written down as follows:

$$\frac{d(\overline{\Delta P_{x,z}^2})}{d\tau} = \frac{4\pi N_i Z_i^2 e^4 L_c}{v} = Z_i m_e \frac{L_c}{L_i}\mathcal{P}_{fr} \; ;$$
$$\frac{d\overline{(\Delta E)^2}}{d\tau} = \Delta E_{max}\cdot\frac{2\pi N_e e^4}{m_e v} = \Delta E_{max}\cdot\mathcal{P}_{fr}/L_i \; ; \qquad (5)$$

where $L_c = \ln\left(\frac{\theta_{max}}{\theta_{min}}\right) \simeq 15$ and ΔE_{max} is the maximum energy transfer to a target electron:

$$\Delta E_{max} = \frac{2m_e\left(\frac{E^2}{c^4} - M^2\right)c^2}{2m_e E/c^2 + M^2 + m_e^2} = \begin{cases} 4\frac{m_e}{M}E_k, & v \ll c \\ 2\frac{m_e}{M}\gamma E, & 1 \ll \gamma \ll \frac{M}{2m_e} \\ E, & \gamma \gg \frac{M}{2m_e} \end{cases}$$

(no aperture limitation is assumed). If each decrement is regarded to be made equal to one third of the sum of decrements (3) as a result of the choice of the magnetic structure, the equilibrium values of the transverse angles and of the energy spread are then determined by the following expressions (with the excitation of transverse oscillations by the fluctuations of energy losses taken into account):

$$\left.\begin{aligned} \left(\frac{\Delta P_{x,z}}{P}\right)^2 &= \frac{3}{2} L_c Z_i \frac{m_e}{M} + \frac{3}{4}\frac{\psi_{x,z}^2}{\beta_{x,z}^2}\frac{m_e}{M} \\ \left(\frac{\Delta P_\parallel}{P}\right)^2 &= \left(\frac{1}{2}\frac{\sigma_E}{E_k}\right)^2 = \frac{3}{4}\frac{m_e}{M} \end{aligned}\right\} v \ll c \; ;$$

$$\left.\begin{aligned}\left(\frac{\Delta P_{x,z}}{P}\right)^2 &= \frac{3}{2}\frac{L_c}{L_i}Z_i\frac{m_e}{\gamma M}+\frac{3}{2}\frac{m_e}{M}\frac{\gamma}{L_i}\,\frac{\psi_{x,z}^2}{\beta_{x,z}^2} \\ \left(\frac{\Delta P_{\parallel}}{P}\right)^2 &= \left(\frac{\sigma_E}{E}\right)^2=\frac{3}{2}\,\frac{m_e}{M}\cdot\frac{\gamma}{L_i}\end{aligned}\right\}\; 1\ll\gamma\ll\frac{M}{2m_e} \qquad (6)$$

where $\beta_{x,z}$ are the values of the beta-function in the target region and σ_E is the energy spread in the beam.

Thus, one can obtain a small value of the equilibrium transverse emittance

$$\Omega_{x,z}=\left(\frac{\Delta P_{x,z}}{P}\right)^2\cdot\beta_{x,z}$$

by choosing the magnetic structure so that the minimum of the beta-function take place in the target region (assuming $\psi_{x,z}\leq\beta_{x,z}\sqrt{\varepsilon L_i/\gamma}$)

The equilibrium geometrical longitudinal emittance of the beam will be equal to

$$\Omega_{\parallel}=\sigma_E\cdot\frac{\ell_\varphi}{v}\cdot\frac{1}{\gamma M v} \qquad (7)$$

$$\Omega_{\parallel}\approx\frac{\ell_\varphi\sigma_E}{E}\;,\; v\to c$$

where ℓ_φ is the mean azimuthal deviation of the particles in the bunch from the equilibrium particle, which is proportional to σ_E. Let us assume that compensation of the ionization losses takes place on a q-harmonic of the revolution frequency ω_s in the phase stability regime. The relation between ℓ_φ and σ_E is then given by the expressions

$$\ell_\varphi=\frac{R}{\omega_\varphi}\,\frac{d\omega}{dE}\cdot\sigma_E$$

where R is the mean radius of the accelerator, $\omega_\varphi=\sqrt{eU_0 q\sin\varphi_s\,\omega_s\frac{d\omega}{dE}/2\pi}$ is the frequency of phase oscillations, eU_0 is the amplitude of energy gain from the accelerating element per turn, φ_s is the equilibrium phase (note that $eU_0\cos\varphi_s$ equals the ionization losses per turn) and, finally, $d\omega/dE$ characterizes the variation rate of the particle revolution frequency in the accelerator as the energy varies and tends to zero at the so-called 'critical energy', dependent on the accelerator magnetic structure. Hence, we obtain

$$\Omega_{\parallel}=\left(\frac{\sigma_E}{E}\right)^2\cdot R\frac{E}{\omega_\varphi}\cdot\frac{d\omega}{dE}\sim\frac{3}{2}\frac{m_e}{M}\cdot\frac{\gamma}{L_i}\,R\frac{E}{\omega_\varphi}\frac{d\omega}{dE}\;,\; 1\ll\gamma\ll\frac{M}{2m_e}\,. \qquad (8)$$

Thus, in order to obtain as small equilibrium longitudinal emittance of the beam as possible at a given energy of cooling, it is necessary to maximally decrease the effective 'longitudinal focal length' $R\frac{E}{\omega_\varphi}\cdot\frac{d\omega}{dE}$ of the accelerator, in particular, trying to increase the harmonic number of the applied compensating RF voltage in the single-bunch regime, it is necessary to strive for working at an energy close to a critical one.

After cooling one can take advantage of the smallness of the equilibrium longitudinal emittance (due to smallness of the equilibrium length of the bunch) for a sharp monochromatization of the beam. To do this, it is sufficient, for example, to eliminate first the ionization losses adiabatically slowly, smoothly reducing the thickness of targets, and then to decrease the accelerating voltage. This results in lengthening the bunch, thereby gaining, proportionally, in monochromaticity. The adiabatic lengthening of the bunch can also be made in an additional matched accelerating track, or using a bending expander with further compensation of the energy gradient along the bunch.

3. Let us now consider for what particles the ionization cooling is reasonable to use /3/.

For electrons and positrons, the ionization cooling is not applicable at all. At low energies the multiple scattering takes place more rapidly than the ionization deceleration, and at high energies the main energy losses are the radiation ones which occur by large 'portions' because the bremsstrahlung spectrum is uniform up to the quanta energies of the order of the initial energy of electrons.

When cooling the protons and antiprotons, the major obstacle at not too low energies is a strong (nuclear) interaction with the target nuclei. (It is worth emphasizing that the necessity for confining the particles, scattered in the target at an angle several times larger compared with the equilibrium angular spread, offers the possibility of neglecting the particle losses because of the single Coulomb scattering). In this case, the particle loss cross section is nearly equal to the total nuclear cross section. The estimates show that the proton beams can be cooled only at 100 MeV even for the hydrogen target /2/.

For antiprotons, the nuclear cross section at low energies is much larger and, hence, the ionization cooling is inapplicable for them, except, possibly, the region of very low energies where the cross section for antiprotons is unknown.

The most interesting and promising is the application of ionization cooling to muon beams since there are no, in practice, radiation losses and nuclear interaction (except the Coulomb scattering) for these beams. The lifetime of the muon beam is limited by the muon decay with the time $\gamma\tau_\mu$ even under the conditions when the accelerator confines the beam with the equilibrium emittance with a sufficient reserve. Therefore, the time of cooling has to be several times shorter compared with the decay time:

$$\left(\Sigma\delta\right)^{-1}\ll\gamma\tilde{\tau}_\mu$$

In our further consideration we will restrict ourselves to the case $1\ll\gamma\ll\mu/2m_e$, where μ is the mass of a muon, since at nonrelativistic energies the equilibrium angle proves to be too large - of the order of unity, whereas at higher energies the equilibrium energy spread becomes too high. The condition for a sufficiently high rate of cooling takes the form

$$\frac{1}{3}\cdot\frac{2\mathcal{P}_\mu}{E}>\frac{\mu c^2}{E\tau_\mu}\to\left\langle\frac{dE}{dx}\right\rangle_{orbit}>\frac{3}{2}\,\frac{\mu c}{\tau_\mu}\approx 1.5\ \text{keV/cm}.$$

If we use, for example, lithium targets, then an admissible fraction ζ of the orbit, used for a target, should be as follows:

$$\zeta \gg \frac{1.5\cdot 10^{3}}{2\cdot 10^{6}} \simeq 0.7\cdot 10^{-3},$$

i.e. the target substance has to occupy of the order of 1% of the accelerator orbit.

Importance is of the fact that the several phase oscillations must occur during the damping time, and, hence, the frequency of phase oscillations should be substantially higher than .

It is worth noting that the cooling can be made both in the cyclic (the above formulae are addressed just to this case) and quasi-linear accelerators with a total energy gain several times higher compared with the energy of cooling; the accelerator has to include sections with bending magnetic field. In this case, the energy dispersion function in the target region needs to be chosen correctly in order to carry out the longitudinal cooling as well. The general estimates of the equilibrium emittances remain valid.

4. Let us briefly discuss, according mainly to Refs /4-6/, possible applications of ionization-cooled muon beams. To produce intense, completely pure and deeply-cooled muon beams, the following steps are necessary to make:

1) to produce a pion beam with as low emittance as possible at an energy E_c of about 1 GeV, by using the most intense proton beams with an energy of hundreds of GeV and higher and the nuclear cascade in the conversion target;

2) to let the pions decay in a strongly-focusing straight-line channel; this ensures the minimum increase of the transverse emittance;

3) to perform the ionization cooling of the produced muon beam;

4) to accelerate, to the necessary energy, the produced muon beam in a linear or cyclic accelerator with a rate of cooling several times higher, per unit length, than $\mu c/\tau_\mu$; this provides the smallness of intensity losses because of the muon decay.

These muon beams is possible to use either for a direct study of the interaction of muons with nucleons and nuclei or for obtaining, after the injections of muons into a special magnetic track, a generator of electron and muon neutrinos and antineutrinos up to the total energy with a very small angular spread. The latter can be made close to $\mu c^2/E_\mu$, where E_μ is the energy of accelerated muons. With E_μ = 1 TeV and a 300-m-thick shield, this will enable one to have the transverse sizes of the neutrino beam of the order of 3 cm. This circumstance will permit the setting up of neutrino experiments to be simplified to a considerable extent.

However, the most interesting possibility is to make the colliding muon beams experiments. If the aberrations do not increase, during acceleration, the emittance of muon beams gathered in two very short bunches with the number of particles $N_{\mu^+} = N_{\mu^-} = N_\mu$, then having injected them into the ring with a strong magnetic field H (to increase the number of collisions N_t during the lifetime of accelerated muons), one can obtain the luminosity

$$\mathcal{L} = \frac{N_\mu^2}{4\pi\Omega_\perp\beta_0\left(\frac{E_c}{E_\mu}\right)}\cdot N_t\cdot f = N_\mu^2\frac{E_\mu}{6\pi Z_i m_e c^2\beta_c\beta_0}\cdot\frac{eH\tau_\mu}{2\pi\mu c}$$

where β_c and β_0 are the values of the beta-functions in the region of ionization targets and in the interaction point, respectively; and f is the repetition frequency of injection cycles.

If we set N_μ = 10^{11}, E_μ = 1 TeV, H = 100 kG, f = 10 Hz and $\beta_c = \beta_0$ = 1 cm, we obtain the luminosity exceeding 10^{31} cm^{-2}s^{-1}. As shown in Ref. /6/, close parameters can be achieved with the help of proton klystrons, by utilizing the intense proton beams of the modern and future accelerators at ultimately high energies.

References

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