MURA-38

R. G. SACHS

On the Application of Very High Energy Machines

The following is a transcription of some informal remarks made at the request of the Midwestern Accelerator group. The desire of that group was to establish some basis for comparing the possible relative merits of a proton accelerator and an electron accelerator at very high energy. No attempt is made here to decide which type of machine is the more desirable. The answer to that question will depend on many circumstances beyond the special knowledge of the theorist, as well as on the special interests of the group responsible for the development of the machine. The remarks presented here are concerned with some of the possible uses to which such machines may be put, and with some guesses as to the performance of the machines when so used.

For the sake of definiteness, most of the discussion is centered around accelerators operating at an energy of 30 Bev or larger. In this region, there is available from cosmic ray experience a limited amount of information concerning the processes that can occur. Such experimental information has been used whenever possible. However, it has been necessary to extrapolate from low energies in many cases and to make use of out-and-out guesses in others. In particular, the estimates of cross sections given in Section 2 may be very much in error. On the other hand, any results based on purely kinematical considerations should be quite reliable.

The discussion is concerned with four more-or-less distinct subjects, namely 1) Kinematical considerations, 2) Estimates of cross sections, 3) The accelerator as a source, and 4) The nature of the information to be gained from experiments. Each of these is taken up in order.

1) Kinematical Considerations

Insofar as any reactions it may produce are concerned, the effective energy of the beam is the center of mass energy. Therefore, for a given type of particle of given laboratory energy, we must calculate the energy in the center of mass system. We consider the collision between an incident particle of mass M_1 and energy E_1 with a target particle of mass M_2 with energy E_2. Kinetic energies of the two particles are denoted by T_1 and T_2 respectively, and the momenta by $\vec{p}_1$ and $\vec{p}_2$. We work with natural units, c = 1. The total energy available in the center of mass system is given by

$$E' = \sqrt{M_1^2 + M_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2)} \qquad (1)$$

It can be seen from this equation that for energies much greater than the rest energy of the particles, the available total energy E' is proportional to the square root of the incident energy, so at very high energy, the gain with increasing size of a machine is rather slow. The CM energy is somewhat increased if the target particle has an appreciable momentum in a direction opposite to the beam direction. For example, in the collision of a very high energy particle with a nucleus, the nuclear particles will have an average kinetic energy around 25 Mev. The maximum energy available in such a collision will then be obtained when the nucleon is moving against the beam, and the increase in the available energy is of the order of the geometric mean of the energies of the two particles.

The available kinetic energy T' in the CM system is tabulated below, both for a photon-nucleon collision (photon beam) and for a nucleon-nucleon collision. The quantity T'^* is the maximum kinetic energy when the target particle (nucleon) has a kinetic energy (lab) of 25 Mev. The energies are given in terms of the rest energy M of the nucleon.

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It is of some interest to remark on the energy available in a collision between a photon and an electron or between two electrons. Then we find

$$E' \approx \sqrt{2mE}, \qquad (2)$$

where m is the electron mass. This result obtains for energies larger than m.

We see that it is very hard indeed to obtain a large effective energy in such a collision between light particles. In order to have E' = 1 Bev, we must have $E_1 = 10^3$ Bev. In this case, target motion will add appreciably to the energy only if the kinetic energy of the target electron is considerably greater than m.

On the basis of Eq. (1), it is possible to establish the thresholds for the production of various particles in a collision between two particles. In every case we will assume that the target is a nucleon. We give the threshold kinetic energy (T_1) when the target is at rest and the threshold kinetic energy (T_1^*) when the target nucleon is assumed to have 25 Mev kinetic energy. The binding energy of the nucleon in the nucleus is neglected. Results for two interesting cases, the production of nucleon pairs and of a K-meson are given in the following table:

produced particles / beam	nucleon pairs		K-meson	
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The determination of the threshold for a pion beam has been based on the assumption that the pion is absorbed in the process. When these figures are compared, it must be kept in mind that the shape of the excitation curve near threshold will be different for different processes. At threshold, one may expect the excitation curve to be proportional to $(E'-E'_0)^{\frac{3n-5}{2}}$ where n is the number of particles in the final state and E'_0 is the (CM) threshold energy. Therefore, it may be necessary to use an energy considerably above threshold to detect the many particle process, such as the nucleon-nucleon production of a nucleon pair ($n = 4$). If it is necessary to go as high as $E' \approx 2E'_0$ to obtain an appreciable cross section, then the laboratory energy is far greater than the estimate given in the table, $E_1 \approx 4E$ or $\frac{T_I}{M} \approx 27$. It will probably not be necessary to go to energies as high as twice threshold in the CM system to detect the process, but it must be kept in mind that, whatever the required factor, it is squared in going over into the laboratory system.

Another purely kinematical remark that may be of some interest concerns the angular spread of the products of a very high energy collision. If the angular distribution in the CM system is isotropic, the mean square angular spread is given by

$$\overline{\Theta^2} = 2\frac{M_2}{E_1}\left(\ln\frac{E_1}{2M_2} - 1\right) \qquad (3)$$

at high energy when the target is at rest. It is to be noticed that only the target mass enters this expression. The result shows that the products of a very high energy reaction will be closely correlated in angle so that there may be some difficulty in separating light from heavy particles.

2) Estimates of Cross Sections

We consider here the cross section for the production of certain species of particles by photons, electrons, or protons. It is assumed that the energy is well above threshold so that the excitation function is reasonably flat. The estimate is meant to include multiple production in all orders of the particular species under consideration. The results are presented in the following table in units of the geometrical cross section of the target nucleus,

$$\sigma_{geom} = 60\, A^{\frac{2}{3}}\ mb \qquad . \tag{4}$$

Species Produced	Photon Beam	Electron Beam	Proton Beam
photon	direct	$\frac{1}{10} Z^2 A^{-\frac{2}{3}}$	$\frac{1}{2}$
electron	———	direct	$\frac{1}{137} \times \frac{1}{2}$
pion	$\frac{1}{2} 10^{-3} A^{\frac{1}{3}}$	$\frac{1}{137} \times 10^{-3} A^{\frac{1}{3}}$	$\frac{1}{2}$
proton	$\frac{1}{2} \times 10^{-3} A^{\frac{1}{3}}$	$\frac{1}{137} \times \frac{1}{2} \times 10^{-3} A^{\frac{1}{3}}$	direct
neutron	$\frac{1}{2} \times 10^{-3} A^{\frac{1}{3}}$	$\frac{1}{137} \times \frac{1}{2} \times 10^{-3} A^{\frac{1}{3}}$	$\frac{1}{2}$
K meson	$\frac{1}{2} \times 10^{-4} A^{\frac{1}{3}}$	$\frac{1}{137} \times \frac{1}{2} \times 10^{-4} A^{\frac{1}{3}}$	$\frac{1}{20}$
hyperon	$\frac{1}{2} \times 10^{-4} A^{\frac{1}{3}}$	$\frac{1}{137} \times \frac{1}{2} \times 10^{-4} A^{\frac{1}{3}}$	$\frac{1}{20}$
antinucleon	$\frac{1}{4} \times 10^{-5} A^{\frac{1}{3}}$	$\frac{1}{137} \times \frac{1}{4} \times 10^{-5} A^{\frac{1}{3}}$	10^{-2}

These estimates have been obtained as follows:

For photon production by electrons, we give the high energy total bremstrahl cross section for a nucleus of charge Z and mass number A

The photoproduction cross section for pions has been estimated in two different ways. First, we extrapolated from 1 Bev on the assumption that the total cross section is roughly proportional to $\sqrt{\omega}$ at high energy where ω is the photon energy in the laboratory system. The $\sqrt{\omega}$ law is obtained as follows. We denote the CM photon energy by ω'. The photoelectric cross section for production of electrons from atoms is proportional to $1/\omega'$ in the extreme relativistic case. However, the pion current operator is proportional to the momentum, while the electron current is essentially independent of momentum. This means that the pion cross section should be ω'^2 times the electron cross section at high energy. In other words, it should be proportional to ω'. However, since the photon energy in the CM system is proportional to the square root of the laboratory energy, we obtain a cross section proportional to $\sqrt{\omega}$.

At 500 Mev, the cross section for photo production of a single pion from hydrogen is decreasing faster than ω^{-2}, and has a value of about 0.1 mb (CIT work reported at Rochester, 1954). We have assumed that this may be extrapolated to the value of 0.01 mb at 1 Bev, and from there we assume that the $\sqrt{\omega}$ law sets in. Extrapolation to 30 Bev gives 0.06 mb = $10^{-3}\sigma_{\text{geom.}}$

Compton wave length of the K meson to that of the pion. Thus, the cross section should be smaller in the ratio of the square of the masses, or about $\frac{1}{10}$ of the pion cross section.

The production of hyperons is assumed to be correlated directly with K meson production, so that its cross section is the same as the K meson cross section.

To estimate the cross section for photoproduction of antinucleons, two methods have again been used. First, we might guess that the cross section is $(\frac{1}{1840})^2$ times smaller than the cross section for electron pair production. However, in the electron case, the recoil momentum is taken up by the coulomb interaction with the nucleus, while the nucleon pair is subject to a direct nuclear interaction. This can be expected to introduce a factor 137 in the cross section of hydrogen if the specific nuclear coupling constant is of the order of unity. This leads to a cross section of 2×10^{-4} mb for nucleon pair production by photons incident on a nucleon.

An alternative method for estimating the cross section is to assume that the process occurs through the absorption of photopions. In other words, the process is similar to the photonucleon process. In this case, we assume that virtual pairs occupy a region of the size of a nucleon Compton wave length in the nucleon proper field. Then the cross section should be about $\frac{1}{50}$ of the photonucleon cross section. This leads to a value of 10^{-4} mb in very good agreement with our previous estimate.

The estimates of production of nuclear particles by the electron beam are simply based on the assumption that the process occurs through virtual photon emission. Hence, each cross section should be about $\frac{1}{137}$ of the cross section for production of the same particle by photons.

All of the cross section for processes due to the proton beam are based on the observation that cross sections for nuclear processes at high energy are of the order of $\frac{1}{2}$ the geometrical cross section. In particular, the pion production cross section is taken to be just of this order.

The production of photons will occur through the decay of π°'s, so this cross section should be of the same order as the pion production cross section.

The production of neutrons may again be taken to be of the order of $\frac{1}{2}\sigma_{geom}$.

The estimates of K meson and hyperon production have been based on the same argument used before, that they will be smaller than the pion cross section by a factor equal the square of the mass ratio.

The production of antinucleons also has been assumed to be smaller than the pion cross section by the square of the ratio of the pion to the nucleon mass, since this would be expected to be the geometrical factor to be taken into account.

3) The Accelerator as a Source

In this section we consider the various possible types of beam which may be produced either directly or indirectly by the accelerator for the purpose of performing experiments. We may hope to extract the primary beam and use it directly, or we may insert a target in the accelerator and produce a beam of secondary particles. These may be photons, pions, K mesons, nucleons, antinucleons, or hyperons. The fundamental experiments would then be performed with the secondary beam. Our guess as to the relative intensity of a given type of secondary beam will be based on the cross section estimates of Section 2. This requires that the energy in the primary beam be far above the threshold for production of each type of particle.

Our comparison of electron and proton accelerators will be made on the basis of equal primary currents for the two machines.

We consider first the use of electron accelerators. It may be possible to extract the electron beam and use it directly, but it is by no means certain (see Section 4) that this would have any advantage over the use of a secondary photon beam produced at a target inside the accelerator. The production of bremstrahl is very efficient, so the photon beam may be produced with high flux. However, the photon energy distribution is expected to have the usual ω^{-1} dependence on energy. Therefore, the really high energy part of the photon beam will constitute a small fraction, possiblely $\frac{1}{10}$ of the total. Since electrons are less efficient by a factor of $1/37$ than photons for the production of particles, it will be assumed that the bremstrahl beam is used to produce the desired (tertiary) beam of particles. That is, we assume that the bremstrahl beam is intercepted by a target which acts as a radiator of pions, K mesons, etc. In this case, the cross sections in the first column of the table in Section 2 are applicable.

Because the cross sections are all small in magnitude, it is expected that a rather thick radiator must be used to obtain efficient conversion. It also means that the production of cascade showers will occur much more readily than will the production of the desired beam. Thus, the beam will be thoroughly contaminated by low energy electrons, photons, photonucleons, and photopions. Some reduction in the contamination can be attained by using a thick source. This procedure will have the obvious disadvantage that the source of the experimental beam will be large, so that geometrical conditions involved in the analysis of the beam will be poor.

It is clear that the proton accelerator will be considerably less efficient as a source of photons or electrons. However, it is to be kept in mind that the photon intensity (due to π°'s) will not be negligible, and it may be high enough to make photo experiments possible. The protons are much more efficient for the production of nuclear particles than are photons. Furthermore, the contamination problem will not be nearly as serious in this case. Therefore, thinner radiators may be used, and the geometrical conditions are thereby expected to be much better.

The relative efficiency for production of high energy nuclear particles of different kinds will be about the same for the electron and the proton accelerator according to our considerations of Section 2, and may be estimated from the cross section table.

The only exception would occur for proton production if the proton beam can be extracted and used directly. The use of the cross section table is possible only when the primary energy is well above threshold, and even then there may be large errors in the estimates.

The energy spectrum of the secondary or tertiary beam produced by the accelerator is expected to be continuous. At very high energy, multiple production processes will occur with high frequency. This will accentuate the low energy end of the spectrum of the particles being produced. Therefore, one expects a rather low efficiency for the production of a secondary beam of high energy with a strong increase in intensity toward the low energy end.

The other method for estimating this cross section is based on the results of E. P. George for the cross section for production of pions by very high energy muons. For an average muon energy of 14 Bev, he finds a cross section in emulsion of 6 x 10^{-3} mb which is about $\frac{1}{2}$ x $10^{-5}\sigma$ geom. Presumably, this cross section is due to electromagnetic effects. We can expect that the photon cross section would be about 137 times greater. This leads to a figure of the same order as the extrapolated cross section obtained by the first method.

For photon energies below 500 Mev, the nuclear cross section appears to be geometrical, and this is taken to mean that the photopions are absorbed within the nucleus with a consequent emission of nucleons. At much higher energies, we may expect that the pions will produce pions as well as nucleons, and that about half of the emitted particles are pions while the other half are nucleons. In this case, the cross section for production of pions of all energies may be about A/2 times the cross section of hydrogen, and the photonucleon cross section would be about the same. Those are the estimates given in the table. Under these assumptions, the photoproduction of very high energy pions will have a cross section somewhat smaller in magnitude, and it would be expected to be proportional to $A^{2/3}$.

The cross section for photoproduction of K mesons has been obtained by assuming that the mechanism is about the same as that for pion and nucleon production, but that the size of the K meson cloud in the nucleon is smaller in the ratio of the

4) The Nature and Interpretation of Experiments

It may be worthwhile to make a few remarks concerning the possibility of investigating the limits of our knowledge of quantum electrodynamics of the electron-positron system by means of very high energy collision. Deviations from the present theories might be expected to show up in a collision between light particles with a relative (CM) wave length somewhat smaller than the classical radius of the electron. This would require a CM energy around 300 Mev[*] or larger. Since we would be dealing with a collision between light particles, the corresponding laboratory energy would be larger than 100 Bev. Machines designed to operate at so high an energy are not being contemplated at the present time, so we need give this problem no immediate consideration. However, it may be worthwhile to call attention to the fact that accurate quantitative measurements would be required for this purpose, and that these would be especially difficult in the presence of the cascade showers that would be produced at such energies.

In considering the application of a high energy machine to the physics of heavy particles (that is, heavier than electrons) we may divide the problem into two parts: A. Questions of particle physics which are essentially qualitative, and B. Quantitative measurements of cross sections. Each of these is described in turn.

[*] The momentum in the CM system is $\frac{1}{2}E'$ for the extreme relativistic case and we have taken $\bar{\lambda}$ to be $\frac{1}{2}$ the classical electron radius.

A. Particle Physics

The purpose of experiments of this type is to produce and identify various particles, to measure branching ratios and multiplicities and to establish correlations between the various species. An external beam is usually preferred for the work because it allows a greater control of the energy and easier reduction of the contamination of the beam. There are, of course, certain emulsion experiments that can be done internally. Except for these special experiments, we may assume that the work is to be done externally. Then it will either be necessary to extract the primary beam from the accelerator or to make use of a secondary beam to investigate the particle production. In the latter case, the bremstrahl beam from an electron accelerator would be used, while in the proton accelerator it would probably be best to use a secondary pion beam.

Our concern here will be the relative efficiencies of the two types of machine for the production of the particles to be studied. The comparison will be made for electron and proton accelerators operating at the same primary current.

We consider the production of K mesons, hyperons, and antinucleons. According to our table of cross sections, the direct proton beam may be expected to produce about 10^3 times as many of these particles as would a photon beam having the same flux. The photon flux capable of producing the desired particles will probably be comparable to the electron flux if the maximum energy is quite large (say $>$ 30 Bev) compared to threshold.

At lower maximum energy, the intense low energy component of the photon beam will, of course, be useless. When the secondary beam is to be used, it is necessary to take into account the relative efficiency for the production of pions by protons compared to the production of photons by electrons, which is about $\frac{1}{40}$ for a Pb radiation. Therefore, the use of the pion beam for a proton accelerator would require a considerably larger radiator (to obtain comparable flux) with a correspondingly much poorer geometry. The effective pion flux might be estimated at 10^{-1} to 10^{-2} the photon flux so the advantage of the proton accelerator for particle production would be reduced to a factor 100 or even to 10. It has been assumed here that a pion has about the same cross section for K meson or antinucleon production as has the nucleon.

It is again important to notice that there will be associated with the production of the interesting particles a large number of contaminants if we use the photon or electron beam. The number of photons and electrons will be enormous, and the photoproduction of pions will be some ten times more efficient than the production of K mesons. Furthermore, the threshold for K meson production is higher, so the number of low energy pions will be even greater in proportion to the number of K mesons.

B. Cross Section Measurements

In order to judge the worth of extracting the electron beam to use it externally, it is of some interest to compare the relative merits of the electron and photon beams for direct use in quantitative measurements. We expect that electron cross sections will be smaller than the photon cross sections by a factor of $\frac{1}{137}$. The electron energy is unique, while the photon spectrum is continuous. On the other hand, the photon is absorbed in the production process, while the electron is not. This means that the kinematics of the photon experiment are much simpler than those of the electron experiment. For example, in the photoproduction of a pion from a nucleon, the momentum of each of the three particles can be determined by measuring the energy and the direction of the pion, while in the corresponding electron induced process, four quantities must be measured even if the initial electron energy is given.

It may be somewhat more difficult to make absolute cross section measurements with the photon beam because of calibration difficulties. However, the theoretical interpretation of the photon experiment will usually be much simpler because it involves the electromagnetic interaction directly. Presumably, the electron interacts by means of virtual photons, so that the electromagnetic interaction enters indirectly in the form of an integral over the virtual photon spectrum. In order to obtain the proper weight factors occurring in this integral, it may be necessary to assume a rather specific model of the system being investigated. In that case, the interpretation of the experiment will be difficult and most indirect.

If we consider now the information to be gained from studies of photoprocesses (either photons or electrons), we find that their great advantage over nuclear processes lies in the fact that they involve a weak interaction which therefore causes a minimal disturbance of the system being probed. Of course, for the same reason the cross sections are so small that it will be difficult to measure them accurately. Nevertheless, it is an advantage from the point of view of theoretical interpretation to deal with an undisturbed system.

Another advantage of the electromagnetic process is that we have some _a priori_ knowledge of the electromagnetic interaction with any system. To the extent that this interaction is known, the cross sections for photoprocesses provide information concerning the structure of the system, or more precisely, concerning the structure of the wave function. Unfortunately, the electromagnetic interaction is not known completely because it is influenced by the strong interaction between the heavy particles. According to our present understanding of heavy particle interactions, we may expect an electric current (interaction current) to flow between the interacting heavy particles. This current can only be deduced from a complete theory of heavy particle interactions. Therefore, the interpretation of photoprocesses will be ambiguous until more detailed knowledge (concerning the interaction currents) is available. The experiments would probably be used as a source of information concerning interaction currents and thereby as an indirect means for studying the heavy particle interactions. It will actually be a difficult undertaking to untangle the effects of the known (free particle) currents and the interaction currents.

Experiments performed directly with a heavy particle beam will certainly give the most direct information concerning heavy particle interactions. The interpretation will be somewhat dependent upon questions of structure, although any analysis in terms of structure will be less direct than in the case of the photoprocesses since the system is profoundly disturbed by the strong interaction. The analysis will also be confused by multiple production of heavy particles. Of course, a study of the multiple production processes may in itself shed an entirely new light on the nature of the interaction.

This report was prepared with the aid of support from the AEC.

R. G. Sachs

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It can be seen from this equation that for energies much greater than the rest energy of the particles, the available total energy E' is proportional to the square root of the incident energy, so at very high energy, the gain with increasing size of a machine is rather slow. The CM energy is somewhat increased if the target particle has an appreciable momentum in a direction opposite to the beam direction. For example, in the collision of a very high energy particle with a nucleus, the nuclear particles will have an average kinetic energy around 25 Mev. The maximum energy available in such a collision will then be obtained when the nucleon is moving against the beam, and the increase in the available energy is of the order of the geometric mean of the energies of the two particles.

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at high energy when the target is at rest. It is to be noticed that only the target mass enters this expression. The result shows that the products of a very high energy reaction will be closely correlated in angle so that there may be some difficulty in separating light from heavy particles.

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photon	direct	$\frac{1}{10}\ Z^2\ A^{-2/3}$	$\frac{1}{2}$
electron	———	direct	$\frac{1}{137} \times \frac{1}{2}$
pion	$\frac{1}{2}\ 10{-}3\,A^{1/3}$	$\frac{1}{137} \times 10^{-3}A^{1/3}$	$\frac{1}{2}$
proton	$\frac{1}{2} \times 10^{-3}A^{1/3}$	$\frac{1}{137} \times \frac{1}{2} \times 10^{-3}A^{1/3}$	direct
neutron	$\frac{1}{2} \times 10^{-3}A^{1/3}$	$\frac{1}{137} \times \frac{1}{2} \times 10^{-3}A^{1/3}$	$\frac{1}{2}$
K meson	$\frac{1}{2} \times 10^{-4}A^{1/3}$	$\frac{1}{137} \times \frac{1}{2} \times 10^{-4}A^{1/3}$	$\frac{1}{20}$
hyperon	$\frac{1}{2} \times 10^{-4}A^{1/3}$	$\frac{1}{137} \times \frac{1}{2} \times 10^{-4}A^{1/3}$	$\frac{1}{20}$
antinucleon	$\frac{1}{4} \times 10^{-5}A^{1/3}$	$\frac{1}{137} \times \frac{1}{4} \times 10^{-5}A^{1/3}$	10^{-2}

These estimates have been obtained as follows:

For photon production by electrons, we give the high energy total bremstrahl cross section for a nucleus of charge Z and mass number A.

The photoproduction cross section for pions has been estimated in two different ways. First, we extrapolated from 1 Bev on the assumption that the total cross section is roughly proportional to $\sqrt{\omega}$ at high energy where ω is the photon energy in the laboratory system. The $\sqrt{\omega}$ law is obtained as follows. We denote the CM photon energy by ω'. The photoelectric cross section for production of electrons from atoms is proportional to $1/\omega'$ in the extreme relativistic case. However, the pion current operator is proportional to the momentum, while the electron current is essentially independent of momentum. This means that the pion cross section should be ω'^2 times the electron cross section at high energy. In other words, it should be proportional to ω'. However, since the photon energy in the CM system is proportional to the square root of the laboratory energy, we obtain a cross section proportional to $\sqrt{\omega}$.

At 500 Mev, the cross section for photo production of a single pion from hydrogen is decreasing faster than ω^{-2}, and has a value of about 0.1 mb (CIT work reported at Rochester, 1954). We have assumed that this way this may be extrapolated to the value of 0.01 mb at 1 Bev, and from there we assume that the $\sqrt{\omega}$ law sets in. Extrapolation to 30 Bev gives 0.06 mb = $10^{-3}\sigma_{\text{geom.}}$

The other method for estimating this cross section is based on the results of E. P. George for the cross section for production of pions by very high energy muons. For an average muon energy of 14 Bev, he finds a cross section in emulsion of 6 x 10^{-3} mb which is about $\frac{1}{2}$ x $10^{-5}\sigma_{\text{geom.}}$ Presumably, this cross section is due to electromagnetic effects. We can expect that the photon cross section would be about 137 times greater. This leads to a figure of the same order as the extrapolated cross section obtained by the first method.

For photon energies below 500 Mev, the nuclear cross section appears to be geometrical, and this is taken to mean that the photopions are absorbed within the nucleus with a consequent emission of nucleons. At much higher energies, we may expect that the pions will produce pions as well as nucleons, and that about half of the emitted particles are pions while the other half are nucleons. In this case, the cross section for production of pions of all energies may be about A/2 times the cross section of hydrogen, and the photonucleon cross section would be about the same. Those are the estimates given in the table. Under these assumptions, the photoproduction of very high energy pions will have a cross section somewhat smaller in magnitude, and it would be expected to be proportional to $A^{2/3}$.

The cross section for photoproduction of K mesons has been obtained by assuming that the mechanism is about the same as that for pion and nucleon production, but that the size of the K meson cloud in the nucleon is smaller in the ratio of the Compton wave length of the K meson to that of the pion. Thus, the cross section should be smaller in the ratio of the square of the masses, or about 1/10 of the pion cross section.

The production of hyperons is assumed to be correlated directly with K meson production, so that its cross section is the same as the K meson cross section.

To estimate the cross section for photoproduction of antinucleons, two methods have again been used. First, we might guess that the cross section is $(\frac{1}{1840})^2$ times smaller than the cross section for electron pair production. However, in the electron case, the recoil momentum is taken up by the coulomb interaction with the nucleus, while the nucleon pair is subject to a direct nuclear interaction. This can be expected to introduce a factor 137 in the cross section of hydrogen if the specific nuclear coupling constant is of the order of unity. This leads to a cross section of 2×10^{-4} mb for nucleon pair production by photons incident on a nucleon.

An alternative method for estimating the cross section is to assume that the process occurs through the absorption of photopions. In other words, the process is similar to the photonucleon process. In this case, we assume that virtual pairs occupy a region of the size of a nucleon Compton wave length in the nucleon proper field. Then the cross section should be about $\frac{1}{50}$ of the photonucleon cross section. This leads to a value of 10^{-4} mb in very good agreement with out previous estimate.

The estimates of production of nuclear particles by the electron beam are simply based on the assumption that the process occurs through virtual photon emission. Hence, each cross section should be about 1/137 of the cross section for production of the same particle by photons.

All of the cross section for processes due to the proton beam are based on the observation that cross sections for nuclear processes at high energy are of the order of 1/2 the geometrical cross section. In particular, the pion production cross section is taken to be just of this order.

The production of photons will occur through the decay of π o's, so this cross section should be of the same order as the pion production cross section.

The production of neutrons may again be taken to be of the order of $\frac{1}{2}$ σ geom.

The estimates of K meson and hyperon production have been based on the same argument used before, that they will be smaller than the pion cross section by a factor equal the square of the mass ratio.

The production of antinucleons also has been assumed to be smaller than the pion cross section by the square of the ratio of the pion to the nucleon mass, since this would be expected to be the geometrical factor to be taken into account.

3) The Accelerator as a Source

In this section we consider the various possible types of beam which may be produced either directly or indirectly by the accelerator for the purpose of performing experiments. We may hope to extract the primary beam and use it directly, or we may insert a target in the accelerator and produce a beam of secondary particles. These may be photons, pions, K mesons, nucleons, antinucleons, or hyperons. The fundamental experiments would then be performed with the secondary beam. Our guess as to the relative intensity of a given type of secondary beam will be based on the cross section estimates of Section 2. This requires that the energy in the primary beam be far above the threshold for production of each type of particle.

Our comparison of electron and proton accelerators will be made on the basis of equal primary currents for the two machines.

We consider first the use of electron accelerators, It may be possible to extract the electron beam and use it directly, but it is by no means certain (see Section 4) that this would have any advantage over the use of a secondary photon beam produced at a target inside the accelerator. The production of bremstrahl is very efficient, so the photon beam may be produced with high flux. However, the photon energy distribution is expected to have the usual ω^{-1} dependence on energy. Therefore, the really high energy part of the photon beam will constitute a small fraction, possible 1/10 of the total. Since electrons are less efficient by a factor of 1/137 than photons for the production of particles, it will be assumed that the bremstrahl beam is used to produce the desired (tertiary) beam of particles. That is, we assume that the bremstrahl beam is intercepted by a target which acts as a radiator of pions, K mesons, etc. In this case, the cross sections in the first column of the table in Section 2 are applicable.

Because the cross sections are all small in magnitude, it is expected that a rather thick radiator must be used to obtain efficient conversion. It also means that the production of cascade showers will occur much more readily than will the production of the desired beam. Thus, the beam will be thoroughly contaminated by low energy electrons, photons, photonucleons, and photopions. Some reduction in the contamination can be attained by using a thick source. This procedure will have the obvious disadvantage that the source of the experimental beam will be large, so that geometrical conditions involved in the analysis of the beam will be poor.

It is clear that the proton accelerator will be considerably less efficient as a source of photons or electrons. However, it is to be kept in mind that the photon intensity (due to π^o's) will not be negligible, and it may be high enough to make photo experiments possible. The photons are much more efficient for the production of nuclear particles than are photons. Furthermore, the contamination problem will not be nearly as serious in this case. Therefore, thinner radiators may be used, and the geometrical conditions are thereby expected to be much better.

The relative efficiency for production of high energy nuclear particles of different kinds will be about the same for the electron and the proton accelerator according to our considerations of Section 2, and may be estimated from the cross section table.

The only exception would occur for proton production if the proton beam can be extracted and used directly. The use of the cross section table is possible only when the primary energy is well above threshold, and even then there may be large errors in the estimates.

The energy spectrum of the secondary or tertiary beam produced by the accelerator is expected to be continuous. At very high energy, multiple production processes will occur with high frequency. This will accentuate the low energy end of the spectrum of the particles being produced. Therefore, one expects a rather low efficiency for the production of a secondary beam of high energy with a strong increase in intensity toward the low energy end.

4) The Nature and Interpretation of Experiments

It may be worthwhile to make a few remarks concerning the possibility of investigating the limits of our knowledge of quantum electrodynamics of the electron-positron system by means of very high energy collision. Deviations from the present theories might be expected to show up in a collision between light particles with a relative (CM) wave length somewhat smaller than the classical radius of the electron. This would require a CM energy around 300 Mev* or larger. Since we would be dealing with a collision between light particles, the corresponding laboratory energy would be larger than 100 Bev. Machines designed to operate at so high an energy are not being contemplated at the present time, so we need give this problem no immediate consideration. However, it may be worthwhile to call attention to the fact that accurate quantitative measurements would be required for this purpose, and that these would be especially difficult in the presence of the cascade showers that would be produced at such energies.

In considering the application of a high energy-machine to the physics of heavy particles (that is, heavier than electrons) we may divide the problem into two parts: A. Questions of particle physics which are essentially qualitative, and B. Quantitative measurements of cross sections. Each of these is described in turn.

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* The momentum in the CM system is $\frac{1}{2}E'$ for the extreme relativistic case and we have taken $\bar{\lambda}$ to be $\frac{1}{2}$ the classical electron radius.

A. Particle Physics

The purpose of experiments of this type is to produce and identify various particles, to measure branching ratios and multiplicities and to establish correlations between the various species. An external beam is usually preferred for the work because it allows a greater control of the energy and easier reduction of the contamination of the beam. There are, of course, certain emulsion experiments that can be done internally. Except for these special experiments, we may assume that the work is to be done externally. Then it will either be necessary to extract the primary beam from the accelerator or to make use of a secondary beam to investigate the particle production. In the latter case, the bremstrahl beam from an electron accelerator would be used, while in the proton accelerator it would probably be best to use a secondary pion beam.

Our concern here will be the relative efficiencies of the two types of machine for the production of the particles to be studied. The comparison will be made for electron and proton accelerators operating at the same primary current.

We consider the production of K mesons, hyperons, and antinucleons. According to our table of cross sections, the direct proton beam may be expected to produce about 10^3 times as many of these particles as would a photon beam having the same flux. The photon flux capable of producing the desired particles will probably be comparable to the electron flux if the maximum energy is quite large (say ≥ 30 Bev) compared to threshold. At lower maximum energy, the intense low energy component of the photon beam will, of course, be useless. When the secondary beam is to be used, it is necessary to take into account the relative efficiency for the production of pions by protons compared to the production of photons by electrons, which is about $\frac{1}{40}$ for a Pb radiation. Therefore, the use of the pion beam for a proton accelerator would require a considerably larger radiator (to obtain comparable flux) with a correspondingly much poorer geometry. The effective pion flux might be estimated at 10^{-1} to 10^{-2} the photon flux so the advantage of the proton accelerator for particle production would be reduced to a factor 100 or even to 10. It has been assumed here that a pion has about the same cross section for K meson or antinucleon production as has the nucleon.

It is again important to notice that there will be associated with the production of the interesting particles a large number of contaminants if we use the photon or electron beam. The number of photons and electrons will be enormous, and the photoproduction of pions will be some ten times more efficient than the production of K mesons. Furthermore, the threshold for K meson production is higher, so the number of low energy pions will be even greater in proportion to the number of K mesons.

B. Cross Section Measurements

In order to judge the worth of extracting the electron beam to use it externally, it is of some interest to compare the relative merits of the electron and photon beams for direct use in quantitative measurements. We expect that electron cross sections will be smaller than the photon cross sections by a factor of $\frac{1}{137}$. The electron energy is unique, while the photon spectrum is continuous. On the other hand, the photon is absorbed in the production process, while the electron is not. This means that the kinematics of the photon experiment are much simpler than those of the electron experiment. For example, in the photoproduction of a pion from a nucleon, the momentum of each of the three particles can be determined by measuring the energy and the direction of the pion, while in the corresponding electron induced process, four quantities must be measured even if the initial electron energy is given.

It may be somewhat more difficult to make absolute cross section measurements with the photon beam because of calibration difficulties. However, the theoretical interpretation of the photon experiment will usually be much simpler because it involves the electromagnetic interaction directly. Presumably, the electron interacts by means of virtual photons, so that the electromagnetic interaction enters indirectly in the form of an integral over the virtual photon spectrum. In order to obtain the proper weight factors occurring in this integral, it may be necessary to assume a rather specific model of the system being investigated. In that case, the interpretation of the experiment will be difficult and most indirect.

If we consider now the information to be gained from studies of photoprocesses (either photons or electrons), we find that their great advantage over nuclear processes lies in the fact that they involve a weak interaction which therefore causes a minimal disturbance of the system being probed. Of course, for the same reason the cross sections are so small that it will be difficult to measure them accurately. Nevertheless, it is an advantage from the point of view of theoretical interpretation to deal with an undisturbed system.

Another advantage of the electromagnetic process is that we have some *a priori* knowledge of the electromagnetic interaction with any system. To the extent that this interaction is known, the cross sections for photoprocesses provide information concerning the structure of the system, or more precisely, concerning the structure of the wave function. Unfortunately, the electromagnetic interaction is not known completely because it is influenced by the strong interaction between the heavy particles. According to our present understanding of heavy particle interactions, we may expect an electric current (interaction current) to flow between the interacting heavy particles. This current can only be deduced from a complete theory of heavy particle interactions. Therefore, the interpretation of photoprocesses will be ambiguous until more detailed knowledge (concerning the interaction currents) is available. The experiments would probably be used as a source of information concerning interaction currents and thereby as an indirect means for studying the heavy particle interactions. It will actually be a difficult undertaking to untangle the effects of the known (free particle) currents and the interaction current.

Experiments performed directly with a heavy particle beam will certainly give the most direct information concerning heavy particle interactions. The interpretation will be somewhat dependent upon questions of structure, although any analysis in terms of structure will be less direct than in the case of the photoprocesses since the system is profoundly disturbed by the strong interaction. The analysis will also be confused by multiple production of heavy particles. Of course, a study of the multiple production processes may in itself shed an entirely new light on the nature of the interaction.

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