Methods of cooling beams of charged particles

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Fiz. Elem. Chastits At. Yadra **12**, 557–613 (May–June 1981)

The main methods of cooling beams of charged particles (radiation, ionization, electron, and stochastic) in storage rings and accelerators are reviewed. The possibilities offered by these methods in experiments on elementary-particle and nuclear physics are discussed.

PACS numbers: 29.20.Dh, 29.25.Fb

INTRODUCTION

In elementary-particle and nuclear physics, the basic experimental investigations involve the use of beams of accelerated charged particles. In almost all cases, it is important for these beams to be monochromatic and well collimated. This requires that in the comoving system, moving with the mean velocity of the beam particles, the particles must have low velocities, i.e., the beam must have a low temperature. It is therefore important to be able to "cool" beams of charged particles. By this we do not mean adiabatic cooling associated with "spreading of the beam," i.e., an increase in its size, but a decrease in the six-dimensional phase space occupied by the beam in the space of its generalized coordinates and conjugate momenta; it is necessary to increase the phase density of the beam. Besides this spatial, or *trajectory*, cooling, spin cooling, which requires a separate discussion, is becoming increasingly important.

Cooling of beams of charged particles permits one not only to compress and monochromatize the beams but also to accumulate particles by adding new particles in the cleared regions of phase space. This is particularly important when one wishes to obtain intense beams of secondary particles, particularly positrons and antiprotons, and is of fundamental importance in experiments with colliding beams.

In addition, cooling makes it possible to suppress various "warming" diffusion processes such as multiple scattering on residual gas in colliding-beam experiments and on the material of the target in experiments using internal targets in storage rings; one can also significantly weaken the quasistochastic part of the collision effects associated with the essentially nonlinear influence of the coherent field of colliding bunches on the particle motion. Moreover, it is not only spreading of the beam in storage rings that is prevented—the beam lifetime is also lengthened considerably.

An increase in the beam phase density cannot be achieved by any given (i.e., independent of the motion of the individual beam particles) external electromagnetic fields. For in this case (a special case of Liouville's theorem) the phase density of the beam particles must be a constant, determined by the "initial conditions." Focusing and acceleration in any combination can only change the shape of the phase space of the beam but cannot change its value. Moreover, any aberration can strongly distort the shape of the phase space, making it very complicated and thereby lowering the effective phase density. To *increase* the density, it is necessary to introduce forces of a dissipative nature.

Various diffusion processes, which are partly associated with the actual principle of the particular cooling method and partly have an "external" nature, generate a certain beam heating power. The balance between the heating and cooling powers determines the steady temperatures and sizes of the beams.

Four essentially different cooling methods have so far been recognized. At present, the most fruitful and best mastered method is radiation cooling,[1-3] which comes about because of the fact that when relativistic charged particles move along curved trajectories they emit synchrotron radiation and are damped by the corresponding radiative reaction force, which has a direction opposite to the total velocity of the particle. If the mean energy losses of equilibrium particles are compensated by an external energy source, and the magnetic structure of the storage ring is correctly chosen, the deviations from the equilibrium motion are steadily damped. Such cooling is most widely used in storing electrons and positrons and in experiments with colliding electron–positron beams.

Another method, also associated with deceleration of the general motion of the particles and compensation of the energy loss of the equilibrium particles, is *ionization cooling*.[4,5] As is clear from the name, the energy losses are introduced by placing fairly dense targets in the path of the beam. This method has not hitherto been developed experimentally or used, though, as we shall see, it opens up very interesting prospects. Ionization cooling involves a transfer of the energy of the general motion of the beam particles to the electrons in the fixed target. If instead of target electrons one uses an electron beam moving with the mean velocity of the particles which are to be cooled, the efficiency with which the energy associated with the deviations from equilibrium motion is transferred to the electrons is sharply increased, and although the density of electron beams is many orders of magnitude lower than the density of ordinary targets and corresponds to the density of the residual gas in a high vacuum, the time of such *electron cooling* can be very short. Proposed by Budker,[6] the method of electron cooling has been well developed theoretically, investigated, and mastered experimentally.[7-40] Its use in elementary-particle and nuclear physics makes possible numerous essentially new and very interesting experiments.[63-68]

Entirely different in its physical principle is the method of *stochastic cooling*, proposed by van der Meer. In this method, information about the deviation of individual particles from the equilibrium motion is obtained from their electromagnetic field by means of pick-up electrodes; the electric signal, amplified electronically, is sent to special plates or magnets, which partly compensate the deviation responsible for the signal. The presence of the remaining beam particles and the unavoidable noise in the electronic system limits the attainable gain and, accordingly, the cooling rate, and also sets a limit to the attainable temperatures.

1. RADIATION COOLING

The synchrotron radiation emitted when a relativistic charged particle moves along a curvilinear trajectory is associated with a damping electric field at such a particle, this field being directed along the velocity of the particle and localized in a region with characteristic longitudinal (with respect to the velocity) dimension of the order of the characteristic wavelength of the radiation. The damping force can be estimated from the equality of the damping power and the power carried away by the radiation, the basic dependences of which are well known. The synchrotron radiation power P_{SR} for a particle moving along a section of the trajectory with radius of curvature R in a magnetic field H perpendicular to the velocity is

$$P_{SR}=\frac{2}{3}\frac{e^2c}{R^2}\gamma^4; \quad (1)$$

$$P_{SR}=\frac{2}{3}\frac{e^4}{m^2c^3}\gamma^2H^2=\frac{2}{3}r_0^2c\gamma^2H^2; \quad (2)$$

$$P_{SR}=\frac{2}{3}\frac{e^4}{m^4c^7}E^2H^2, \quad (3)$$

where e is the charge of the particle, m is its mass, c is the velocity of light; r_0 is the classical electromagnetic radius of the particle, $\gamma=E/mc^2$ is the relativistic factor, and E is the energy of the particle. Whenever possible, we shall assume in all the equations that the particle velocity is equal to the velocity of light. The synchrotron radiation is concentrated in a narrow cone around the instantaneous direction of the particle's velocity with an opening angle of order $1/\gamma$.

Equations (1)–(3) hold for the case of a magnetic field constant along the trajectory. However, Eq. (2) also remains valid in an arbitrary case after averaging $\langle H^2\rangle$, and only the radiation spectrum is changed. For example, if there is a periodic alternation of the sign of the magnetic field along the trajectory with a spatial period L_0, we still have a quasiwhite "synchrotron" spectrum with characteristic wavelength

$$\lambda_{SR}\approx\frac{R}{\gamma^3}=\frac{mc^2}{e}\frac{1}{H\gamma^2} \quad (4)$$

provided L_0 is much greater than the formation length $R/\gamma=mc^2/(eH)$ of the radiation; in the opposite limiting case, when $L_0\ll mc^2/(eH)$, the radiation spectrum is at shorter wavelengths, being shifted to the region $\lambda=L_0/2\gamma^2$ and simultaneously becoming relatively narrower.

In the majority of cases of practical interest, there are many other particles in the characteristic region, of dimension R/γ^3, in which the damping electric field of the radiation reaction acts. Thus, each particle is subject to not only the reaction field of its own radiation but also the reaction fields produced by neighboring particles. However, if there are no microscopic correlations between the positions of the particles, then the damping force averaged over the time (or the ensemble) is determined solely by the self-radiation (incoherent) of the particle. Indeed, the instantaneous radiation power of the ensemble of particles is proportional to $\oint(\Sigma E_i)^2dS$, where the integral is taken over a surface surrounding the radiating system in the wave (far) zone. Averaging this power and bearing in mind that the time-average intensity $\langle E_i\rangle$ in the wave zone is necessarily zero, and $\langle E_iE_k\rangle=\langle E_i\rangle\langle E_k\rangle$ for $i\neq k$ in the case of completely random relative positions of two particles i and k, we immediately find that the total power in this case is, on the average, simply the sum of the radiation powers of the individual particles. Therefore, on the average, each particle is damped by an amount determined solely by its own radiation.

If the particles are grouped in bunches of length L_b, radiation with wavelengths $\lambda\geq L_b$ becomes coherent, and the damping field associated with this part of the radiation that acts on each particle is multiplied by the number of particles. However, in a first approximation this field does not affect the relative motion of the particles and does not contribute to the cooling.

As was shown earlier, the damping force of the radiation reaction, which is in the direction opposite to the instantaneous velocity of the particle, is

$$|\mathbf{F}_R|=P_{SR}/c; \quad (5)$$

$$|\mathbf{F}_R|=\frac{2}{3}\frac{e^2}{R^2}\gamma^4; \quad (6)$$

$$|\mathbf{F}_R|=\frac{2}{3}r_0^2\gamma^2H^2. \quad (7)$$

If the energy losses of an equilibrium particle are compensated (on the average at least), then the particles with velocities deflected in direction will be subject to only the difference force, which is directed in the opposite direction to the local deflection of the particle's velocity and is proportional to this deflection.

We denote by z and x the coordinates of the particle relative to the equilibrium trajectory in the plane perpendicular to the local velocity; then the velocity deviations will be the time derivatives $\dot{z}$ and $\dot{x}$, respectively. Hence, the force in which we are interested will be the projection of the total force $\mathbf{F}_R$ onto the corresponding axis and equal, for example, for the z direction to

$$F_R^z = -F_R\dot{z}/c = -P_{SR}\dot{z}/c^2. \quad (8)$$

It can be seen immediately that this force has the nature of a force of viscous friction for the z direction. If the beam moves in a focusing structure, this friction must lead to a gradual damping of the transverse oscillations of the particles. However, in analyzing the effect of the frictional force on the total motion of the particles, it is necessary to take into account the real dynamics in the fields of the considered accelerator or storage ring.

We consider a storage ring for charged particles in which a closed equilibrium orbit in the magnetic fields is planar. We shall assume that the "energy motion" of a particle takes place under the influence of a high-frequency accelerating system, as is the case in storage rings. A closed orbit for which the revolution frequency is exactly equal to the frequency of the accelerating system or a submultiple of it will be an equilibrium orbit, and the energy of a particle moving in such an orbit will be an equilibrium energy. The deviations from the equilibrium motion can be represented in a first approximation as a superposition of three independent motions. One motion, associated with the change in the particle's energy, consists of a deviation of the closed orbit from the equilibrium orbit and takes place, without external perturbations, slowly compared with the revolution period. The other two are transverse oscillations of the particle about the equilibrium orbit with conservation of the total momentum. We decompose the transverse oscillations into the radial ones, in the plane of the orbit (x oscillations), and vertical, or axial ones, which are associated with deviations of the particles from the plane of the closed orbits (z oscillations).

It is simplest to estimate the damping rate of the z oscillations, since the only factor besides the focusing forces exerted by the magnetic fields of the storage ring, which ensure the stability of the particle motion, is the z projection of the radiation reaction force:

$$\left.\begin{array}{l} \gamma m\ddot{z} + \gamma m\omega_z^2 = -P_{SR}\dot{z}/c^2; \\ \ddot{z} + \dot{z}P_{SR}/E + \omega_z^2 z = 0, \end{array}\right\} \quad (9)$$

where ω_z is the frequency of the axial oscillations. It can be immediately seen that the presence of the synchrotron radiation leads to damping of the amplitude of the z oscillations with damping rate δ_z equal to

$$\delta_z = P_{SR}/E, \quad (10)$$

and cooling time

$$\tau_z = \frac{E}{P_{SR}} = \frac{3}{2}T_0\frac{R}{r_0}\gamma^{-3} = \frac{3}{2}\frac{m^4c^7}{e^4}\frac{1}{EH^2}. \quad (11)$$

As we see, the damping time is the "de-excitation" time of the total energy of the particle. In cases of practical importance nowadays, it varies from seconds to milliseconds. This time is also characteristic of the damping of the deviations with respect to the remaining degrees of freedom. The damping rates may be significantly changed and even reverse their sign, when the oscillations are no longer damped but have amplitudes which grow exponentially.

We now consider the damping of the particle deviations from the equilibrium energy, making the assumption that the energy losses through radiation are compensated by a high-frequency accelerating element and that the energy deviations oscillate (autophasing regime). We shall also assume that the energy acquisition on passage through the accelerating element depends only on the time, or, rather, on the phase of the accelerating voltage at the time of passage, and not on the energy and the radial position of the particle corresponding to it. Then it is obvious that the energy oscillations will be damped if the ratio of the radiation energy loss per revolution to the energy loss of an equilibrium particle increases with increasing energy of the particle and vice versa, i.e., the sign of the damping rate is determined by the sign of the derivative of the radiation loss per revolution with respect to the energy of the particle. Hence, the damping rate of the energy oscillations will be in sign and magnitude

$$\delta_E = f_0\, d\Delta E/dE,$$

where ΔE is the energy loss per revolution, f_0 is the revolution frequency of an equilibrium particle, and the total derivative is taken with respect to the deviation of the energy from the equilibrium value with allowance for the dependence of the radial position of the orbit on the energy. Substituting $\Delta E = P_{SR}2\pi R/c$, we obtain

$$\delta_E = \frac{1}{R}\frac{d(P_{SR}R)}{dE} = \frac{P_{SR}}{E}\left(\frac{E}{P_{SR}}\frac{dP_{SR}}{dE} + \frac{E}{R}\frac{dR}{dE}\right). \quad (12)$$

If the curvature is not constant along the equilibrium orbit, it is necessary to differentiate the product of the averaged quantities.

When the energy oscillations are damped, so are the phase oscillations, i.e., the length of the bunches of charged particles decreases. The effect of the radiation reaction on the radial transverse x oscillations, which take place in the plane of the equilibrium orbit, consists of two parts. The first corresponds exactly to the frictional force for the z oscillations [see (9)]. The second is due to the circumstance that when the particle executes radial oscillations it comes into fields different from those on the equilibrium orbit, and the energy of the particle is modulated with the frequency of the transverse oscillations. This modulation $\Delta\varepsilon$ gives rise to a modulation $\Delta r = \Delta\varepsilon dR/d\varepsilon$ of the instantaneous position of the orbit determined by focusing of the radial deviations of the particle with the frequency of these oscillations. This resonance modulation may make either a positive or a negative contribution to the radiation damping rate of the radial oscillations. Note that there is no analogous contribution to the damping rate of the z oscillations, because modulation of the energy does not shift the instantaneous orbit in the z direction.

Let us estimate this modulation of the energy. The deviation in energy is equal to the difference between the energy obtained from the accelerating system, which we shall assume is uniformly distributed around the perimeter of the storage ring, and the energy loss in the form of synchrotron radiation:

$$\Delta\varepsilon(t)=\int\{\Delta E_0 f[r_b(t)]-\mathscr{P}_{E_0}[r_b(t)]\}\,dt$$
$$=\int\left\{[\Delta E_0 f_0-\mathscr{P}_{E_0}(0)]+\Delta E_0 f_0\frac{1}{f_0}\frac{\partial f}{\partial r_b}r_b-\frac{\partial\mathscr{P}}{\partial r_b}r_b\right\}dt.$$

Here, ΔE_0 is the energy acquired (in the equilibrium phase) during a revolution, and $f[r_b(t)]$ is the instantaneous revolution frequency, which depends on the radial position $r_b(t)$ taken up by the particle in its x oscillations. The expression in the square brackets vanishes for a particle moving in an equilibrium orbit in the equilibrium phase of the accelerating voltage. Accordingly, $\Delta E_0 f_0=\mathscr{P}E_0(0)$ and $(r_b/f_0)(\partial f/\partial r_b)=(-r_b/R_0)$. Therefore,

$$\Delta\varepsilon(t)=-\int\left(\frac{\mathscr{P}_E}{R_0}+\frac{\partial\mathscr{P}}{\partial r_b}\right)r_b(t)\,dt. \tag{13}$$

We now estimate the influence of this effect on the x oscillations. With allowance for the energy modulation, the equation takes the form

$$\ddot{x}+\omega_x^2(x-\Delta r)=0. \tag{14}$$

Making the substitution $x-\Delta r=r$, we obtain $\ddot{r}+\ddot{x}-d^2\Delta r/dt^2$, and Eq. (14) after the substitution $\Delta r=\Delta\varepsilon dR/dE$ becomes

$$\ddot{r}+\omega_x^2 r=-\frac{dR}{dE}\frac{d^2\Delta\varepsilon}{dt^2}.$$

Substituting $\Delta\varepsilon$ from (13), we finally find

$$\ddot{r}+\left\{\frac{\mathscr{P}_{E_0}}{R}\frac{dR}{dE}-\frac{\partial\mathscr{P}}{\partial r_b}\frac{dR}{dE}\right\}\dot{r}_b+\omega_x^2 r=0.$$

The expression in the curly brackets is the contribution of the considered effect to the damping rate of the x oscillations. The total damping rate is

$$\delta_x=\frac{\mathscr{P}}{E}\left\{1-\left(1+\frac{R}{\mathscr{P}}\frac{\partial\mathscr{P}}{\partial r_b}\right)\frac{E}{R}\frac{dR}{dE}\right\}. \tag{15}$$

If the magnetic structure of the storage ring does not have azimuthal symmetry, the quantities in the expression must be averaged along the equilibrium orbit. An interesting result is obtained by adding δ_E and δ_x:

$$\delta_E+\delta_x=\frac{\mathscr{P}}{E}\left\{1+\frac{E}{\mathscr{P}}\frac{\partial\mathscr{P}}{\partial E}\right\}, \tag{16}$$

where the partial derivative with respect to the energy is taken for fixed magnetic field. It can be seen that the sum of the damping rates does not depend on the particular magnetic structure of the storage ring. If the equilibrium orbit is not planar or there exist dynamic couplings of the z oscillations to other degrees of freedom, the sum of all three damping rates is still universal:

$$\delta_z+\delta_x+\delta_E=2\frac{\mathscr{P}}{E}+\frac{\partial\mathscr{P}}{\partial E}; \tag{17}$$
$$\delta_z+\delta_x+\delta_E=\frac{\mathscr{P}}{E}\left(2+\frac{E}{\mathscr{P}}\frac{\partial\mathscr{P}}{\partial E}\right). \tag{18}$$

In the relativistic case, this formula holds for any form of the energy losses. For synchrotron radiation [see (2)],

$$(E/\mathscr{P})(\partial\mathscr{P}/\partial E)=2 \tag{19}$$

and the sum of the damping rates is

$$\delta_z+\delta_x+\delta_E=4P_{SR}/E. \tag{20}$$

We now consider radiation damping in some important types of magnetic structures in storage rings. As we have seen above, the damping of the z oscillations is the same for all ideal structures and it is only necessary to consider δ_E and δ_x.

An azimuthally symmetric storage ring can be characterized by just one structural characteristic—the exponent n with which the magnetic field decreases:

$$n=-\frac{R}{H}\frac{dH}{dR}.$$

Transverse oscillations are simultaneously stable for $0<n<1$. In this case, the coefficient of spatial increase in the density of the orbits is

$$\alpha=\frac{E}{R}\frac{dR}{dE}=\frac{1}{1-n},$$

and

$$\frac{R}{P_{SR}}\frac{\partial P_{SR}}{\partial r_b}=\frac{H}{\mathscr{P}}\frac{\partial\mathscr{P}}{\partial H}\frac{R}{H}\frac{\partial H}{\partial R}=2n.$$

It follows from (15) that

$$\delta_x=\frac{\mathscr{P}_{SR}}{E}\left\{1-\frac{1}{1-n}+\frac{2n}{1-n}\right\}=\frac{\mathscr{P}_{SR}}{E}\frac{n}{1-n}. \tag{21}$$

For δ_E we also obtain from (20)

$$\delta_E=4\frac{P_{SR}}{E}-\delta_z-\delta_x=\frac{\mathscr{P}_{SR}}{E}\left(4-1-\frac{n}{1-n}\right)=\frac{\mathscr{P}_{SR}}{E}\frac{3-4n}{1-n}.$$

Thus, in an azimuthally symmetric storage ring the transverse oscillations are radiation damped in the entire stability range of these oscillations. The phase-energy oscillations are damped only for $n<3/4$, and above this value $\delta_E<0$, and these oscillations grow exponentially for synchrotron radiation.

For storage rings with sharp ($\omega_x,\omega_z\gg\omega_0$) focusing, in which the simultaneous stability of the x and z deviations is ensured by alternating sections with $n\gg1$, which focus with respect to z and defocus with respect to x, and sections with $n<0$ for $|n|\gg1$, which defocus with respect to z and focus with respect to x, the distribution of the damping rates may be different depending on the relationship between the magnetic fields on the equilibrium orbit in the radially focusing and defocusing sections and the bending (homogeneous) magnets. In the special case of a magnetic field which is constant around the entire storage ring on the equilibrium orbit, making the purely formal substitution $n/(1-n)|_{n\to\infty}=-1$ in (21) we obtain $\delta_x=-P_{SR}/E$. From (20), we then find $\delta_E=4P_{SR}/E$. In this case, the radial transverse oscillations are strongly "antidamped," and a facility with such a structure cannot be used to store electrons or positrons.

The simplest, but not unique, structure of a storage ring that combines very sharp focusing and normal radiation damping of all types of deviation is a system with separated functions. Here, the bending sections are made homogeneous and hardly influence the focusing of the transverse deflections, while the focusing (and defocusing) sections are made in the form of quadrupole lenses and a magnetic field which is zero on the equilibrium orbit.

In this case, the radiation in the lenses can (in the linear approximation) be ignored, while in the bending sections the radiation does not depend on the radial position. Accordingly, the two transverse directions become equivalent, $\delta_x = \delta_z = \mathscr{P}_{SR}/E$, and the damping rate of the energy oscillations is $\delta_E = 2\mathscr{P}_{SR}/E$. The strong dependence of δ_x and δ_z on the relationship between the fields in the sections with different focusing has the consequence that these damping rates are rapidly redistributed when the equilibrium orbit is changed. This effect is particularly important for rf electron–positron storage rings, in which, to reduce the energy loss through synchrotron radiation, it is necessary to use low values of the guide magnetic field [see (2)] with high gradients in the focusing elements, i.e., the lenses. The range of admissible variations of the mean radius of the equilibrium orbit can in practice become much less than the useful radial aperture of the storage ring.

Thus, if the magnetic structure of the storage ring is correctly chosen, the deviations from the equilibrium motion of a synchrotron radiating particle are gradually damped, and the beam emittance is reduced by a factor e during approximately the radiation time of the total energy. However, there are always effects that limit the contraction of the beam. For example, there may be external diffusion effects such as multiple scattering on atoms of the residual gas or a very thin internal target, or the so-called *collision effects* (for colliding beams), when the strongly nonlinear fields of the colliding bunches have a quasidiffusive effect on the motion of the beam particles. However, there is also a mechanism limiting the beam contraction in the actual synchrotron-radiation mechanism—the quantization of the energy loss of the radiating particle. At high electron and positron energies, it is this effect that in the majority of cases determines the steady size of the beam and the energy spread.

Let us first estimate the steady energy spread. On the average, the emitted photons [see (4)] have energy

$$E_\gamma = \hbar\omega_0\gamma^3 = (\hbar c/R_0)\,\gamma^3 = e\lambda\!\!\!^{-}_c H\gamma^2, \qquad (22)$$

where $\lambda\!\!\!^{-}$ is the Compton wavelength of the particle. During the damping time, the particle radiates the energy E, and E/E_γ photons are emitted. The emission of the photons is statistically independent. Therefore, the dispersion of the number of emitted photons will be characterized by $\sqrt{E/E_\gamma}$ and, accordingly, the steady spread of the energy of the particles in the beam by

$$\langle\Delta E\rangle \approx E_\gamma\sqrt{E/E_\gamma} \approx \sqrt{E_\gamma E}, \qquad (23)$$

so that the relative energy spread is

$$\langle\Delta E\rangle/E \approx \sqrt{E_\gamma/E} \approx \gamma\sqrt{\lambda\!\!\!^{-}/R_0} = \sqrt{\hbar e\gamma H/(m^2c^3)}\,. \qquad (24)$$

For a more accurate estimate, it is necessary to use (12) for the damping time and to average correctly over the azimuth of the storage ring and the radiation spectrum.

The stationary width of the beam in the plane of the orbit, i.e., the radial width, can be readily estimated directly from (24). It is made up of two parts: first, the energy spread gives directly a radial width $\alpha R\langle\Delta E\rangle/E$, where $\alpha \approx 1/\nu_x^2$ is the coefficient of the spatial density increase of the orbits in the storage ring; secondly, a contribution of about the same magnitude derives from the excitation of a jump in the energy of the particle on the emission of a quantum of the x oscillations of the particle. The energy of the particle changes instantaneously with respect to the period of the transverse oscillations, and during this time the particle cannot change its position; accordingly, a particle moving, for example, initially strictly along the equilibrium orbit for the instantaneous value of the energy is deflected from the new equilibrium orbit and radial betatron oscillations arise. The final steady radial width of the beam is

$$\langle\Delta r\rangle_{st} \approx \sqrt{2}\alpha R\,\langle\Delta E\rangle/E \approx \gamma\sqrt{\lambda\!\!\!^{-}_c R/\nu_x^2}. \qquad (25)$$

With regard to the steady axial z spread, it is readily seen that in an ideal storage ring with planar closed orbits it must be much less. In this case, the z oscillations will be excited because the photon is emitted not only along the instantaneous velocity but in an angle of order $1/\gamma$. As a result of this, the particle acquires in each radiation event a recoil momentum of order $E_\gamma/\gamma c$ and z oscillations are excited with amplitude

$$\frac{R}{\nu_z}\frac{1}{\gamma}\frac{E_\gamma}{E},$$

so that during the damping time a z spread is acquired that is $\sqrt{E_\gamma/E}$ times greater, reaching the value

$$\Delta z \approx \frac{1}{\gamma}\frac{R}{\nu_z}\frac{\langle\Delta E\rangle}{E}.$$

The steady z spread is then $\gamma\nu_z/\nu_x^2 \approx \gamma/\nu_x$ times less than the radial spread. At high energies, it should become very small. Because real storage rings are not ideal and there is, in particular, a dynamical coupling of the transverse oscillations, the damping of the z oscillations is halted much earlier. The influence of this and other factors of a diffusion nature on the steady characteristics of a beam in the presence of damping can be estimated quantitatively from the balance of the heating and cooling powers.

Radiation cooling has now become a customary method and is very important for high-energy physics. Its main application is in the accumulation of large electron and positron currents and in experiments with colliding electron and positron beams. It is no exaggeration to say that the existence of this "natural" cooling was decisive in the rapid development of this direction, which today gives a considerable fraction of all fundamental information in elementary-particle physics and which was initiated by the experiments on electron–electron interaction at Stanford and Novosibirsk (1965) and electron–positron interaction at Novosibirsk (1967).

There is a steady development of the use of radiation cooling in elementary-particle and nuclear physics experiments employing "superthin" internal targets in electron and positron storage rings.[41] Such experiments are facilitated by the possibility of arbitrary "stretching out" in time of the investigated reactions and the thinness of the target, which makes it possible to detect even very slow reaction products, for example, nuclear fragments. At the same time, the effect of multiple scattering and fluctuations of the ionization losses can (if the mean energy losses are compensated by an external energy source) be suppressed by radiation cooling, and the steady size of the beam, its angular spread, and, most important, monochromaticity can remain very good. At the same time, the luminosity $\mathscr{L}$ of the experiment, defined as the number of interaction events of the beam particles with the target particles per unit time for unit cross section of the process, can be made very high, being limited solely by the destruction of the beam particles in single interactions with the target particles and the output of the "beam generator": $\mathscr{L}=\dot{N}/\delta_L$, where $\dot{N}$ is the flux of particles entering the storage ring, and σ_L is the cross section for processes of interaction with the target which take the beam particles outside the range of admissible (for the given storage ring) deviations from the equilibrium parameters (with respect to the angle or energy). At energies of order 1 GeV and above, bremsstrahlung is the decisive process. The presence of strong radiative friction also eases the problem of ensuring coherent stability of a high particle current in the storage ring, which is needed to ensure a high luminosity and monochromaticity simultaneously. It is also very important to work with high currents when electron storage rings are used as sources of synchrotron radiation. For the majority of applications, the main characteristic of the source is its brightness, which increases proportionally when the cross-sectional area of the beam decreases, such contraction being ensured by radiation cooling.

The already existing electron storage rings, developed and constructed for high-energy physics, ensure a brightness which is greater (on the time average) by several orders of magnitude and in the entire range of photon energies from 10 eV to hundreds of kiloelectron-volts than any other sources of electromagnetic radiation hitherto developed.[42] A further increase in the brightness and extension of the spectral range will be made possible by the development of special magnetic radiating elements and specialized storage rings. The use of synchrotron radiation is of great importance for many branches of science and technology.

Nevertheless, the main field in which radiation cooling is at present used is in experiments with colliding electron–positron beams. In such experiments one can not only accumulate the necessary positron and electron currents and compress the beam to the very small transverse (down to 0.5×10^{-2} mm^2) and longitudinal (down to a few centimeters) bunch dimensions that are needed. The cooling makes it possible to overcome the "warming" effect of scattering on the atoms of the residual gas and neighboring particles in high-density bunches. In addition, the cooling makes it possible to raise appreciably the threshold current of the colliding beams above which the transverse motion of the particles in the strongly nonlinear fields of the colliding bunches acquires a quasistochastic nature, which leads to spreading of the beams and, thus, to a loss of luminosity or even the rapid destruction of the particles.

The unavoidable transition to linear colliding electron–positron beams at superhigh (hundreds of giga-electron-volts) energies in place of the now traditional experiments with colliding beams in storage rings[43]—a transition made necessary by the catastrophic increase in the synchrotron-radiation losses in the case of motion in curvilinear trajectories—will increase even more the importance of radiation cooling in high-energy physics. To obtain the necessary luminosity, it is necessary to compress high-density bunches of electrons and positrons at the point of collision to fractions of a square micron. This can be achieved by injecting into linear accelerating systems bunches of particles that have previously been deeply cooled in storage facilities with a very special structure.

Radiation cooling does not yet play any part in studies with heavy particles. Even for the largest realized projects, the cooling time is many days. However, it may be important for implementing electron cooling of colliding proton–antiproton beams even at a few hundred giga-electron-volts, making it possible to obtain electron beams with the necessary parameters.[44] Such cooling may be helpful in maintaining a high luminosity.

At even higher energies, beginning around 10 TeV and bearing in mind future progress in obtaining high magnetic fields in storage rings, radiation cooling of the heavy particles themselves will begin to "work," which will be a great help in experiments with colliding proton–antiproton beams at high luminosity.

2. IONIZATION COOLING

We now consider ionization cooling, which is based on the use of ionization energy losses of accelerated charged particles to produce a frictional force with compensation of the energy loss of an equilibrium particle by an external source. Many properties of the process and the basic equations are the same or similar to those derived above for radiation cooling. However, for ionization cooling the nonrelativistic region is also of interest.

For simplicity, let us consider the case of rectilinear motion of the cooled beam in a medium with ionization losses. The characteristic features of ionization cooling are manifested in this case too, and the sum of the damping rate does not depend on the motion of the cooled particles, in particular, on the presence of focusing. Therefore, from the obvious equation for the transverse (relative to the direction of the force which makes good the energy losses) momentum component p_z,

$$\dot{p}_z = -\mathcal{F}_{fr} p_z/p = -p_z \mathcal{P}_{fr}/pv = p_z \partial \mathcal{F}_{fr}/\partial p_z,$$

where $\mathcal{F}_{fr}$ and $\mathcal{P}_{fr}$ are the force and power of the ionization damping, we readily obtain an expression for the damping rate:

$$\delta_z = \mathcal{F}_{fr}/p = \mathcal{P}_{fr}/pv = \partial \mathcal{F}^z_{fr}/\partial p_z = \begin{cases} \mathcal{P}_{fr}/2E_{\text{kin}}, & v \ll c, \\ \mathcal{P}_{fr}/E, & v \to c. \end{cases} \quad (26)$$

Similar expressions hold for δ_x. For the deviations of the longitudinal momentum $\Delta p_{\parallel}$ from the equilibrium value for which the ionization losses are exactly compensated by the external longitudinal electric field, we obtain in the linear approximation

$$\frac{d\,\Delta p_{\parallel}}{dt} = -(\partial \mathcal{F}^{\parallel}_{fr}/\partial p_{\parallel})\,\Delta p_{\parallel} = \frac{\partial}{\partial p_{\parallel}}(\mathcal{P}_{fr}/v)\,\Delta p_{\parallel}.$$

Hence, the damping rate of the longitudinal motion is

$$\delta_{\parallel} = \frac{\partial \mathcal{F}^{\parallel}_{fr}}{\partial p_{\parallel}} = \frac{\partial}{\partial p_{\parallel}}\left(\frac{\mathcal{P}_{fr}}{v}\right) = -\frac{\mathcal{P}_{fr}}{v^2}\frac{dv}{dp_{\parallel}} + \frac{1}{v}\frac{\partial \mathcal{P}_{fr}}{\partial p_{\parallel}}. \quad (27)$$

For the sum of the damping rates, we obtain

$$\begin{aligned} \delta_x + \delta_z + \delta_{\parallel} &= \frac{\partial \mathcal{F}^z_{fr}}{\partial p_z} + \frac{\partial \mathcal{F}^x_{fr}}{\partial p_x} + \frac{\partial \mathcal{F}^{\parallel}_{fr}}{\partial p_{\parallel}} = \operatorname{div}_p \mathcal{F}_{fr} \\ &= \frac{2\mathcal{P}_{fr}}{pv}\left(1 - \frac{p_{\parallel}}{2v}\frac{\partial v}{\partial p_{\parallel}}\right) + \frac{1}{v}\frac{\partial \mathcal{P}_{fr}}{\partial p_{\parallel}} \\ &= \begin{cases} 2\dfrac{\mathcal{P}_{fr}}{E} - \dfrac{\partial \mathcal{P}_{fr}}{\partial E}, & v \to c; \\ \dfrac{\mathcal{P}_{fr}}{2E_{\text{kin}}} - \dfrac{\partial \mathcal{P}_{fr}}{\partial E_{\text{kin}}}, & v \ll c, \end{cases} \end{aligned} \quad (28)$$

where E is the total energy of the cooled particle, and E_{kin} is its kinetic energy.

We now obtain estimates for the ionization losses for $\mathcal{F}_{fr}$ and $\mathcal{P}_{fr}$ having the form

$$\mathcal{F}_{fr} = 4\pi N_e e^4 L_i/(m_e v^2); \quad \mathcal{P}_{fr} = 4\pi N_e e^4 L_i/(m_e v),$$

where N_e is the electron density, $L_i = \ln(2m_e\gamma^2 v^2/ZI) = 8-12$, Z is the atomic number of the material of the target, and I is the effective ionization potential. In the estimates, we shall assume that the target fills the entire orbit.

In the nonrelativistic case, the rapid decrease of the force of friction with increasing velocity leads to a negative longitudinal damping rate and an appreciable reduction in the sum of the damping rates:

$$\delta_z + \delta_x + \delta_{\parallel} = \mathcal{P}_{fr}/E_{\text{kin}}(L_i/2) \quad \text{for } v \ll c, \quad (29)$$

which is 3–5 times less than the sum of the transverse damping rates alone. The magnetic structure of the ionization cooler and the shape of the target must ensure a "transfer" of the damping rates from the two transverse degrees of freedom to the longitudinal degree of freedom to prevent a growth of the energy spread, if, of course, this is necessary in the particular experiment.

In the relativistic region, in which the dependence of the ionization losses on the particle energy virtually disappears, the sum of the damping rates is

$$\delta_z + \delta_x + \delta_{\parallel} = 2\mathcal{P}_{fr}/E = 8\pi N_e e^4 L_i/(\gamma m_e M c^3), \quad v \to c. \quad (30)$$

To ensure damping of the energy spread, it is sufficient to place the target in a section where the position of the closed orbit of a particle does not depend on its energy, the thickness of the target being made variable (a greater thickness of the target corresponding to higher energy).

We now estimate the limits to which beams of charged particles can be cooled by the ionization method. The principal "warming" factor is multiple scattering on the nuclei and electrons of the target itself, and also fluctuations of the energy losses.

The mean square of the angle of the particles with respect to the equilibrium orbit in the z direction in the target section can be found from the balance between the rates of cooling and heating:

$$d\langle\theta_z^2\rangle/dt = -\delta_z\theta_z^2 + 4\pi N_i Z_i^2 e^4 L_c/(\gamma^2 M^2 v^3) = 0,$$

where the second term is the rate of increase of the z projection of the angle of multiple scattering during motion in the material of the target, N_i is the density of the nuclei in the target with charge Z_i, and $L_c = \ln(r_{\max}/v_{\min}) \approx 15$ is the Coulomb logarithm. If it is assumed that the damping rate is made equal to one third of the sum $\Sigma\delta_i$ of the damping rates as determined earlier [see (30)], then the steady mean square z angle is

$$\langle\theta_z^2\rangle = \frac{4\pi N_i Z_i^2 e^4 L_c}{\gamma^2 M^2 v^3 \left(\sum \delta_i/3\right)} = \begin{cases} 1.5 L_c Z_i \dfrac{m_e}{M}, & v \ll c, \\ 1.5\dfrac{L_c}{L_i} Z_i \dfrac{m_e}{\gamma M}, & v \to c. \end{cases} \quad (31)$$

Thus, the steady angle in the target section does not depend on the focusing structure of the accelerator (provided it is the same at all sections with targets). At the same time, the steady z emittance in the case of ionization cooling is

$$\Omega^2_{eq} = \langle\theta_z^2\rangle\beta_z^0 = \begin{cases} 1.5 L_c Z_i \dfrac{m_e}{M}\beta_z^0, & v \ll c; \\ 1.5\dfrac{L_c}{L_i} Z_i \dfrac{m_e}{\gamma M}\beta_z^0, & v \to c, \end{cases}$$

where β_z^0 is the value of the β function of the accelerator in the region of the target (the analog of the local focal length of the magnetic structure, equal in the azimuthally homogeneous case to R/ν_z), and is smaller, the greater is the local sharpness of the focusing. Therefore, the magnetic structure must be chosen to ensure that the β function has a sharp minimum in the region of the target. Experience in, for example, the construction of electron–positron storage rings shows that in practice it is possible to make the local β function of the order of a centimeter. At the same time, the length of the target should not exceed this amount. For such a sparse distribution of the targets in the storage facility, the cooling time naturally increases in proportion to their spacing. Note that the maintained transverse emittance (admittance) of the accelerator must be several times greater than $\Omega^{\perp}_{eq}$.

In the relativistic region, the stationary transverse phase space decreases in proportion to the increase in the energy. However, when the energy is raised, electron scattering with energy losses becomes more and more troublesome, which increases the steady energy spread.

In a collision with an electron at rest, the maximal energy transfer is bounded by

$$\Delta E_{\max} = \frac{2m(E^2/c^4 - M^2)}{2mE/c^2 + M^2 + m^2}$$

$$= \begin{cases} 4\frac{m}{M}E_{\rm kin}, & v \ll c; \\ 2\frac{m}{M}\frac{E^2}{Mc^2}, & Mc^2 \ll E \ll \frac{M}{2m}Mc^2; \\ E - M\frac{M}{2m} \approx E, & E \gg \frac{M}{2m}Mc^2, \end{cases}$$

which approaches the total energy of the particle in the extreme relativistic domain.

We estimate the steady energy spread in an ionization-cooled beam, assuming that the admissible deviations of the energy are much greater than $\Delta E_{\max}$. As for the transverse motion, the steady energy spread σ_E can be found from the balance between the rate of cooling of the square $\langle\varepsilon^2\rangle$ of the energy spread, whose damping rate we assume is $\delta_E = \Sigma\delta/3$, and the rate of growth of the square of the energy deviation due to the ionization losses:

$$\frac{d\langle\varepsilon^2\rangle}{dt} = -\delta_E\langle\varepsilon^2\rangle + \int\limits_0^{\Delta E_{\max}} \varepsilon^2 \frac{2\pi N_e e^4}{mv}\frac{d\varepsilon}{\varepsilon^2}$$

$$= -\frac{1}{3}\sum\delta\langle\varepsilon^2\rangle + \frac{2\pi N_e e^4 \Delta E_{\max}}{mv} = 0.$$

Hence $\sigma_E^2 = 6\pi N_e e^4 \Delta E_{\max}/(mv\Sigma\delta_i)$. It is obvious that σ_E is determined by the nature of the interaction of the particles with the target electrons. Substituting the expressions for $\Delta E_{\max}$ and $\Sigma\delta_i$, we obtain:

a) for $v \ll c$ $\quad \sigma_E^2 = 3(m/M)E_{\rm kin}^2;\ \sigma_E/E_{\rm kin} = \sqrt{3m/M}$;

b) as $v \to c$

$$\sigma_E^2 = \begin{cases} \frac{3}{2}\frac{m}{M}\frac{E^3}{L_i Mc^2}, & Mc^2 \ll E \ll \frac{M}{2m}Mc^2; \\ \frac{3}{4}\frac{E^2}{L_i}, & E \gg \frac{M}{2m}Mc^2; \end{cases}$$

$$\frac{\sigma_E}{E} = \begin{cases} \sqrt{\frac{3}{2}\frac{m}{M}\frac{\gamma}{L_i}}, & 1 \ll \gamma \ll \frac{M}{2m}; \\ \sqrt{\frac{3}{4L_i}}, & \gamma \gg \frac{M}{2m}. \end{cases}$$

We see that for $\gamma \gtrsim M/2m$ the steady spread becomes the same order of magnitude as the total energy, and the energy cooling effectively ceases to work. At lower energies, the steady spread is much greater than $\Delta E_{\max}$, and the accelerator must retain particles with energy deviations from equilibrium that are a few σ_E. We shall be interested in only this region of not too high energies: $\gamma \ll M/2m$.

We now find the steady geometrical longitudinal phase space of the beam:

$$\Omega^{\parallel}_{eq} = \sigma_E \frac{l_\varphi}{v}\frac{1}{\gamma Mv} = \begin{cases} \frac{1}{2} l_\varphi \frac{\sigma_E}{E_{\rm kin}}, & v \ll c; \\ l_\varphi \frac{\sigma_E}{E}, & v \to c, \end{cases}$$

where l_φ is the rms azimuthal deviation of the beam particles from an equilibrium particle, and it is proportional to σ_E. We shall assume that the ionization losses are made good in the autophasing regime at the first harmonic of the revolution frequency. The connection between l_φ and σ_E is given by

$$l_\varphi = \frac{R}{\omega_\varphi}\frac{d\omega}{dE}\sigma_E,$$

where R is the mean radius of the accelerator, $\omega_\varphi = \omega_s\sqrt{eU_0 \sin\varphi_s \mathcal{K}/(2\pi E_s)}$ is the frequency of the phase oscillations, eU_0 is the amplitude of energy acquisition from the accelerator element in one revolution, φ_s is the equilibrium phase, $eU_0\cos\varphi_s$ being equal to the ionization losses during a revolution; $\mathcal{K} = d\ln\omega/(d\ln E)$ characterizes the rate of change of the frequency of revolution of a particle in the accelerator with respect to the energy and for the so-called *critical energy*, which depends on the magnetic structure of the accelerator, tends to zero. We immediately obtain

$$\Omega^{\parallel}_{eq} = \begin{cases} \frac{R}{\omega_\varphi}\frac{d\omega}{dE}\frac{1}{2}\frac{\sigma_E^2}{E} = \frac{3}{2}R\frac{\omega_s}{\omega_\varphi}\mathcal{K}\frac{m}{M}, & v \ll c; \\ \frac{3}{2}R\frac{\omega_s}{\omega_\varphi}\mathcal{K}\frac{m}{M}\frac{\gamma}{L_i}, & 1 \ll \gamma \ll \frac{M}{2m}. \end{cases}$$

Thus, to obtain the smallest possible steady longitudinal emittance of the beam at the given cooling energy, it is necessary to reduce as much as possible the effective "longitudinal focal length" $R\omega_s/\omega_\varphi$ of the accelerator, and also raise the frequency multiplicity of the employed rf compensating voltage in the single-bunch regime, working simultaneously as near the critical energy as possible.

After cooling, one can use the small value of the steady longitudinal emittance (the short steady bunch length) for sharp monochromatization of the cooled beam. For this, it is sufficient, for example, to begin by removing adiabatically slowly the ionization losses (reducing the target thickness smoothly) and then reduce the accelerating voltage, increasing the length of the bunch and gaining monochromaticity in proportion. This adiabatic bunch lengthening can also be done in an additional matched accelerating tract.

Let us now consider the particles for which it is sensible to use ionization cooling. It must be borne in mind that the presence of a target necessarily imposes a limitation on the lifetime of the particles in the storage facility, and one must therefore consider what happens first—compression of the beam under the influence of ionization cooling or ejection of particles from the beam due to interaction with target nuclei and electrons? A convenient characteristic is the ratio of the particle lifetime τ_L to the growth time of the 6-dimensional phase space, i.e., $(\Sigma\delta_i)^{-1}$: $\xi = \tau_L\Sigma\delta_i$. Cooling is effective if $\xi \gg 1$.

For electrons and positrons, ionization cooling is certainly not appropriate. At low energies, multiple scattering proceeds faster than ionization damping, and at high energies the main energy losses are radiation losses, which occur in large "portions"—the bremsstrahlung spectrum is uniform to photon energies of the order of the initial electron energy.

For cooling of protons and antiprotons, the main hindering factor at not too low energies is the strong (nuclear) interaction with the target nuclei. (Note that it is necessary to retain particles scattered in the target through an angle several times greater than the steady angular spread and simultaneously ensure the possibility of ignoring the loss of particles through single Coulomb scattering). The cross section for the loss of particles is effectively equal to the total nuclear cross section σ_{tot}, and therefore

$$\xi=\frac{\sum\delta_i}{N_i v\sigma_{tot}}=\begin{cases}\frac{\mathscr{F}_{fr}}{N_i\sigma_{tot}E_{kin}(L_i/2)}\approx\frac{2.4\cdot10^{-27}Z_i}{\beta^4\sigma_{tot}}, & v\ll c;\\ \frac{2\mathscr{F}_{fr}}{N_iE\sigma_{tot}}=\frac{10^{-26}Z_i}{\gamma\sigma_{tot}}, & v\to c.\end{cases}$$

A rough empirical estimate for the nuclear cross section of protons is as follows: for hydrogen

$$\sigma^p_{tot}=\begin{cases}4\cdot10^{-27}/\beta^2\ \text{cm}^2, & v<0.5c;\\ 40\cdot10^{-27}\ \text{cm}^2, & v\to c;\end{cases}$$

and for the remaining nuclei

$$\sigma^p_{tot}=\begin{cases}\frac{15Z_i10^{-27}}{\beta^2}\ \text{cm}^2, & v<0.5c;\\ 80Z_i10^{-27}\ \text{cm}^2, & v\to c.\end{cases}$$

Thus, we obtain for hydrogen

$$\xi=\begin{cases}0.6/\beta^2, & v<0.5c;\\ 0.25/\gamma, & v\to c;\end{cases}$$

and for other materials

$$\xi=\begin{cases}0.15/\beta^2, & v<0.5c;\\ 0.01/\gamma, & v\to c.\end{cases}$$

If we assume that the condition for ionization cooling to be useful is, for example, $\xi\geqslant3$, we see that even for a hydrogen target proton beams can be cooled only for $E\leqslant100$ MeV.

For antiprotons, the nuclear cross section at low energies is appreciably higher and even for a hydrogen target and energy 50 MeV the cross section is about 300 mb, and $\xi=0.8$. Thus, ionization cooling cannot be used for antiproton beams, except, perhaps, at extremely low energies, where the nuclear cross section for antiprotons is unknown.

Of great interest and promise is the application of ionization cooling to muon beams. For them there are virtually no radiation losses and no nuclear interaction (apart from Coulomb scattering). The lifetime of a muon beam, even under conditions when an accelerator retains with sufficient margin a beam with steady emittance, is limited by muon decay in the time $\gamma\tau_\mu$. Therefore, the cooling time must be several times shorter than the decay time:

$$(\Sigma\delta_i)^{-1}\ll\gamma\tau_\mu.$$

In the following, we shall restrict ourselves to the case $1\ll\gamma\ll\mu/2m$, where μ is the muon rest mass, because at nonrelativistic energies the steady angle is too large, of the order of unity, and at higher energies the equilibrium energy spread becomes large. For the indicated energy range, the basic expressions become

$$\sum\delta_i=2\mathscr{F}_{fr}/E=8\pi N_e e^4L_i\eta/(\gamma m\mu c^3);$$
$$\sqrt{\langle\theta_z^2\rangle_{eq}}=\sqrt{1.5\,(L_c/L_i)\,Z_i\,(m/\gamma\mu)};$$
$$\Omega^{\perp}_{eq}=1.5\,(L_c/L_i)\,Z_i\,(m/\gamma\mu)\,\beta^0;$$
$$\sigma_E/E=\sqrt{(1.5/L_i)\,(m/\mu)\,\gamma};$$
$$\Omega^{\parallel}_{eq}=(1.5/L_i)\,(m/\mu)\,\gamma R\,(\omega_s/\omega_\varphi)\,\mathscr{K},$$

where η is the fraction of the accelerator orbit occupied by targets with electron density N_e.

The condition of sufficiently fast cooling becomes

$$\frac{1}{3}\frac{2\mathscr{F}_{fr}}{E}\gg\frac{\mu c^2}{E\tau_\mu}\to\left\langle\frac{dE}{dx}\right\rangle\gg\frac{3}{2}\frac{\mu c}{\tau_\mu}=1.5\ \text{keV/cm}.$$

If one uses, for example, lithium targets, then the admissible fraction of the employed orbit is $\eta\gg1.5\times10^3/(2\times10^6)=0.7\times10^{-3}$, i.e., the material of even a condensed target must occupy about 1% of the accelerator orbit.

An important point is that during the damping time several phase oscillations must occur, and, accordingly, the frequency of the phase oscillations must be appreciably greater than $(\gamma\tau_\mu)^{-1}$. At the minimal rate of ionization energy losses that is still admissible, which is several times larger than $\mu c/2\tau_\mu$, we obtain for the steady longitudinal emittance

$$\Omega^{\parallel}_{eq}=\frac{1.5}{L_i}\frac{m}{\mu}\gamma\frac{2\pi R}{eU_0\sin\varphi_s}E\frac{\omega_\varphi}{\omega_s}\approx\frac{3}{L_i}\gamma^2\frac{mc^2}{eH\tan\varphi_s}$$

under the condition $\omega_\varphi\gg(\gamma\tau_\mu)^{-1}$, where H is the intensity of the average magnetic field of the accelerator.

Note that cooling can be done in cyclic accelerators (and it was for this case that the above expressions were written down) as well as in a quasilinear accelerator to an energy several times higher than the energy at which the cooling is done. In this case, it is necessary to choose correctly the energy dispersion function in the region of the targets in order to realize longitudinal cooling. The general estimates of the steady emittances remain valid.

We now consider briefly, following mainly Refs. 7 and 70, the possible applications of ionization-cooled muon beams. To obtain intense, completely pure, and deeply cooled muon beams one must proceed as follows:

1) using intense proton beams with an energy of hundreds of giga-electron-volts or higher obtain through a nuclear cascade in a conversion target a pion beam with the smallest possible emittance at an energy around 1 GeV;

2) allow the pions to decay in a strongly focusing rectilinear channel, which ensures the minimal growth of the transverse emittance;

3) carry out ionization cooling of the obtained muon beam;

4) accelerate the obtained muon beam to the necessary energy in a linear or cyclic accelerator with a rate of acceleration per unit length several times greater than $\mu c/\tau_\mu$, which ensures that the intensity losses due to muon decay are small.

These muon beams can be used either directly to study the interaction of muons with nucleons and nuclei, or one can, injecting the muons into a special magnetic tract, obtain a generator of electron and muon neutrinos and antineutrinos up to a total energy (with very small angular spread) that can be made close to $\mu c^2/E_\mu$, where E_μ is the energy of the accelerated muons. At $E_\mu = 1$ TeV behind a shield of thickness 300 m this permits one to obtain transverse dimensions of the neutrino beam of order 3 cm, which would greatly simplify neutrino experiments.

However, the most interesting possibility is to make experiments with colliding muon beams. If during acceleration aberration does not increase the emittance of muon beams collected into two bunches with number of particles $N_{\mu^+} = N_{\mu^-} = N_\mu$, then, injecting them into a tract with a strong magnetic field (to increase the number of collisions during the lifetime of the accelerated muons), it is possible to obtain the luminosity

$$\mathscr{L}_{\mu\mu} = \frac{N_\mu^2}{4\pi\Omega_{eq}^{\perp}\beta^0 (E_{\text{cool}}/E_\mu)} N_\tau f$$

$$= N_\mu^2 \frac{E_\mu}{6\pi (L_0/L_i) Z m_e c^2 \beta_z^0 \beta^0} \frac{eH\tau_\mu}{2\pi\mu c} f,$$

where β_z^0 and β^0 are the values of the β functions in the region of the ionization targets and the position of the collision, N_τ is the number of collisions of the muon bunches during the luminosity decay time, which is equal to half the lifetime of the accelerated muons, H is the intensity of the average magnetic field in the accelerator with the colliding beams, and f is the repetition frequency of the injection cycles. If we take $N_\mu = 10^{11}$, $E_\mu = 1$ TeV, $H = 100$ kG, $f = 10$ Hz, and $\beta_z^0 = \beta^0 = 1$ cm, we obtain a luminosity exceeding 10^{31} cm$^{-2}\cdot$sec^{-1}. As is shown in Ref. 70, parameters of this order can be found if, for example, the intense proton beams of present-day and future accelerators to maximally high energies are used to excite linear accelerators.

3. ELECTRON COOLING

General description of electron cooling

In its simplest form, the idea behind the method of electron cooling is as follows. In one of the straight sections of a storage ring in which a beam of heavy particles, for example, protons, is circulating, an intense beam of electrons with the same mean velocity and small momentum spread is arranged to run parallel to the proton beam. Then in the common section of the trajectory, in the rest frame, the "hot" proton gas is in a "cold" electron gas and is cooled by Coulomb collisions. The cooling process continues until the proton temperature in the center-of-mass system is equal to the electron temperature. If the electron velocity distribution is the same for all degrees of freedom, the angular spread in the proton beam θ_p will be $\sqrt{M/m}$ times less than the angular spread of the electrons:

$$\theta_p = \sqrt{m/M}\,\theta_e. \tag{32}$$

Since θ_e may be of order 10^{-3}, the steady angular spread for protons or antiprotons can be reduced to 10^{-5}.

The characteristic features of the process of electron cooling and the problems which arise in its practical realization depend strongly on the energy region in which the cooling is done.

1. At comparatively low energies, corresponding to electron energies up to 2–3 MeV (proton or antiproton energies up to 4–6 GeV), it is most natural to use straight electrostatic acceleration of the electrons to the necessary energy and then introduce them into the cooling section and use an electron collector after extraction. At the lowest energies (electron energies up to a few keV) the power required from the high-voltage source and dissipated in the collector presents essentially no problem, but at electron energies of hundreds of kilo-electron-volts and higher the necessary power becomes one of the principal technical problems. For this region, it is natural to use energy recuperation, i.e., to decelerate the electrons to the lowest possible energy before they reach the collector, which is connected to the source of the accelerating voltage U_0 through a rectifier with a small positive voltage U_c. The voltage U_c must be sufficient for the collection of the complete cooling electron current I_e on the collector (limitation with respect to the space charge). The product $U_c I_e$ effectively determines the active power required from the grid and dissipated on the collector. The voltage U_c can be reduced to the order of a kilovolt and, accordingly, the ratio of the active power to the reactive $U_0 I_e$ to the level 1% or less.

The current requirement from the source of the main, accelerating voltage is determined by the energy losses of the electrons through scattering on the residual gas, ionization in the accelerating and decelerating sections, the defects of the electron optics, and the imperfection of the collector. The consumption can be reduced to the level 10^{-4}. This very low load on the main source is very convenient, since it must meet very high requirements of voltage stability and absence of pulsations.

If a grouped beam of heavy particles is cooled, to reduce the mean electron current one can switch off the electron gun during the entire time that there are no particles in the cooling section, this giving a corresponding gain as regards the mean electron current. It is only necessary to ensure that the modulation of the electron current does not lead to a modulation of the electron energy and an increase in their temperature.

2. An important task is to transport the intense electron beam over large distances (long cooling sections), keeping a low effective temperature of the electrons. Compensation of the transverse repulsion of the electrons requires distributing focusing with short focal length. External fields of various configurations can be introduced. Whatever the axial focusing, it can suppress the appearance of additional transverse velocities only for a given value of the electron current; if the current is changed, the focusing must be readjusted. In addition, the external axial focusing on the straight cooling section cannot be homogeneous along the length of the section, and it is necessary to use a lens of quadrupole type. The variable sign of the ob-

tained focusing leads to the appearance of additional bends in the electron beam.

It is much more rational to use a homogeneous (except at the positions of injection and extraction) longitudinal magnetic field accompanying the electron beam from the cathode to the point at which it leaves the cooling section; of course, the effect of the longitudinal field on the protons must be taken into account in the focusing structure and the corrections of the storage facility. The transverse electron velocities produced by the effect of the space charge of the electron beam are then smaller, the stronger is the longitudinal magnetic field, and they can be made less than the thermal velocities with comparative ease. For this, the longitudinal field $H_{\parallel}$ must satisfy the condition

$$H_{\parallel} > \pi e n_e r_0/(\beta\gamma^2\theta_e),$$

where e is the electron charge, n_e is the density of the electron beam, r_0 is the radius of its transverse section, β is the ratio of the particle velocity to the velocity of light, and $\gamma = (1-\beta^2)^{-1/2}$.

If the accompanying magnetic field is sufficiently strong, the transverse electron velocities are determined mainly by the cathode temperature and the imperfection of the gun optics. We note that the transverse velocities in the comoving system are conserved in the case of electrostatic acceleration. With regard to the spread of the longitudinal electron velocities in the comoving system, the contribution of the initial electron temperature (which is of the order of the cathode temperature T_c) decreases strongly in the case of potential scattering, since the energy spread is conserved[29]:

$$T_{\parallel} \approx \frac{T_c^2}{2\gamma^2\beta^2 mc^2} \to T_c \frac{T_c}{4E_{\text{kin}}}\Big|_{\beta\ll 1} \ll T_c. \tag{33}$$

The transverse velocities of the electrons introduce a contribution of the same order into their longitudinal temperature. If for any reason the transverse temperature is higher than the cathode temperature, then its contribution to the longitudinal temperature increases. At not too low energies, other sources of spread of the longitudinal velocities become decisive: pulsations of the accelerating voltage, coherent instabilities in the intense electron beam, and, in very long cooling sections, collisions of the electrons with one another with conversion of some of the transverse momenta into longitudinal momenta (there is a tendency for the longitudinal and transverse temperatures to be equalized).

We consider separately the occurrence for a high electron current of a very appreciable potential difference within the beam and, accordingly, dependence of the electron energy on the distance from the center of the beam. This circumstance affects the cooling efficiency and may require, for example, compensation of the electron charge by light positive ions.

3. At proton energies of order 4–6 GeV and above (electron energies 2–3 MeV and above), it is no longer sensible to use the method described above for producing the electron beam, and it is necessary to go over to a closed tract, into which an electron beam is injected from an external source with a definite value of the instantaneous current and density. The magnetic field configuration must ensure cyclic motion of the electrons and sufficiently strong transverse focusing. At low energies, it is again sensible to focus by means of an accompanying longitudinal magnetic field, making the field closed and toroidal. An intense circulating beam will be heated fairly rapidly, in the first place due to coherent instabilities. To prevent excessive heating of the beam at electron energies around 10 MeV and below, it is necessary to replace the beam by a new portion of cold electrons, which is, evidently, the only possibility. Then the mean power used by the system will be reduced in proportion to the ratio of the electron circulation time in the torus to the heating time of the electron beam.

At even higher energies, it becomes possible to use radiation cooling of the electrons. One can choose the structure of the electron storage facility in such a way that at least the "single-particle" electron temperature will be fairly low. If the cooled proton beam is grouped into short bunches, then one can use electron bunches of the same length, with a corresponding gain for the mean circulating current of the electrons for the same cooling effect. In the case of a closed electron beam, the decisive factors are evidently the relationships between the frequencies of the orbital motion of the protons and the cooling electron beams and it may be possible to achieve a resonance acceleration of the cooling.

Consideration has recently been given to the possibility of using colliding proton–antiproton beams to maintain a high rate of cooling by a circulating electron beam.[7,44] Another way of introducing a mean friction at high energies is through periodic short-term lowering of the energy of the cooled particles and the use of electron cooling at a comparatively low energy. The phase space of the beam remains constant when the energy is reduced, and its emittance and size are increased but nevertheless the cooling time (for a fixed cooling current or fixed density of it) decreases rapidly, and the friction can be made more effective.

Kinetics of electron cooling

We consider in detail the simple case when an electron passes through the cooling region only once. This is currently also the most important case, since it is used for storing antiprotons and in other "first echelon" experiments; in addition, it has been the case that has been most fully investigated theoretically and has been realized and studied experimentally.

It is well known that in the case of Coulomb interaction the exchange of momentum and energy of the colliding particles diverges logarithmically in the region of large impact parameters. Therefore, in each problem one must find a $\rho_{\max}$ beyond which the interaction is effectively reduced compared with the purely Coulomb interaction. It is clear that in collisions of heavy particles with electrons in a magnetic field under conditions when

$$r_L \ll \rho_{\max} \tag{34}$$

(r_L is the Lamor radius of the electrons) an important contribution to the collision integral can be made by the region of impact distances ρ satisfying the condition

$$r_L < \rho < \rho_{\max}. \quad (35)$$

If at these impact parameters the proton velocity with respect to the Lamor circle, which is

$$\mathbf{u}_A = \mathbf{v} - \mathbf{v}_{e\parallel}, \quad (36)$$

where $\mathbf{v}$ is the proton velocity in the system moving with the electron beam, does not exceed the electron velocity $\mathbf{v}_{e\perp}$ at right angles to the magnetic field, then the protons interact not with free electrons but with the Lamor circles, since the collision duration

$$\tau = \rho/|\mathbf{u}_A| \gg r_L/|\mathbf{v}_{e\perp}| \quad (37)$$

exceeds the Lamor period of the electrons.

Because of the small spread of the electrons with respect to the longitudinal velocities (for electrostatic acceleration), there is a rapid increase in the cooling efficiency at low proton velocities. Therefore, the frictional force $\mathbf{F}$ and the momentum-diffusion tensor

$$d_{\alpha\beta} = \frac{d}{dt}\langle \Delta p_\alpha \Delta p_\beta \rangle$$

for collisions in a strong magnetic field can be represented as sums[39]:

$$\mathbf{F} = \mathbf{F}_A + \mathbf{F}_0; \quad d_{\alpha\beta} = d^A_{\alpha\beta} + d^0_{\alpha\beta}, \quad (38)$$

where the subscripts 0 and A denote the contributions of the ordinary (fast) and the adiabatic collisions. The expressions for $\mathbf{F}_0$ and $d^0_{\alpha\beta}$ are well known[8]:

$$\left.\begin{aligned} \mathbf{F}_0 &= -\frac{4\pi n_e Z^2 e^4}{m} \int L^0(u) \frac{\mathbf{u}}{u^3} f(\mathbf{v}_e)\, d^3 v_e; \\ d^0_{\alpha\beta} &= 4\pi n_e Z^2 e^4 \int L^0(u) \frac{u^2 \delta_{\alpha\beta} - u_\alpha u_\beta}{u^3} f(\mathbf{v}_e)\, d^3 v_e, \end{aligned}\right\} \quad (39)$$

where Ze is the charge of the ion moving in the electron beam, $f(\mathbf{v}_e)$ is the electron velocity distribution, and

$$L^0(u) = \ln(r_L m u^2/Ze^2) \quad (40)$$

is the Coulomb logarithm, in which the Lamor radius is taken as the maximal impact parameter and the minimal impact parameter corresponds to the largest possible deflection of an ion in a scattering:

$$\rho_{\min} = Ze^2/(mu^2). \quad (41)$$

In Ref. 39, expressions are obtained for $\mathbf{F}_A$ and $d^A_{\alpha\beta}$:

$$\left.\begin{aligned} \mathbf{F}_{\perp A} &= -\frac{2\pi Z^2 e^4}{m} \int L^A \frac{\mathbf{v}_\perp u_\parallel}{u_A^3} \frac{\partial f}{\partial v_{e\parallel}}\, d^3 v_e; \\ F_{\parallel A} &= \frac{2\pi Z^2 e^4}{m} \int L^A \frac{v_\perp^2}{u_A^3} \frac{\partial f}{\partial v_{e\parallel}}\, d^3 v_e; \\ \alpha^A_{\alpha\beta} &= 4\pi n_e Z^2 e^4 \int L^A \frac{u_A^2 \delta_{\alpha\beta} - u_{A\alpha} u_{A\beta}}{u_A^3} f(v_e)\, d^3 v_e, \end{aligned}\right\} \quad (42)$$

where

$$\left.\begin{aligned} L^A(u) &= \ln(\rho^A_{\max}/\rho^A_{\min}); \\ \rho^A_{\min} &= \max\left\{r_L; \frac{e^2}{m u_A^2}\right\}; \\ \rho^A_{\max} &= \min\left\{r_0; \frac{u_A l}{\beta c}; \frac{u_A}{\omega_0}\right\}; \end{aligned}\right\} \quad (43)$$

l is the length of the cooling section.

The rate of cooling of the oscillations is characterized by the damping rate, which is equal to the ratio of the power of the dissipative energy losses averaged over a period to the energy of the oscillations. It is simplest to find the damping rate δ_z of the oscillations perpendicular to the plane of the proton orbit in the storage ring. In this case, there is no need to take into account the effects of coupling between the transverse and longitudinal motion of the proton. Using the expressions given above for the frictional force, we can calculate the energy loss during an oscillation period, assuming, to be specific, that the electron energy distribution has a quasi-Maxwellian form:

$$f = \frac{\exp(-v^2_{\perp e}/2\Delta^2_{\perp e} - v^2_{\parallel e}/2\Delta^2_{\parallel e})}{(2\pi)^{3/2}\Delta^2_{\perp e}\Delta_{\parallel e}}, \quad (44)$$

where $\Delta_{\perp e}$ and $\Delta_{\parallel e}$ are the transverse and longitudinal spread of the electron velocities in the comoving system.

We consider first the damping of an ion beam with velocity spread $\Delta_i > \Delta_{\perp e} > \Delta_{\parallel e}$. In this case, the damping rate is

$$\delta_z = \frac{4\pi}{3}\frac{Z^2 e^4 n_e L}{mM\Delta_i^3}; \quad L = L^0(\Delta_i) + L^A(\Delta_i). \quad (45)$$

Then, once Δ_i becomes less than the transverse electron spread $\Delta_{\perp e}$, the contribution to the damping rate from the fast collisions remains constant, while the friction from the adiabatic collisions will continue to increase rapidly with decreasing Δ:

$$\delta_z = \frac{4\pi Z^2 e^4 n_e}{mM}\left[\frac{L^0(\Delta_{\perp e})}{\Delta^3_{\perp e}} + \frac{L^A(\Delta_i)}{\Delta_i^3}\right] \quad (46)$$

for $\Delta_{\perp e} > \Delta_i > \Delta_{\parallel e}$. The first term in this expression, responsible for the fast collisions, no longer depends on the ion velocity, which corresponds to constancy of the damping rate (without allowance for magnetization effects). However, because of adiabatic collisions the damping rate continues to increase as the ion velocity decreases down to velocities equal to the transverse velocities of the electrons. With further decrease in the spread of the ion velocities, the damping rate reaches the maximal value

$$\delta_z^{\max} = \frac{4\pi Z^2 e^4 n_e L^A}{mM\Delta^3_{\parallel e}}, \quad \Delta_i \leqslant \Delta_{\parallel e}, \quad (47)$$

after which the damping continues with constant damping rate until the ion temperature for all the degrees of freedom is equal to the longitudinal electron temperature:

$$T_i^{\lim} \approx T_{\parallel e}. \quad (48)$$

Under real experimental conditions, the longitudinal spread of the electron velocities may be extremely small. For example, if the electron energy is 30 keV and the cathode temperature is 0.2 eV, then it follows from (33) that

$$\Delta_{\parallel e}/\beta c \approx T_c/2E_{\text{kin}} \approx 3\cdot 10^{-6}. \quad (49)$$

Therefore, it is necessary to take into account the influence of the errors in matching the mean velocities of the electrons and protons in the cooling section.

Thus, it is possible to have a situation when there is a mean angle $\boldsymbol{\alpha}$ between the direction of motion θ_p of the protons and the direction of the magnetic field in the cooling section, along which the electrons move. We consider the situation when $\alpha \gg \Delta_{\parallel e}/(\gamma\beta c)$.

From the expression (42), we can obtain the frictional force in this case[39]:

$$\mathbf{F}_{\perp A} \approx \frac{2\pi Z^2 e^4 n_e L^A}{m(\gamma\beta c)^3 \alpha^3} [\boldsymbol{\alpha} - \boldsymbol{\theta} + 3\mathbf{n}(\mathbf{n}\boldsymbol{\theta})], \quad (50)$$

where $\mathbf{n} = \boldsymbol{\alpha}/|\boldsymbol{\alpha}|$. If $\boldsymbol{\alpha}$ is directed along the normal degree of freedom, for example, $\alpha_x = 0$, then the rate of energy loss is

$$\dot{\varepsilon}_z \sim 2\theta_z^2; \quad \dot{\varepsilon}_x \sim -\theta_x^2, \quad (51)$$

i.e., the oscillations in the direction $\boldsymbol{\alpha}$ are excited, while those in the transverse direction are damped, and the sum of the damping rates is negative. Excitation occurs under these conditions until the transverse proton velocities are equal to the mean error in the electron velocity. However, if $\boldsymbol{\alpha}(s)$ oscillates along the cooling section, passing through small values, the instability disappears, but in this case the parameter

$$\Delta v = \gamma\beta c \sqrt{\overline{\langle\alpha^2\rangle}/2} \quad (52)$$

plays the part of the longitudinal spread of the electron velocities.

A quite different effect with a cooling electron beam, relevant only for positive cooled particles (protons, ions), is the capture of electrons in bound states (recombination), this being a process that leads to the formation of fast neutral hydrogen atoms or ions with charge reduced by unity.

The main form of recombination at the moderate electron beam densities used today is radiative recombination with emission of a photon of the corresponding energy. The low-lying levels of the hydrogen atom or ion are generally populated, this being true, at least, if the influence of the accompanying magnetic field is ignored. The fast neutral hydrogen atoms formed after recombination, which retain the angular spread in the proton beam, leave the storage ring. The probability of their disintegration in the magnetic field, which bends the protons, will be small in this case if the proton energy is not too high.

The proton lifetime against radiative recombination at relative velocities low compared with the atomic velocities at the low-lying levels, which is characteristic for a cooled beam, is proportional to the temperature Δ_e of the electron velocity and decreases strongly with increasing charge of the ion:

$$\tau_{\text{rec}} \approx \frac{\gamma^2 \Delta_e}{20 \alpha r_e^2 c^2 Z^2 \eta n_e \ln(Z\alpha c/\Delta_e)}. \quad (53)$$

The recombination lifetime τ_{rec} for a proton beam usually exceeds by several orders of magnitude the damping time of small oscillations, but for heavy ions the excess becomes slight, since the cooling time is proportional to the mass of the ion. Therefore, one can cool a beam of heavy ions but one cannot keep it in the "cold" state for a long time. The use of a special device, namely, "proton cooling," i.e., cooling by a cold proton beam,[7] completely eliminates this difficulty.

At very high electron and proton densities, triple collisions may play the principal part in recombination. In this case, higher levels are mainly populated. The subsequent fate of the produced neutrals is then complicated and depends strongly on the particular conditions.

Besides recombination, which leads to the strongest restriction on the lifetime of positive ions in the case of electron cooling, particles (with any charge) can be ejected from the beam by interaction with atoms of the residual gas.

Without cooling, the main process responsible for losses is multiple Coulomb scattering on nuclei of atoms of the residual gas, which leads to a gradual increase in the amplitudes of the betatron oscillations to the maximal admissible value. If cooling is effective, this diffusion process is suppressed, and there remains single Coulomb scattering on the nuclei (the lifetime with respect to this process is several times longer than in the previous case) and nuclear interaction with the nuclei.

New effects arise when intense beams of heavy particles are cooled. In this case, even the well-known "large-current effects" in the presence of electron cooling must be reanalyzed.

Thus, at exceptionally low currents, Coulomb repulsion between the beam particles may play a part, since the steady size and energy spread of the cooled beam may be very small.[34] The influence of the interaction between the protons within the beam can be most readily observed in the change in the noise induced by the beam in electrodes surrounding it.

For example, the voltage induced in an annular pickup is proportional to the local beam density:

$$\left.\begin{aligned} \rho(\theta, t) &= \sum_{a=1}^{N} \delta(\theta - \theta_a(t)) = \sum_{n=-\infty}^{\infty} \frac{\exp[in\theta]}{2\pi} A_n(t); \\ A_n(t) &= \sum_{a=1}^{N} \exp[-in\theta_a(t)], \end{aligned}\right\} \quad (54)$$

where $\theta_a(t)$ is the azimuthal position of particle a, θ is the azimuth, N is the number of particles in the beam, and δ is a periodic δ function. If the motion of the beam particles is not correlated, it is easy to see that the amplitudes of the density harmonics satisfy

$$\overline{A_n} = 0; \quad \overline{|A_n|^2} = N. \quad (55)$$

It was shown in Ref. 37 that allowance for the interaction between the particles leads to the following expression for the amplitudes of the harmonics:

$$\overline{|A_n|^2} = \frac{N N_{\text{th}}}{N + N_{\text{th}}} = \begin{cases} N, & N \ll N_{\text{th}} \\ N_{\text{th}} = \pi R_0 (\delta\omega)^2/(e^2 \omega_s \omega_s' Z), & N \gg N_{\text{th}}, \end{cases} \quad (56)$$

where $\delta\omega$ is the spread of the revolution frequencies in the beam, $\omega_s' = d\omega_s/dp$, R_0 is the radius of the storage ring, and Z is the impedance of the chamber with respect to the beam.

It can be seen from this expression that at a small number of particles the dispersion of the amplitudes of the harmonics, i.e., the power of the noise signal, is proportional to the number of particles and does not depend on the spread; this is the so-called *shot noise*. In the opposite limiting case $N \gg N_{th}$, the noise power does not depend on the number of particles and is proportional to the beam temperature: $\delta\omega^2 \sim T_{\parallel}$ (thermal noise). Under these conditions, the noise spectrum is also significantly distorted, since fluctuations of the density propagate in the form of waves with a velocity that depends on the number of particles. Therefore, the spectrum of the noise signal consists of two peaks around the harmonics $n\omega_s$ of the revolution frequency, the distances between the peaks being

$$\Delta\omega_n = \pm n\delta\omega \sqrt{N/N_{th}} \gg n\delta\omega, \quad (57)$$

i.e., the behavior of the beam under these conditions resembles that of a solid or a liquid rather than an ideal gas. Thus, at an energy above the critical energy of the storage ring $\omega' < 0$ and at the threshold particle number $N \approx N_{th}$ the dispersions of the density harmonics becomes infinite, which corresponds to self-bunching of the beam of particles (negative-mass effect). Electron cooling makes it possible to obtain these states of a dense proton beam both below and above the critical energy.

The interaction of the particles in the transverse direction leads to a weakening of the focusing and may shift the frequencies of the betatron oscillations to dangerous "machine" resonances and in principle may limit further compression of the beam. Compensation of this shift of the betatron frequencies by rearrangement of the focusing structure of the storage facility is ineffective because of the strong dependence of the shift on the amplitudes of the oscillations within the beam. The frequency shift for a beam of particles of length l in the longitudinal direction must be less than the distance to the nearest resonance ($\Delta\nu_{max}$):

$$\Delta\nu = \frac{NR_0 r_p}{\Omega l \beta^2\gamma^3} < \Delta\nu_{max}, \quad (58)$$

where Ω is the phase space of the protons. This phenomenon limits the minimal possible phase space of the cooled beam.

Another effect manifested in an intense proton beam, especially one that is maximally cooled, is scattering of protons by other protons in the beam; this is the internal scattering effect. If the revolution frequency of the protons in the storage ring (for unchanged magnetic field) increases with increasing proton energy (such a situation is similar to the case of rectilinear motion of a proton beam with the same characteristics), internal scattering leads only to equalization of the temperatures with respect to all the degrees of freedom, this leading in the laboratory system to the appearance of a longitudinal momentum spread γ times greater than the transverse spread.

An entirely different situation arises if the revolution frequency of the protons in the storage ring (at unchanged magnetic field) decreases with increasing energy. Two protons executing, for example, radial betatron oscillations with strictly equilibrium energy can change their energies (the sum of the energies is of course conserved) discontinuously after scattering; this simultaneously excites additional radial betatron oscillations and, if the energy is above the "critical" value, the resulting betatron oscillations will on the average be greater than the initial ones. Therefore, one has not simple equalization of the temperatures with respect to all the degrees of freedom but, as it is called, "self-heating" of the proton beam, which can be suppressed only by the presence of friction, for example, electron cooling.

Experimental investigation of electron cooling

For the first realization and experimental study of electron cooling, the special proton storage facility NAP-M was developed together with a system to produce an electron beam on the cooling section with the necessary parameters.[19-26]

1. The facility NAP-M was made as a prototype antiproton storage facility, as invisaged in 1970, which explains its acronym: *n*akopitel'*a*ntiprotonov, *m*odel'. This explains the overall size of the facility and its structure with very long sections and purely edge focusing; the edges of the planar bending magnets are aligned strictly toward the symmetry center of the facility to ensure that the focusing is independent of the mean radius of the equilibrium orbit of the protons. The general scheme is shown in Fig. 1, and the basic parameters are as follows:

Energy of accelerated particles	up to 100 MeV
Injection energy	1.5 MeV
Perimeter	47 m
Number of magnets and straight sections	4 each
Radius of curvature	3 m
Length of straight sections	7.1 m
Useful aperture in bending magnets	4×7 cm^2
Frequencies of betatron oscillations ν_x	1.2
Frequencies of betatron oscillations ν_z	1.4
Duration of acceleration cycle	30 sec
Accelerating high-frequency voltage (harmonic I)	10 V
Stability of magnetic field in cooling process	1×10^{-5}
Mean pressure of residual gas (with electron beam)	5×10^{-10} torr

One of the straight sections contains the electron-beam facility, the arrangement of which is shown in Fig. 2. The main parameters of the electron cooling system are as follows:

Length of cooling section	1 m
Energy of the electrons in the experiments	up to 50 keV
Current of electrons	up to 1 A
Relative transverse velocity of the electrons	$\pm3\times10^{-3}$
Energy stability	$\pm1\times10^{-5}$
Intensity of accompanying magnetic field	1 kG

The facility has three straight sections, two of which contain the gun and the collector, while the third is the

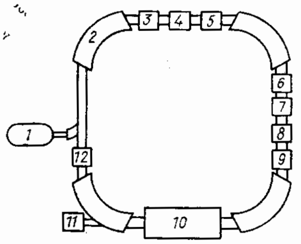

FIG. 1. Arrangement of the storage ring NAP-M: 1 is the injector, 2 is the magnet of the storage ring, 3 and 4 are horizontal and vertical plates for exciting betatron oscillations, 5 is the accelerating station, 6 is a quartz filament for measuring the beam profile, 7 and 8 are horizontal and vertical magnesium jets, 9 is a scintillation counter, 10 is the facility with the electron beam, 11 is the facility for measuring the hydrogen atoms, and 12 is a special pick-up for measuring the beam noise.

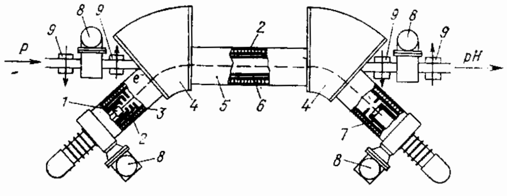

FIG. 2. Arrangement of the electron-beam facility: 1 is the electron gun, 2 is the winding of the electromagnet, 3 are the anodes of the electron gun, 4 is the section in which the electron beam is bent by the toroidal magnetic field, 5 is the cooling section, 6 is the vacuum chamber, 7 is the collector, 8 are vacuum pumps, and 9 are correcting magnets.

cooling section, in which the protons move together with the electron beam. The electron beam is formed and transported by the accompanying magnetic field, in which the electron gun is placed. The straight sections are connected by two sections with a toroidal magnetic field, which serve to inject the electrons into the cooling section and extract them from it. The centrifugal drift in the bends is eliminated by applying a transverse magnetic field, which "guides" the electrons along a trajectory whose curvature is equal to the curvature of a line of force of the longitudinal field.[14-18]

The facility employs recuperation of the electron energy, so that the power used by the high-voltage source does not exceed a few percent of the reactive power $U_e I_e$ of the beam.

The working cycle of the complex is as follows. Protons with energy 1.5 MeV are injected after one revolution into the storage ring. After 30 sec, the magnetic field intensity is raised to the necessary value. The rf accelerating system ensures that the increase in the proton energy is matched to the increase in the magnetic field. When the necessary level is reached, the increase in the magnetic field and the change in the frequency of the accelerating voltage are stopped, and the protons, which are grouped into a bunch whose length is about one quarter of the perimeter of the orbit, can "live" for many hundreds of seconds in the storage ring, being gradually ejected by scattering on the residual gas. If it is desirable to work with a continuous beam, the rf voltage is switched off. After acceleration, the electron beam is switched on and the actual process of electron cooling begins.

2. The experimental realization of electron cooling required the solution of a number of complicated technical problems. For example, in a storage ring with a vacuum chamber of length about 50 m and intense internal electron beam with active power of several kilowatts, a mean vacuum of about 5×10^{-10} torr is maintained.[22] The stability of the magnetic field and of the electron energy is 1×10^{-5}.[24] The entire process of acceleration and the transition to the cooling regime is realized automatically by means of a computer. The same computer is used to read out the experimental information, process it rapidly, and present it in a convenient form on a display for the experimentalists and on a printer for subsequent storing.[26,27]

The electron cooling experiments used various means of observation—pick-ups (integral and differential) to measure the time structure and position of the grouped beam, ferromagnetometers to measure the circulating current, "killing" probes to measure the integrated amplitude distribution of the betatron oscillations, and micron filaments, which rapidly intercept the beam and permit measurement (through detection of protons scattered through the aperture angle) of the density distribution of the proton beam with high resolution. Particularly helpful is the magnesium-jet method and detection of the fast hydrogen atoms produced by radiative recombination.

The magnesium-jet method (Fig. 3) is based on detection of the ionization electrons produced when the proton beam intersects a thin jet of magnesium vapor. The ionization electrons are accelerated and collected on a phosphor, whose luminescence is measured by a photomultiplier. The experiments used a strip jet measuring 0.5×20 mm^2 (broad side along the direction of motion of the protons) and a vapor pressure of about 10^{-8} torr. Such a jet introduces virtually no additional scattering. A horizontal jet can be displaced along the vertical (i.e., one can scan); the photomultiplier signal is then proportional to the vertical density distribution of the proton beam. Similarly, a vertical jet makes it possible to obtain information about the radial distribution of the protons. The jets can be stopped anywhere, and this makes it possible to follow the change in time of the density of the proton beam.

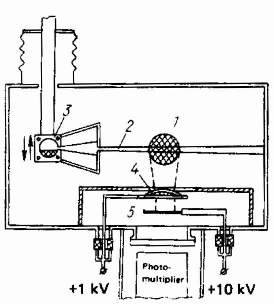

FIG. 3. Arrangement for measuring the proton beam profile: 1 is the proton beam, 2 is the magnesium vapor jet, 3 is the container with the magnesium, 4 is the collecting electrode, and 5 is the phosphor.

Observing the flux of fast hydrogen atoms, one can estimate the mean relative velocity of the protons and electrons from the total flux of these atoms [see (20)] and measure the steady size and angular spread of the proton beam with very high resolution.

3. If the electron beam is not switched on, then after acceleration the transverse dimensions of the proton beam (in other words, the amplitudes of the betatron oscillations of the protons) gradually increase through scattering on atoms of the residual gas. This is clearly revealed in the increasing width and decreasing amplitude of the secondary-electron signal obtained by scanning the transverse section of the storage-ring chamber by a thin filament (Fig. 4). The beam size increases until it reaches the aperture restrictions (mean diameter of the proton beam up to 1.5 cm). If an electron beam is then switched on with correctly chosen mean velocity, the proton beam contracts to fractions of a millimeter, the small amplitudes being damped first and only gradually the large ones.

One can measure the damping rate of the betatron oscillations and establish the corresponding functional dependences particularly conveniently by stopping a magnesium jet in the center of the proton beam and observing the density as a function of the time after applying a special deflector, which excites betatron oscillations in the cooled beam that are the same for all protons (Fig. 5). In this case, if the proton current is small, the measured cooling time does not depend on this current, i.e., the cooling proceeds completely incoherently, and the amplitude of the oscillations of each proton decreases independently. The proton damping rate measured in this way for different values of the relative velocity of the protons and electrons is shown in Fig. 6.[36] The relative velocity was produced by exciting betatron oscillations of the protons (Δv_p) or inclining the electron beam relative to the proton trajectory (Δv_e). The average electron density with allowance for the fraction of the length of the electron beam in the proton orbit $[\langle n_e\rangle = l n_e/(2\pi R_0)]$ is denoted by $\langle n_e\rangle$. In the same figure, we give the results obtained in electron cooling experiments at CERN.[69]

As can be seen in Fig. 6, the damping rate increases with decreasing relative velocity as $v_\perp^{-2}$. At cathode temperature 2000°K, the velocity of Lamor gyration is $v_{\perp e} \approx 3\times10^7$ cm/sec; the increased efficiency of the proton–electron interaction for $v_\perp < v_{\perp e}$ indicates that the protons interact with the Lamor circles.

The efficiency of the damping of the longitudinal velocity spread is characterized by the longitudinal fric-

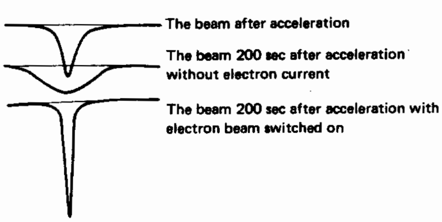

FIG. 4. Measurements of the transverse distribution of the proton beam by the quartz filament.

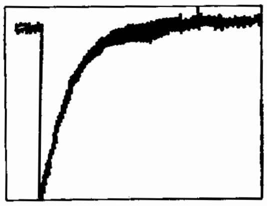

FIG. 5. Measurements of the proton-beam density by a thin jet of magnesium vapor after expansion of the proton beam. One horizontal division corresponds to 0.1 sec.

tional force. This was found by measuring the rate of radial displacement of the proton beam and by creating a discontinuity of the difference between the longitudinal velocities of the protons and electrons:

$$dx/dt = R_0\psi v_{\parallel} F_{\parallel}/(\beta^2 E).$$

The dependences of the longitudinal frictional force on the difference between the longitudinal velocities, the transverse velocities, and the velocity of Lamor gyration are shown in Fig. 7. It can be seen that the velocity of Lamor gyration begins to influence the longitudinal force only at comparatively large values.

The obtained dependences of the transverse damping rate and the longitudinal frictional force can be combined in the following empirical expressions, which are written down in the comoving system:

$$\delta = \frac{66 r_p r_e \langle n_e\rangle c^4}{[(\alpha v_0)^2 + \Delta v_\perp^2 + 11\Delta v_\parallel^2]\sqrt{v_L^2 + \Delta v_\perp^2 + \Delta v_\parallel^2}};$$

$$F_\parallel = \frac{12\pi r_e^2 \langle n_e\rangle m c^4}{\sqrt{[(\alpha v_0/2)^2 + \Delta v_\perp^2 + (\Delta v_\parallel)^2][(v_L/2)^2 + \Delta v_\perp^2 + (\Delta v_\parallel)^2]}},$$

where α is a coefficient that takes into account the distortions of the lines of force of the longitudinal magnetic field and is approximately equal to the angle between the lines of force and the equilibrium orbit of the protons, and r_e and r_p are the classical electron and proton radii. The "natural" distortions of the magnetic field at energy 65 MeV corresponded to $\alpha \approx 4\times10^{-4}$.

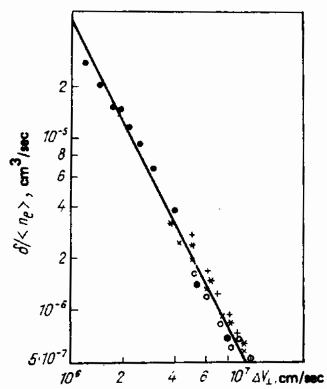

FIG. 6. Dependence of the ratio of the cooling damping rate to the orbital mean of the electron-beam density on the relative velocities of the electrons and protons. The black circles are for proton energy 1.4 MeV, electron beam current 2 mA, and relative velocity Δv_p; the pluses are for 65 MeV, 500 mA, and Δv_p; the crosses are for 65 MeV, 300 mA, and Δv_p; the open circles with pluses are for 65 MeV, 300 mA, and Δv_e; the open circles are for 65 MeV, 500 mA, and Δv_e; the stars are for 35 MeV, 100 mA, and Δv_p; and the open circles with a dot are the CERN data.

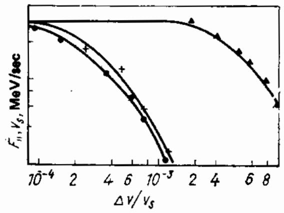

FIG. 7. Dependence of the longitudinal friction power on the difference between the proton and electron velocities. The black circles give the dependence on the difference between the longitudinal velocities, the pluses, the dependence on the transverse velocity difference, and the black triangles, the dependence on the Lamor gyration velocity of the electrons.

The range of the investigated parameters in which these expressions are confirmed experimentally is

$$10^5\ \text{cm/sec} < \Delta v_{\perp} < 7\cdot 10^6\ \text{cm/sec};$$
$$4\cdot 10^5\ \text{cm/sec} \leqslant \Delta v_{\parallel} \leqslant 2\cdot 10^7\ \text{cm/sec};$$

$$4\cdot 10^7\ \text{cm/c} \leqslant v_L \leqslant 10^8\ \text{cm/c};$$
$$10^7\ 1/\text{cm}^2 \leqslant n_e \leqslant 4.8\cdot 10^8\ 1/\text{cm}^3;$$

These expressions should be extrapolated to a wider range of parameters only with care. If the indicated region, the expressions agree with the experiments to an accuracy of not worse than 20%. The minimal cooling time, obtained at energy 65 MeV, is $\tau = 30$ msec.

4. Of great importance for the use of cooled beams is the question of the stationary limiting values of the phase density. The transverse dimensions of the proton beam were measured by two independent methods, namely, scanning with a quartz filament and detection of the fast neutral hydrogen atoms by means of nuclear emulsion or a proportional chamber, and, for large dimensions, by magnesium-jet scanning. At energy 65 MeV and up to intensities 2×10^8, significant changes in the transverse dimensions were not noted, and the beam diameter was about 0.2 mm, which corresponds to angles $\pm 2\times 10^{-5}$ in the proton beam. As can be seen from the expression (58), the intensity of the proton beam has its greatest influence on the steady size at low energy. Indeed, in the experiments at 1.5 MeV the steady diameter of the proton beam was found to increase strongly with increasing beam current (Fig. 8). It can be seen that the cross-sectional area of the beam increases in proportion to the current and a maximal shift of the frequency of the betatron oscillations in the self-field equal to $\Delta\nu \approx 0.15$ is attained.

The energy spread in a freely circulating beam was measured by the spread of the revolution frequencies. For this, measurements were made of the spectra of signals induced by the beam in pick-ups near harmonics of the revolution frequency.[38] The dependence of the power of this signal on the proton current in cooled and uncooled beams is shown in Fig. 9.

This shows that the noise power of an uncooled beam increases linearly with the current (shot noise), while the noise power of the cooled beam is two orders of magnitude less and does not depend on the current up to ~10 μA. In this region, the momentum spread obtained in accordance with the expression (56) is $\Delta p/p \approx \pm 1.4\times 10^{-6}$, and the longitudinal temperature attains the exotically small value of 1°K. If the intensity is raised, heating of the longitudinal degree of freedom at the expense of the transverse temperature as a result of proton–proton scattering begins to occur.

The transverse proton temperature at low currents reaches values $T_{\perp} \sim M\Delta v_{\perp p}^2/2 \approx 2\cdot 10^{-2}$ eV ≈ 200°K. Note that under the conditions of these experiments the transverse temperature of the electron beam was $T_{\perp e} \approx 0.2$ eV ≈ 2000°K, and the longitudinal temperature, defined formally by the expression (33), was $\sim 1.3\times 10^{-6}$ eV (0.013°K). As we have already said, it is most probable that the transverse inflection of the lines of force of the magnetic field is $\alpha \approx 4\times 10^{-4}$ and this determined the effective temperature of the electron Lamor circles:

$$T_e \sim m\Delta v_{\perp e}^2/2 \sim 5.6\cdot 10^{-3}\ \text{eV} \approx 56^\circ\text{K}.$$

As can be seen from the above estimates, the temperature of the proton beam is appreciably lower than the cathode temperature, which also indicates an appreciable change in the kinetics of heat transfer between the protons and electrons in the presence of a strong magnetic field and small longitudinal spread of the electron velocities.

Thus, electron cooling was successfully tested and experimentally investigated in the energy range from 1.5 to 85 MeV. The overall efficiency of the electron cooling can be well characterized by the ratio of the initial 6-dimensional phase space of the beam, which corresponds to an amplitude 1 cm of the betatron oscillations and an energy spread of 0.1%, to the steady phase space with amplitude 0.10 mm of the betatron oscillations and energy spread 1×10^{-5}. This gives an increase in the phase density of $1^4\times 10^{-3}/(2.5\times 10^{-2})^4 \times 10^{-5} \approx 3\times 10^8$ times(!). A further increase in the efficiency of electron cooling is evidently possible. At the least, the cooling time can be reduced (especially for large amplitudes) by several times by filling with electrons a greater fraction of the beam orbit of the heavy particles.

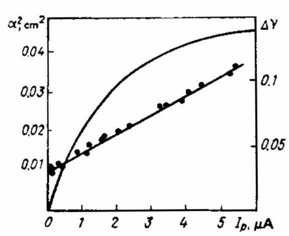

FIG. 8. Dependence on the square a^2 of the transverse radius of the proton beam on the proton current at 1.5 MeV. The continuous curve is the calculated shift of the betatron oscillation frequency.

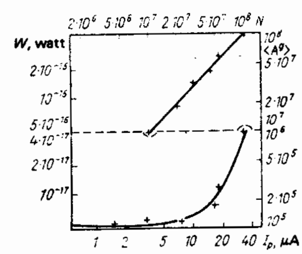

FIG. 9. Dependence of the noise power of the proton beam on the current (energy 65 MeV) before and after cooling.

4. STOCHASTIC COOLING

The method of stochastic cooling, which was proposed by van der Meer,[45] is based on the use of active feedback systems, which interact with the beam. The signal obtained in pick-ups is amplified and sent to electrodes (the kicker), which act on the beam to correct the motion of the particles. As was shown in Ref. 54, the damping of the particle deviations from the equilibrium motion is associated with the effect of fields induced by the particle and amplified by the system on the same particle (self-interaction effect). Naturally, none of the other particles of the beam give signals helpful for damping the oscillations of the given particle, and they can only introduce perturbations that limit the possibilities of damping.

The arrangement of the system for suppressing betatron oscillations of the particles is shown in Fig. 10. Let us consider the processes which take place in this system.

When the particle (with charge e) passes the differential pick-ups with effective input resistance ρ a voltage pulse

$$U = k_g e W \rho x / A \tag{59}$$

with effective duration

$$\Delta t \approx 1/W \tag{60}$$

appears at the amplifier output; here, W is the frequency bandwidth of the amplifier, k is the gain, x is the coordinate of the particle at the time it passes, and A characterizes the sensitivity of the pick-up (in order of magnitude, the aperture size).

To reduce the mutual influence of the particles, it is necessary to have a fairly short duration of the voltage pulse. One of the limiting factors is the aperture of the pick-ups. Even with a wide band of the amplifier, the effective band of the pick-up–amplifier system is limited by

$$W < v/A = \omega_s R/A, \tag{61}$$

where v is the particle velocity, R is the mean radius of the accelerator, and ω_s is the equilibrium revolution frequency.

After amplification, the voltage pulse is delayed in a cable to ensure that it arrives on the correcting plates at the same time as the particle passes. This condition also imposes certain restrictions on the amplifier band, since the different particles in the beam have different revolution frequencies, and the spread in the times of flight of the particles from the pick-up to the correcting plates must not exceed the duration of the voltage pulse; hence

$$W < \frac{\omega_s^2}{\Delta\omega} \frac{1}{\theta_p}, \tag{62}$$

where $\Delta\omega$ is the spread of the revolution frequencies, and θ_p is the azimuthal distance from the pick-ups to the correcting plates. Note that the conditions (61) and (62) are connected. Expressing the necessary aperture of the storage ring in terms of the coefficient of increase in the orbit density and the momentum spread of the beam, we can write these conditions in the form

$$W < \omega_s (\alpha \Delta p/p)^{-1}; \quad W < \frac{\omega_s}{(\alpha - 1/\gamma^2)\,\Delta p/p} \frac{1}{\theta_p}.$$

As a result of the application of the amplified signal to the particle, its transverse momentum is changed (assuming that the force is electrostatic) by

$$\Delta p_\perp = \frac{k_g e^2 \rho}{A^2} x(t-\tau), \tag{63}$$

where $\tau = \theta_p/\omega_s$ is the time of flight of an equilibrium particle and of propagation of the signal from the pick-up to the plates. If the transverse oscillations are represented in the form $x(t) = a\sin(\nu\omega_s t + \varphi)$, where φ is the initial phase of the oscillations, then the change in the square of the amplitude during one flight from electrode to the plates can be written in the form

$$\Delta(a^2) = a^2 [2\varkappa \sin\varphi \cos(\nu\omega_s\tau + \varphi) + \varkappa^2 \sin^2(\varphi - \nu\omega_s\tau)], \tag{64}$$

where

$$\varkappa = k_g e^2 \rho / (A^2 \gamma M \nu\omega_s).$$

After averaging over the phases φ, we obtain

$$\langle \Delta(a^2) \rangle = a^2 (-\varkappa \sin\nu\omega_s\tau + \varkappa^2/2). \tag{65}$$

The maximally rapid suppression of the oscillations will occur if the delay time is equal to one quarter of the period of the betatron oscillations. This result is completely natural, since it is only in such a case that the correction force will be directed counter to the velocity of the transverse motion when the particle passes the correcting plates irrespective of the phase of the oscillations.

Thus, for $\varkappa \ll 1$ and optimal $\nu\omega_s\tau = \pi/2$ we can write the single-particle damping rate in the form

$$\delta = f_0 \langle \Delta a^2 \rangle / a^2 = f_0 \varkappa = k e^2 \rho / (2\pi A^2 \gamma M \nu). \tag{66}$$

It can be seen from this expression that when the gain increases so does the damping rate of the particle oscillations.

However, the actually attainable damping rate is limited by several factors, namely, the requirement of coherent stability of the beam, screening of the signal of the given particle by the induced motion of the neighboring particles, the width of the frequency band, and the noise in the electronic feedback system. Let us estimate the restrictions imposed by these factors on the damping rate.

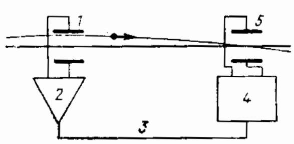

FIG. 10. Transverse stochastic cooling: 1 is the measuring pick-up, 2 is the preamplifier, 3 is the delay cable, 4 is the power amplifier, and 5 are the correcting plates.

1. The number of particles N^* which influence each other during each passage is determined by the duration of the voltage pulse induced by the particle, and

$$N^* = N\omega_s/(2\pi W) = N/n, \tag{67}$$

where N is the total number of particles in the storage ring distributed uniformly with respect to the azimuth; n is the number of harmonics of the revolution frequency in the amplification band of the feedback system.

Let us suppose that all the particles have the same revolution frequency and oscillate coherently with the same amplitude and phase. Then the signal from the pick-ups and, hence, the feedback coefficient for the coherent oscillations is increased in proportion to the number of such particles:

$$\varkappa_c = \varkappa N\omega_s/(2\pi W). \tag{68}$$

As can be seen from Eq. (65), the condition of coherent stability can be written in the form

$$\varkappa_c \leqslant 1, \tag{69}$$

which limits the gain and, hence, the single-particle damping rate:

$$\delta \leqslant W/N. \tag{70}$$

Note that in this case the coherent oscillations of the beam are damped at the maximal rate—during one revolution of the beam!

2. We now consider the influence of the remaining particles on the suppression of the oscillations of the given particle, taking into account the spread of the revolution frequencies, which leads to a finite interaction time t_{ik} of particle pairs:

$$t_{ik} \approx \omega_s/\Delta\omega W. \tag{71}$$

During this time, the signal of the particle gives rise to an induced motion of all the remaining particles in the region of influence, the motion having the opposite phase to that of the given particle; this can be expressed in the form

$$x^*(t) = -\varkappa x(t)\,\omega_s t_{ik}/4\pi. \tag{72}$$

An appreciable weakening (screening) of the particle's signal occurs when the subtracted signals induced by the given particle and all the remainder are equal:

$$x(t) \approx x^*(t)\,N^*. \tag{73}$$

From this condition, we can again write down a restriction on the damping rate in the form

$$\delta \leqslant \frac{\Delta\omega 2\pi W}{\omega_s^2}\frac{W}{N}. \tag{74}$$

This expression differs from (70) in the additional factor

$$G = \Delta\omega 2\pi W/\omega_s^2 = T_0/t_{ik}. \tag{75}$$

The quantity G^{-1} is equal to the number of revolutions during which the particles effectively interact with one another. By virtue of the restrictions (61) and (62) on the maximally possible band W, the value of G is always less than unity but under limiting conditions can be close to it. However, during the process of cooling and decrease in the spread $\Delta\omega$ there is also a decrease in G and, hence, a decrease in the damping rate. Thus, the maximal damping rate may approach the values (70) as determined by the requirement of coherent stability only during the initial stage of the cooling.

In obtaining the restriction (74) on the damping rate, we took into account only the induced motion of the remaining beam particles under the influence of the given particle. The proper motion of the remaining particles also changes the oscillation amplitude of the considered particle in accordance with (62).

The equation of motion for the square of the oscillation amplitude can be written in this case in the form

$$da_k^2/dt = -\varkappa f_0 a_k^2 + \varkappa^2 f_0 \langle a^2\rangle N^* \frac{\omega_s^2}{4\pi\Delta\omega W}. \tag{76}$$

As can be seen from this equation, the maximal value of the damping rate corresponds to (75); we note only that in this case diffusion can heat the particles that have small amplitudes to the mean-square values

$$da_k^2/dt = -\delta(a_k^2 - \langle a^2\rangle/2). \tag{77}$$

3. We now estimate the influence of thermal noise of the system on the stationary beam size. If it is assumed that the amplifier noise is determined solely by the thermal vibrations at the input and that the amplifying elements do not add noise, then in our model the voltage fluctuations at the input are

$$\langle \Delta U\rangle^2 \approx 4kT\rho W, \tag{78}$$

where k is Boltzmann's constant, and T is the temperature. Writing the equation for the steady oscillation amplitude in the form

$$d\langle a^2\rangle/dt = -\delta\langle a^2\rangle + \left(\frac{d\langle\Delta a^2\rangle}{dt}\right)_D = 0,$$

where

$$\left(\frac{d\langle\Delta a^2\rangle}{dt}\right) \approx \left(\frac{k_g \Delta U_r}{\gamma M\omega_s v W}\right)^2 f_0 \tag{79}$$

and using the value of $\langle\Delta U^2\rangle$ from the expression (78), we obtain the steady value of the oscillation amplitude:

$$\frac{\langle a^2\rangle}{R^2} \approx \frac{kT}{v\gamma M v^2}\frac{k_g}{n}.$$

It can be seen from this expression that when the gain k_g is increased the steady size increases despite the proportional growth of the damping rate δ (the noise power at the output increases as k_g^2).

To increase the damping rate without increasing the steady size of the beam, one can use many of the systems described above working in parallel. In this case, both the damping rate and noise power increase in proportion to the number of systems, which makes it possible to accelerate the cooling without deterioration in the parameters of the cooled beam. One can also use one fairly powerful amplifier which receives signals from many pick-ups and returns the amplified signal to many correcting elements.[55]

We give numerical estimates of the possible cooling rates for a proton or antiproton storage ring with radius $R = 20$ m for $\nu \approx 2$, particle momentum $\gamma M v = 1$ GeV/c, aperture $A = 10$ cm, wave resistance $\rho = 50\ \Omega$ of the pick-ups, and amplifier band $W = 500$ MHz. If the beam is cooled to diameter 2 cm ($a = 1$ cm), then from the expression (79) we obtain a restriction on the maximal possible gain, $k_g \leq 3.5\times10^6$, and hence the cooling time will be $\tau = \delta^{-1} = 140$ sec. If we use such a system to store secondary particles, the cooling time must be shorter than the working cycle of the accelerator, which is usually several seconds. To obtain such rapid cooling, it is necessary, as can be seen from our estimate, to use about 100 pick-ups to reduce the contribution of the thermal noise. The limiting number of particles that does not violate the condition $N \lesssim \tau W$ (70) of coherent stability is about 10^9 for the chosen numerical example and $\tau \approx 2$ sec. We complete our example by estimating the part played by the mutual influence of the particles. If the momentum spread of the particle is $\Delta p/p \approx 10^{-3}$, which is characteristic for the final stages in the cooling, then the spread of the revolution frequencies for the chosen parameters is

$$\Delta\omega \approx \omega_s (\alpha - 1/\gamma^2)\, \Delta p/p.$$

Here, α is the coefficient of increase of the orbit density and is equal to $1/\nu^2 = 0.25$.

As a result, the particles under these conditions will influence one another for $G^{-1} \approx 14$ revolutions [Eq. (75)], which imposes additional restrictions on the cooling (75) and (76). The number of particles that can be cooled during $\tau \approx 2$ sec must be less than 10^8.

The power of an amplifier needed for stochastic cooling is determined by the thermal noise and the signals from the particles and has the form

$$P = (4kT + Ne^2\omega_s\rho \langle a^2\rangle/A^2)\, W k_g^2.$$

An amplifier with gain $k_g = 3.5\times10^6$ will have a noise power ~100 W at the output, and to achieve a cooling time of ~1 sec it is necessary to have about 100 such amplifiers or one with power ~10 kW.

In principle, the same system can be used for longitudinal cooling, the deflecting plates being merely replaced by a wide-band accelerating element. In this case, the pick-ups measure the radial deviation of the particles associated with the momentum deviation. If the accelerating gap is placed at a position in the orbit where the value of the dispersion function is zero, radial betatron oscillations will not be excited when the signals are applied to the beam. An interesting scheme of longitudinal damping was proposed by Torndal.[55] It is based on the use of a special filter in the form of a short-circuit cable, as shown in Fig. 11. The signal at the output of such a filter is

$$U_{\text{out}}(t) \approx U(t) - U(t - T),$$

where $U(t)$ is the signal at the filter input, i.e., the system realizes subtraction of the signal which is reflected from the short-circuited end and arrives at the amplifier input after the signal propagation time through the cable, which is chosen to be equal to the revolution period of an equilibrium particle. The measuring pick-up (for example, a ferrite ring surrounding the beam) sends a voltage pulse when the particle passes. If the frequency characteristic of the pick-up and the amplifier are chosen so as to realize differentiation of the signal, the pulses at the output of the amplifier will be proportional to the deviation of the revolution frequency of the particle from the equilibrium value.

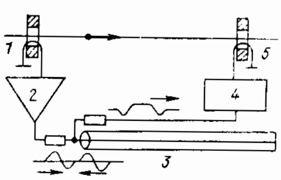

FIG. 11. Stochastic cooling of the spread of the longitudinal momenta on the basis of a filter: 1 is the measuring pick-up, 2 is the preamplifier, 3 is the filter in the form of a short-circuiting cable, 4 is the power amplifier, and 5 is the accelerating gap.

The effect of the thermal noise of the preamplifier on the beam will have a quite different nature from that in the system considered earlier. The noise voltage $U_n(t)$, which changes the particle energy, is again at the accelerating gap after the revolution time with the opposite sign, $-U_n(t)$, and exactly compensates the discontinuity of the energy of an equilibrium particle. The rate of heating of the nonequilibrium particles depends on the accuracy with which the particle arrives after a revolution ($\Delta t_i = \Delta\omega T_s/\omega_s$) and is proportional to $(\Delta U)^2 \approx U_n^2 (W T_s/\omega_s)^2 (\Delta\omega)^2$, i.e., depends on the spread $\Delta\omega$ in the same way as the friction power. This has the consequence that the cooling damping rate is decreased, but the damping can continue to small spreads, which are restricted only by the interaction between the particles. The noise heating increases as k_g^2, and the friction power only as k_g, so that in the case under consideration the noise limits the damping rate.

For a more detailed comparison of the possibilities of longitudinal cooling based on a filter and based on direct measurement by a pick-up of the energy deviation, we give the values of the single-particle damping rate and the restrictions on the damping rate associated with the thermal noise:

$$\left.\begin{array}{l} \delta \approx r_p \rho W^2 k_g \eta \dfrac{1}{\beta^2\gamma} < f_0 \dfrac{Mc^2}{kT} r_p \rho W \quad \text{(filter method)}, \\ \delta \approx r_p \rho W \dfrac{R_0 \psi_p}{A} \omega_s k_g \dfrac{1}{\beta^2\gamma} < f_0 \dfrac{Mc^2}{kT} r_p \rho W \left(\dfrac{\Delta p}{p} \dfrac{\psi_p R_0}{A}\right)^2 \\ \text{(direct method)}, \end{array}\right\} \qquad (80)$$

where ψ_p is the value of ψ in the region of the pick-ups.

As can be seen from these expressions, the restrictions on the damping rate at the start of cooling, $(R_0 \psi_p (\Delta p/p)_i / A) \approx 1$, are approximately the same, but during the process of cooling to the required $(\Delta p/p)_f \ll (\Delta p/p)_i$ the direct method gains significantly. Note that if amplifiers with a band appreciably less than the maximal possible as given by the expression (61) are used, the filter method requires an appreciably greater gain.

The main factors that limit the maximal possible damping rates have the same nature for longitudinal as for transverse cooling, and therefore we shall not repeat them. We note only that during the process of cooling of the momentum spread the interaction between the beam particles increases, and the damping may have a nonexponential nature:

$$d\Delta\omega/dt \approx -\delta\Delta\omega \approx -(\Delta\omega)^2 n^2/N, \quad (81)$$

and, therefore, $\Delta\omega(t) \approx \Delta\omega_0/(1+\Delta\omega_0 n^2 t/N)$.

Stochastic cooling experiments were made at CERN.[48,52,55] The particle cooling rate was investigated in a wide range of beam intensities from several hundred particles to 10^{10}. The dependence of the cooling time τ on the beam intensity obtained in the ICE experiment is shown in Fig. 12.[55] One can see clearly that the limiting cooling time is restricted by the interaction between the particles discussed above. The restrictions at low particle number $\lesssim 8\times10^7$ were due to electronic noise, and the minimal cooling time was about 20 sec. At beam intensity $\gtrsim 10^8$, the main restriction was the interaction between the particles. The cooling time for amplifier frequency band $W = 170$ MHz and spread $\Delta p/p \approx 10^{-3}$ was about 500 sec ($N = 1.2\times10^9$).

Finally, we list the main features of stochastic cooling of intense beams. In this case, the cooling efficiency is inversely proportional to the number of particles and proportional to the spread of the revolution frequencies in the beam, i.e., stochastic cooling is most effective for particles with large spread of the revolution frequencies. It is impossible to obtain high phase densities in this method of cooling because of the strong mutual influence of the particles. Thus, in the experiments with electron cooling densities were obtained at which the interaction between the particles (even without amplifiers and merely by the fields in the chamber) led to an appreciable distortion of the noise spectrum. The interaction of the particles within the beam led to a strong decrease in the signals and shifted the spectrum by an amount greater than the intrinsic spread of the particles in the storage ring. Under these conditions, the signal from a particle induced in the pick-ups does not correspond to the motion of the particle and, therefore, cannot give rise to damping. At the same time, electron cooling for particles with large velocity spread is not so effective and encounters definte technical difficulties, so that in a certain sense these two cooling methods complement each other. Figure 13 shows (very approximately) the regions in

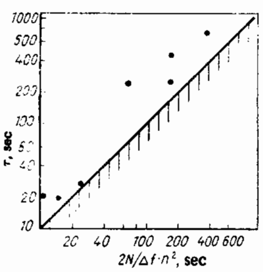

FIG. 12. Cooling time as a function of the proton-beam intensity.

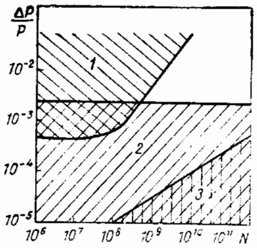

FIG. 13. Approximate regions of the use of stochastic and electron cooling of beams with cooling time 1 sec: 1 is the region of effective stochastic cooling, 2 is the region of electron cooling, and 3 is the region in which intrabeam interaction becomes important.

which one can obtain damping in around 1 sec with an amplifier band of 1 GHz and 100 pick-ups for a storage ring with parameters $R = 20$, $(p/\omega_s)(d\omega/dp) = 0.1$, and $\gamma = 2$. It can be seen from the figure that the developed methods make it possible to cool beams of heavy particles in a wide range of both the beam intensity and beam spread parameters, and in the near future this will make it possible to perform qualitatively new experiments.

5. SOME EXAMPLES OF THE USE OF THE METHODS OF COOLING PARTICLES

It has become very helpful to use cooling of heavy-particle beams to make experiments in the regime of superthin internal targets (see Sec. 1).[63] The steady spread of the transverse and longitudinal momentum in the particle beam (which is often very important in experiments) is determined by the relationship between the rate of diffusion due to collisions with target atoms and the cooling rate. An important feature of electron cooling in the given case is the growth of the friction power as $\delta\Delta p^2$ with decreasing spread Δp. If the diffusion on the target does not depend on the spread ($\Delta p/p \ll 1$), and if the cooling power exceeds the heating power (diffusion), the beam is cooled to very small spreads, which are determined by entirely different diffusion sources, for example, intrabeam scattering. For the stochastic and radiation methods, the cooling power decreases with decreasing Δp, and one therefore obtains a well-defined value of the spread at a cooling power comparable with the heating power. This property of electron cooling makes it possible to have a very small spread in the beam in the case of a sufficiently dense target ($\Delta p/p \approx 10^{-5}-10^{-6}$).

The luminosity in experiments with superthin targets is determined by the rate of accumulation $\dot{N}$ of the particles and the cross section of their destruction on the target: $\sigma\mathscr{L} = \dot{N}/\sigma$. At energies of strongly interacting particles of order 1 GeV and above, the decisive interaction is the nuclear, with a cross section of about 10^{-25} cm^2. Therefore, at an antiproton accumulation rate of $\sim10^8\bar{p}$/sec, the luminosity reaches 10^{33} cm$^{-2}\cdot$sec^{-1}. The number of particles in the storage ring is determined by the particle lifetime, i.e., by the target density (n particles/cm^2): $N = \dot{N}/f_0 n\sigma$, where f_0 is the revolution frequency of the particles. If one is working with gas targets, $n > 10^{15}$, the particle number is not too large, but in the case of a polarized gas target with $n \approx 10^{13} p/\text{cm}^2$ and revolution frequency $f_0 = 6\times10^6$ the number of accumulated antiprotons for achieving the indicated luminosity reaches the appreciable value $N \approx 10^{13}\bar{p}$.

Experiments with superthin internal targets are most effective when one uses either rare particles (for example, antiprotons or antideuterons) or nondense targets which cannot be made dense for physical or technical reasons, i.e., targets such as free neutrons, vapors of elements used to detect fission fragments, or another beam of particles. In the last case, the transparency of the target is compensated by the accumulation of a large number of particles, so that it will clearly be difficult to use stochastic cooling, which has restrictions associated with a large number of particles. For rare particles and a dense target, when the number of accumulated particles is not too high, $N\approx 10^9$, stochastic cooling makes it possible to perform such experiments at virtually any energy accessible in the storage facility.

The methods of cooling heavy particles developed in recent years open up interesting prospects of experiments with accumulated ion beams, especially with ions, primarily heavy, that are difficult to obtain. At a high degree of monochromaticity of an ion beam in a storage facility after cooling there is a natural separation of even the isotopes with nearly equal masses, which makes it possible to work with the rarest isotopes under absolutely pure conditions. Cooling makes it possible to use as target a second beam of ions from which the electron shells have also been removed. In particular, reactions with the emission of nuclear γ rays can be performed with absolutely no background, which may be helpful in studying quantum-electrodynamical phenomena in the critical and "supercritical" electric fields of superheavy compound nuclei. In the case of electron cooling, a complicated circumstance when one is working with heavy ions is the accelerated loss of ions due to recombination with the cooling electrons. If the number of accumulated ions is fairly large and it is difficult to use stochastic cooling, it is possible to use a proton beam cooled, in its turn, by an electron beam to cool the ions.[64,7]

Colliding proton–antiproton beams

One of the most interesting and imminent applications of the developed methods of cooling heavy particles is in performing experiments with colliding proton–antiproton beams (Refs. 56–62, 66, and 67). More and more attention is being paid to the possibility of colliding beams in the proton accelerators with maximally high energies currently in operation or under construction, the creation of the antiproton storage facility requiring comparatively modest expenditure. In fact, all the large physics centers with high-energy accelerators are working in this direction. The most intensive work on the creation of an antiproton source is being done at CERN.[67]

The luminosity of the colliding beams is determined by the particle numbers N_p and $N_{\bar{p}}$, the area s of the beam sections at the collision points, and the revolution frequency f in the storage ring:

$$\mathscr{L}=fN_pN_{\bar{p}}/s. \tag{82}$$

The maximal number of particles at which betatron oscillations are not excited because of the nonlinearity of the frequency shift in the colliding beams can be written in the form

$$N_{\max}\approx\gamma\Delta\nu\Omega/r_p, \tag{83}$$

where Ω is the one-dimensional transverse phase space of the storage ring, γ is the relativistic factor, and $\Delta\nu$ is the admissible frequency shift of the oscillations. This varies from 0.1 to 0.001, depending on the damping time or the existence of the beams, the chosen magnetic system, and the precision of its adjustment. If cooling is used, the beam dimensions will be determined by the collision effects, and it is therefore sensible to work with equal particle numbers, $N_p=N_{\bar{p}}=N$, and equal phase spaces. Accordingly, the maximal luminosity of the storage ring is

$$\mathscr{L}_{\max}=\frac{\gamma^2(\Delta\nu)^2 f}{r^2 p}\frac{\Omega}{\beta_0}, \tag{84}$$

where β_0 is the value of the β function of the storage ring at the collision point. For a storage ring with phase space of the beam $\Omega\approx 10^{-4}$ cm·rad, $f\approx 5\times 10^4$, $\gamma=1000$, $\Delta\nu=10^{-2}$, and sufficiently strong focusing $\beta_0=1$ m at the collision points, the maximal possible total luminosity reaches 2.5×10^{32} cm$^{-2}\cdot$sec^{-1}. Note that at this luminosity the total number of reactions taking place due to the nuclear reaction reaches $dN/dt=\mathscr{L}\sigma_{\rm tot}\approx 10^7$ particles/sec, and the total number of particles in the beams is $N=6\times 10^{12}$. Note that if fewer particles are used the luminosity decreases linearly, since the use of cooling makes it possible to decrease the phase space of the beams in proportion to the limit determined by the admissible shift $\Delta\nu$, which partly compensates the loss of luminosity.

The lifetime of the particle beams also determines the necessary rate of accumulation of antiprotons to sustain the luminosity. At high energies, the losses are due mainly to the nuclear interaction, and the lifetime can be written in the form

$$\tau=1/[(\mathscr{L}/N+n_0c)\sigma_{\rm tot}], \tag{85}$$

where n_0 is the mean density of the nucleons of the residual gas in the chamber, and $\sigma_{\rm tot}$ is the nuclear interaction cross section. For the chosen numerical example and at pressure $p\leq 10^{-8}$ torr(H_2) in the storage ring, the decisive effect will be the destruction of the beam by interaction with the colliding beam, and the lifetime is about 2 days. Hence, the necessary rate of accumulation of antiprotons must be $\dot{N}_{\bar{p}}\approx 10^7\bar{p}$/sec.

The antiproton source is a target irradiated with a proton beam from an accelerator. In the center-of-mass system of the colliding nucleons, the produced antiprotons have low kinetic energies of the order of the rest energy of the pions which are the bulk of the produced particles. Then in the laboratory system the spectrum of the antiprotons emitted from the target will peak at the energy $E_{\rm m}=\sqrt{M_{\rm p}c^2E_{\rm in}}$ and the momenta at $p_\perp\approx\sqrt{2M_\pi M_{\rm p}c^2}$, i.e., the production angles of antiprotons with momentum p are

$$\langle\theta^2_{\bar{p}}\rangle\approx 2M_\pi M_p c^2/p^2. \tag{86}$$

Accordingly, the effective phase space of the antiprotons (one-dimensional) for a small size of the initial proton beam is determined by the length l_c of the converter target and is equal to

$$\Omega_{\bar{p}} = \pi l_c \langle\theta^2\rangle/2 = \pi l_c M_p M_\pi c^2/p^2. \qquad (87)$$

The length of the converter is determined by the efficiency of the interaction of the protons with the material of the target and the absorption of the already created antiprotons and, as a rule, is usually about half the nuclear absorption length of the material of the target. For tungsten, this is about 4 cm.

Thus, the ratio of the useful number of antiprotons in the phase space Ω of the storage ring to the total number of incident protons can be written in the form

$$k = \frac{N_{\bar{p}}}{N_p} = \begin{cases} Fn(p, p_0)\,\dfrac{\Omega}{\Omega_{\bar{p}}}\,\dfrac{\Delta p}{p} & \text{for } \Omega \ll \Omega_{\bar{p}}; \\ Fn(p, p_0)\,\dfrac{\Delta p}{p} & \text{for } \Omega \gg \Omega_{\bar{p}}, \end{cases}$$

where $n(p, p_0)$ is the momentum density distribution of the antiprotons, p is the antiproton momentum, p_0 is the proton momentum, $\Delta p/p$ is the interval of momenta trapped in the storage ring, and F is the efficiency of the target (it takes into account absorption, scattering, etc.). Thus, in the case of conversion of protons with energy 70 GeV into antiprotons with momentum 5 GeV/c the rms angle of spread of the antiprotons will be $\Delta\theta_p \approx 0.1$, the phase space of the conversion $\Omega_p = 6.6\times10^{-2}$ cm·rad, and the momentum density distribution $n(p, p_0) = 10^{-2}$. The phase space of a strongly focusing storage ring with chamber half-aperture equal to 5 cm and mean β function of 10 m is $\Omega = 7.8\times10^{-2}$ cm·rad.

Thus, it is possible to inject created antiprotons without losses due to the transverse phase space. There is a certain technical difficulty in creating lenses with sufficiently short focal lengths in order to match the phase space of the antiprotons to the phase space of the storage ring. Pulsed lithium lenses have now been developed to solve this problem.[58,61] As can be seen from the above example, conversion of $N = 10^{13}$ protons can yield in the storage ring up to $4\times10^9\,\bar{p}$ with momentum spread $\Delta p/p \approx \pm 2\times10^{-2}$ and phase space $\Omega_{\bar{p}} = 6\times10^{-2}$ cm·rad.

However, the cooling of such a large phase space at high energy and comparatively high intensity encounters certain technical difficulties if stochastic or electron cooling is used. Therefore, the modern plans for antiproton sources will give a significantly lower antiproton yield in the first stage.

In the CERN project,[67] which is based on stochastic cooling, two systems of cooling are used to reduce the restrictions associated with the large number of accumulated particles. One of them, the "pre-cooler," reduces the momentum spread of the freshly injected antiprotons, and then these protons are delivered by acceleration to a storage region within the common chamber of the storage ring where the actual storing is done. The main difficulty is evidently associated with ensuring coherent stability of the accumulated portion of the particles in the presence of fairly frequent additions of new portions, which must be cooled in the period between injections. As we discussed above, the limiting number of particles is restricted by the frequency band W of the cooling system and the required cooling time $\tau (N < \tau W)$, which for a band $W \approx 10^9$ Hz and $\tau \approx 2$ sec significantly restricts the accumulation possibilities ($N < 2\times10^9\,\bar{p}$). In the project, it is intended to use a specially introduced, essentially nonlinear dependence of the signal induced by the particles deflected from the equilibrium momentum value in order to lift this restriction. The sensitivity of the pick-ups is chosen in such a way that the particles near the equilibrium orbit give much smaller signals than for a linear system. It is then to be expected that they will not restrict the coherent stability so strongly. However, diffusion from the freshly injected particles and the thermal noise of the cooling system with reduced cooling damping rate in the region of accumulation leads to a broadening of the momentum distribution of the particles (a flatter distribution), which may lead to serious problems.

At Fermilab,[66] a combined system of storing antiprotons has been selected. It is intended to cool the freshly injected antiprotons in the longitudinal direction using stochastic cooling during the time of deceleration of the antiprotons to an energy convenient for electron cooling and storing. In this system, the antiprotons are accumulated in a separate storage facility with electron cooling at antiproton energy 200 MeV with fairly modest and technically readily attainable electron beam parameters.

In the design of the antiproton source proposed by the Institute of Nuclear Physics, Siberian Branch of the USSR Academy of Sciences, for the accelerator and storage complex UNK (Institute of High Energy Physics, Serpukhov) it is intended to use only electron cooling

TABLE I. Parameters of antiproton sources in various projects (stage I).

Center	CERN	Fermilab	Serpukhov
Initial energy of proton beam, GeV	26	80	70
Number of protons per cycle	10^{13}	$1.8\cdot10^{13}$	$5\cdot10^{13}$
Cycle time, sec	2.6	8	7
Antiproton injection acceptance, mm · mrad	100	4.8	60
Momentum spread	$\pm7.5\cdot10^{-3}$	$\pm2\cdot10^{-2}$	$\pm3.2\cdot10^{-2}$
Antiproton momentum at injection, GeV/c	3.57	4.5	5
Number of antiprotons per cycle	$2.5\cdot10^{7}$	$5.7\cdot10^{7}$	$2.5\cdot10^{8}$
Cooling energy, GeV	Stochastic, 2.75	Stochastic, three steps, 4.25, 2.2, and 1.2	—
Cooling in storage ring	Stochastic, 2.6	Electron	Electron
Energy in antiproton storage ring, GeV		0.2	0.4
Energy of electron beam, keV	—	110	220
Electron beam current, A	—	25	47
Length of cooling section, m	—	7	200
Diameter of electron beam, cm	—	5	20
Density of electron beam, A/cm²	—	0.3	0.15
Antiproton accumulation rate, $\bar{p}$/sec	$0.9\cdot10^{7}$	$0.7\cdot10^{7}$	$3.6\cdot10^{7}$

in combination with sharp monochromatization of the beam after injection with lengthening of the initially short antiproton bunches (in the initial stage, it is intended to use only 1/7 of the accelerated protons). In Table I we give the parameters of these antiproton-source projects. These projects ensure a rate of accumulation of antiprotons sufficient to give an adequate luminosity in colliding proton–antiproton experiments.

Further improvements in the storage systems, in particular the methods of cooling of beams of heavy particles, will evidently make it possible to raise the rate of accumulation appreciably. As can be seen from the above estimate, it is perfectly realistic to achieve an accumulation rate of order $10^9\ \bar{p}$/sec. In the case of stochastic cooling, this involves the development of powerful (10 kW) amplifiers with a band greater than 1 kHz, and in the case of electron cooling it requires the creation of 2-MeV electron beams with currents 1 kA (in the recuperation regime!). In principle, these parameters are not too fantastic, although they appear rather difficult at present, so that there is hope of achieving significantly higher rates of accumulation of antiprotons.

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Translated by Julian B. Barbour