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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH
Section A

Theoretical investigation of a flowing heavy metal target for an accelerator-driven neutron source

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The possibility to design a flowing heavy metal target with free surface under medium energy proton beam irradiation within the 10–320 mA current range has been analyzed on the basis of experimental data on longitudinal and radial heat deposition and the solution of the hydrodynamic equation system. Some parameters of the lead–bismuth targets for this current range are given.

1. Introduction

The use of a high-current radio-frequency proton accelerator within the 0.8–1.6 GeV energy range for transmutation of the long-lived radioactive nuclei of nuclear waste into stable or short-lived species has been under consideration in Ref. [1]. The average beam current for a proton accelerator of such a system must be within the 25–250 mA range. This neutron production-transmutation system consists of the proton beam accelerator, a lead target for neutron production, and a blanket surrounding it containing primarily heavy water (D_2O) to moderate neutrons into the thermal range.

The target itself is a flowing liquid heavy metal. The proton beam strikes the target from above in a cylindrical spot producing neutrons which inelastically scatter in the lead before entering the blanket. Multiplication in the target is possible for neutrons with energy above the (n, 2n) threshold. About 60% of the beam power is actually deposited as a heat in the target [2]. The remainder is spent on binding energy during neutron production in inelastic reactions.

The heat load on the target is limited only by the thermal properties of the liquid metal and the flow rate. A 1 m/s flow would handle a 1.6 GeV beam current of 30 mA in the case that the liquid Pb enters at the top at a temperature of about 350°C and exits from the bottom at about 450°C [1]. The diameter of the target is 0.5 m. The temperature of 350°C at the surface of the liquid target will give rise to a substantial evolution of volatile elements into the accelerator vacuum space. The liquid heavy metal target therefore is arranged so that the beam strikes the target vertically from above [3].

In this paper we examine, apparently for the first time and mainly from the hydrodynamics point of view, the possibility to design a liquid heavy metal target with free upper surface under 0.8 GeV proton beam irradiation within the 10–320 mA current range.

2. Physical notions and the target design

The effectiveness and the design features of a flowing heavy metal target are limited by the flowing medium composition and by the hydrodynamics laws.

We investigate in this paper the liquid target where flowing medium is the lead–bismuth eutectic for several reasons. First, the element Bi is a fissile material under the hadron irradiation energy above 60 MeV. This phenomenon increases the overall leakage of the neutrons from the lead–bismuth target surface. Second, the lead–bismuth eutectic melting point is equal to 127°C. If the liquid eutectic enters from the top at a temperature of about 275°C and exits from the bottom at about 475°C, a Pb–Bi target is preferred to lead one from the technical viewpoint. The liquid lead–bismuth eutectic would be continuously recirculated and spallation products would be allowed to accumulate in the target. In addition to the spallation products which have been under consideration in Ref. [1], the element Po will be accumulated in the Pb–Bi medium of the target.

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Now, let us consider the longitudinal and radial heat deposition in a lead–bismuth medium under the 0.8 GeV proton beam effect. As it follows from Ref. [4] about 50% of the beam power is actually deposited as the heat in the target up to the z'_{stop}-point. The decrease in the heat deposition value behind this point is about two orders of magnitude. The main part of the heat is the ionization losses. About 4% of the beam power is lost due to the albedo leakage from the irradiation surface of the target. The laterally integrated heat deposition would be analytically approximated in the form:

$$q(z') = \begin{cases} a_1 \exp\left[-b_1(z'-z'_{max})^2\right], & \text{for } 0 \leqslant z' < z'_{exp}, \\ a_2 \exp[-b_2 z'], & \text{for } z'_{exp} \leqslant z' < z'_{stop}. \end{cases}$$

The z'-axis is the target depth. The coefficients z'_{max} and z'_{stop} of this approximation for the medium under consideration are equal to 0.03 m and 0.4 m respectively. Another coefficients are given in Ref. [5]. The radial heat deposition $q(r')$ for thickless proton beam at z'_{max}-point is shown in the Fig. 1 in logarithmic scale (it was simulated using the MARS93 code [6]). The decrease in the radial energy deposition value is more than two orders of magnitude at the 0.01 m distance from the beam. The conclusion follows from these data that the outer boundary of irradiation spot is about 0.01 m away from the target walls and the total target length is not less than 0.4 m. High energy neutrons fly downwards [7] to the target bottom where the neutron reflector must be arranged to return the neutrons into blanket.

We shall consider the axisymmetric flowing heavy metal target with upper free surface if the flow can be rotated around z-axis to be stabilized. It must be noted that in some cases the flow with free surface is unstable [8] (in particular, the need of stabilization limits the minimal level of the flow velocity at the target's top). The pressure p inside the target must be as low as possible, but pipelining for heavy liquid metal can be realized only for a high pressure p_{pl} in the pipeline. Therefore the liquid metal must be moved from the outer pressure pipe to the top of the working target volume through the hydrodynamic resistance device. The liquid consumption through this device is determined by the Darcy's low: $Q = C_D(p_{pl} - p)$, where C_D is the Darcy's constant. The Reynolds number $Re = VL/\nu$ inside the target is about 10^7 for the conditions under consideration ($V \simeq 1$ m/s; $L \simeq 1$ m and $\nu \simeq 10^{-7}$ m^2/s). But the turbulent Reynolds number $Re_t = VL/\nu_t$ can be more less. It can be estimated from the following formulas (see Ref. [9]):

$$\nu_t = 0.09k^2/\varepsilon, \quad k = \langle V_i' V_i' \rangle /2 \sim V_R^2,$$

$$\varepsilon = \nu < (\partial V_i' / \partial x_k)^2 > \sim \nu V_R^2 l_R^{-2},$$

where the average flow velocity and the space scale of the turbulent pulsations l_R are given at the hydrodynamic resistance outlet; another symbols have their usual in meaning as in turbulent theory. In the present paper we shall assume that the resistance device should be designed in such a way that $Re_t \gg 1$. Moreover the turbulent Prandtl number is equal to $Pr_t \approx 1.2$ for liquid metal medium. Therefore we shall solve the hydrodynamic equations system for the nonviscous and non-heat conductive heavy metal flow. A temperature rise in the target is assumed to

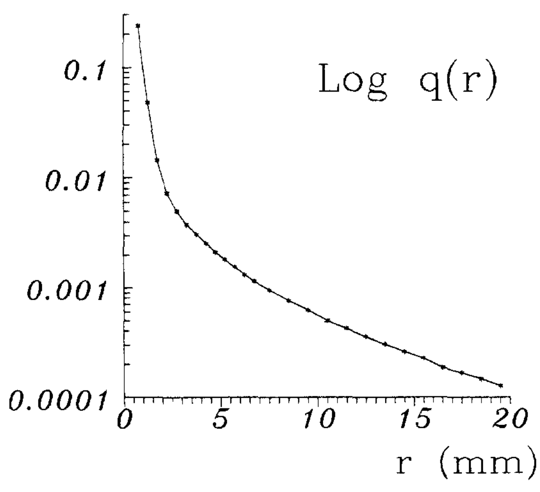

Fig. 1. The radial heat deposition in the lead–bismuth medium under the thickless proton beam operation.

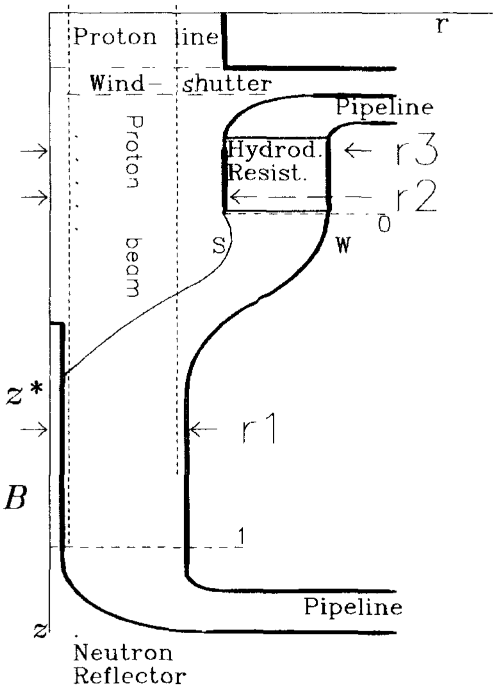

Fig. 2. The scheme of a flowing heavy metal target for an accelerator-driven neutron sourse.

be 200°C and we may consider the medium density ρ as a constant in these equations.

In the case under consideration one must design the liquid target with the cylindrical central body for several reasons. First, any stream line length of a free surface must be limited to avoid a very high level of the flow velocity V_z at the bottom. Second, the circulation $\Gamma = V_\alpha r$ (if any) is a constant along any stream line and the minimal distance between free surface and z-axis must be limited to avoid a very high level of the flow rotational velocity V_α. A free surface should be joined with this cylindrical body to satisfy these two requirements. A central body existence means that the proton beam must be circulated around z-axis and the inner boundary of the irradiation ring spot is about 0.01 m away from the central body.

Let us notice, finally, that the minimal pressure $p_{\min}$ inside the flow must meet the requirement $p_{\min} > 0$ to avoid separation of the stream line from the target wall or the beginning of cavitation. It means that the pressurization of the volume with the free liquid target becomes necessary. That is why we design the liquid target with a supersonic wind-shutter [10] to isolate the target from the accelerator vacuum line.

The assumed scheme of the flowing heavy metal target for an accelerator-driven neutron source is shown in Fig. 2 on the (r,z) plane (the point, in which the free surface is joined with the central body is indicated as z^*).

3. Basic equations for the computer simulation

Using the conditions given above hydrodynamic equations for the heavy metal flow can be represented in the following form (see Refs. [11] and [12]):

$$\partial(V_z r)/\partial z + \partial(V_r r)/\partial r = 0, \tag{1}$$

$$V_z \partial(V_\alpha r)/\partial z + V_r \partial(V_\alpha r)/\partial r = 0, \tag{2}$$

$$V_z \partial V_z/\partial z + V_r \partial V_z/\partial r + \partial P/\partial z = -g, \tag{3}$$

$$V_z \partial V_r/\partial z + V_r \partial V_r/\partial r + \partial P/\partial r = V_\alpha^2/r, \tag{4}$$

$$V_z \partial H/\partial z + V_r \partial H/\partial r = U/\rho, \tag{5}$$

where $P = p/\rho$, $H = c_p T + (V^2 + V_\alpha^2)/2 + gz$, $V^2 = V_z^2 + V_r^2$. In these equations, T, c_p and g are the temperature, the heat capacity and the acceleration due to gravity, respectively. If the proton beam is uniform, one can approximately associate the quantity U in Eq. (5) with the heat deposition $q(z')$ given by the formula $U = q[z_S(r) - z]J_b/S_b$, where J_b, S_b and $z_S(r)$ are the proton current, the beam cross-section and the free surface coordinate, respectively.

A free surface is a stream lines surface. That is why it is useful to transform Eqs. (1)–(5) to the so-called "natural" coordinates. These are the local and orthogonal coordinates (ψ, φ), where the φ coordinate coincides with a stream line direction at any point of the flow. Some advantage of (ψ, φ) coordinates is in the fact that the flow spatial domain between 0-and 1-planes (see Fig. 2) may be reflected to the square domain $(0 \leqslant \psi \leqslant 1,\ 0 \leqslant \varphi \leqslant 1)$. The formulas of such transformation can be written as

$$dr = h_\psi \cos\theta \, d\psi + h_\varphi \sin\theta \, d\varphi,$$

$$dz = -h_\psi \sin\theta \, d\psi + h_\varphi \cos\theta \, d\varphi,$$

where $\mathrm{tg}\,\theta = V_r/V_z$ and h_φ, h_ψ functions are the Lame coefficients. Note that the following equations must be correct:

$$\begin{aligned} &\partial r/\partial\psi = h_\psi \cos\theta, \quad \partial z/\partial\psi = -h_\psi \sin\theta, \\ &\partial r/\partial\varphi = h_\varphi \sin\theta, \quad \partial z/\partial\varphi = h_\varphi \cos\theta, \end{aligned} \tag{6}$$

and moreover

$$\begin{aligned} &\partial(h_\varphi \cos\theta)/\partial\varphi = \partial(h_\varphi \sin\theta)/\partial\psi, \\ &-\partial(h_\psi \sin\theta)/\partial\varphi = \partial(h_\varphi \cos\theta)/\partial\psi. \end{aligned} \tag{7}$$

Now one can rewrite Eq. (1) in the natural coordinates as the finite relationship

$$V = 1/[C(\psi) h_\psi r], \tag{8}$$

where $C(\psi)$ is an arbitrary function. We can integrate Eq. (2) overall to obtain the finite expression

$$V_\alpha r = \Gamma(\psi) \tag{9}$$

with an arbitrary circulation $\Gamma(\psi)$. Invoking Eqs. (6)–(9), one can rewrite Eqs. (3) and (4) in natural coordinates as

$$\partial P/\partial\varphi = (V^2 h_\varphi/h_\psi)\partial\theta/\partial\psi + (V^2/r + \Gamma^2/r^3) h_\varphi \sin\theta - g h_\varphi \cos\theta, \tag{10}$$

$$\partial P/\partial\psi = -(V^2 h_\psi/h_\varphi)\partial\theta/\partial\varphi + (\Gamma^2/r^3) h_\varphi \cos\theta - g h_\varphi \sin\theta. \tag{11}$$

Excluding P from these equations, we obtain

$$\frac{\partial}{\partial\varphi}\left(\frac{V^2 h_\psi}{h_\varphi}\frac{\partial\theta}{\partial\varphi}\right) + \frac{\partial}{\partial\psi}\left(\frac{V^2 h_\varphi}{h_\psi}\frac{\partial\theta}{\partial\psi} + \frac{V^2}{r} h_\varphi \cos\theta\right) + \frac{h_\varphi \sin\theta}{r^3}\frac{\partial\Gamma^2}{\partial\psi} = 0. \tag{12}$$

Finally, Eq. (5) can be rewritten as

$$V\frac{\partial H}{\partial\varphi} = h_\varphi U. \tag{13}$$

Eqs. (6)–(9) and (12) are in the closed form if functions $C(\psi)$ and $\Gamma(\psi)$ are known from the boundary conditions. To compute this system of equations in the case when the free surface is joined with the central body one may use the following boundary conditions:

- the coefficient h_ψ is specified on the O-plane $(\varphi = 0)$;
- the coefficient h_φ is specified on the wall W $(\psi = 0)$;
- the coordinates r and z of some flow point are specified;

- the V distribution is specified on the plane $\varphi = 0$ (in particular, this condition mean that the consumption Q is known);
- the V_α distribution is specified on the plane $\varphi = 0$;
- the function $\theta = \theta_W(l)$ is specified on $\psi = 0$ (argument l is the distance along the wall W and it can be obtained by integrating $dl = h_\varphi\,(0,\varphi)d\varphi$ over φ);
- the function $\theta = \theta_B(l)$ is specified on the central body B;
- the pressure p_S is a constant along the free surface S (the condition for θ on the free surface S may be found from Eq. (10) ;
- $\partial\theta/\partial\varphi = 0$ by $\varphi = 1$ (according to Eq. (11) this condition mean that along the 1-plane the projections of the pressure gradient, the centrifugal force and the force of gravity are in equilibrium).

We must note here that under these conditions the location of z^* joined point is unknown. Its determination is a difficult problem. But one may compute the more simple inverse problem, when z^* location is pointed and the consumption Q and/or the wall W location are determined.

Now we can outline the determination scheme for heavy liquid target parameters. First, the boundary problem for Eqs. (6)–(9) and (12) must be solved. Second, the pressure p inside the target can be found by integration Eqs. (10) and (11) over φ and ψ. Note that using the condition $p_{min} = 0$ inside the flow it is easy to obtain in addition the minimal permissible level p_S^* of the pressure p_S over the free surface. One can find also the Darcy's constant C_D if the pressure p_{pl} within the pipeline is given. Finally, we must use Eq. (13) to find the temperature distribution T inside the target if the initial temperature T_0 at the hydrodynamic resistance outlet and the proton beam current J_b are given.

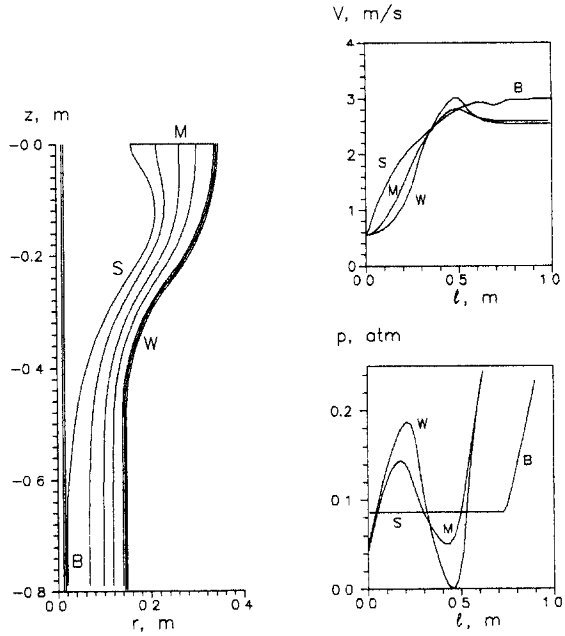

Fig. 4. The stream line configuration, the pressure (p) and velocity (V) for the "whole" flow regime by minimal the eutectic consumption.

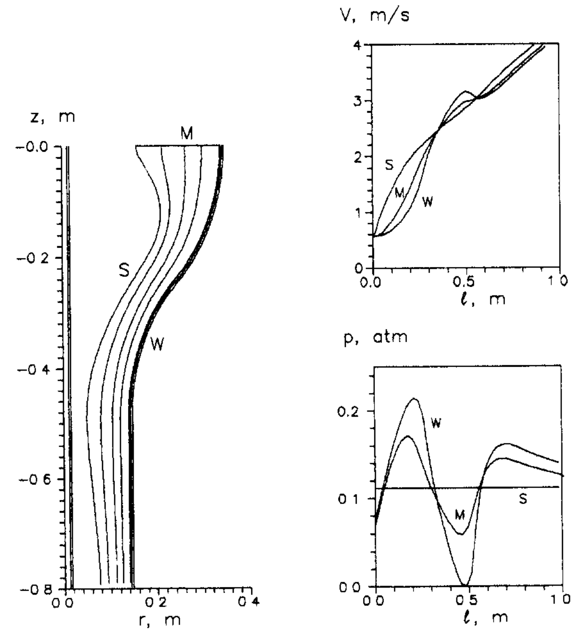

Fig. 3. The stream line configuration, the pressure (p) and velocity (V) for the "hole" flow regime.

The above scheme was realized with the LQD code [13] and applied for calculations of some parameters of the Pb–Bi liquid target. The numerical net technique has been employed to solve the boundary problem for the equations under consideration. In particular, the boundary problem for Eqs. (6)–(9) and (12) has been solved by means of an iteration procedure based on the successive relaxation method [14].

4. Permissible flow regimes

In all cases under consideration below we shall assume that the central body has the radius $r_0 = 0.02$ m and the shape of the middle part of the target wall W consists of two conjugate parts of circumferences with the tangent angle at the conjugate point equal to 45° (see Fig. 2). These circumferences conjugate vertically with the hydrodynamic resistance boundary and the cylindrical target part. The velocities V and V_α by the 0-plane will be assumed to be constant and $V_\alpha/V = \mathrm{tg}\ 30°$ along this plane.

Let us consider consumptional properties of the flow by example of the target with the following parameters: $r_1 = 0.14$ m, $r_2 = 0.16$ m and $r_3 = 0.34$ m.

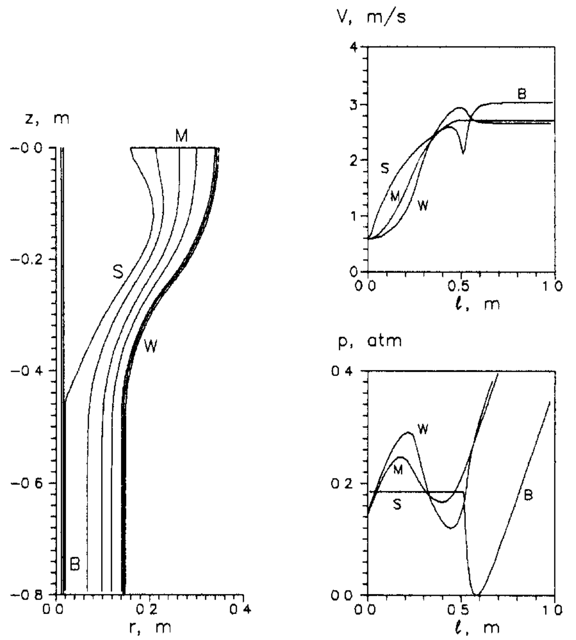

Fig. 5. The stream line configuration, the pressure (p) and velocity (V) for the "whole" flow regime by normal the eutectic consumption.

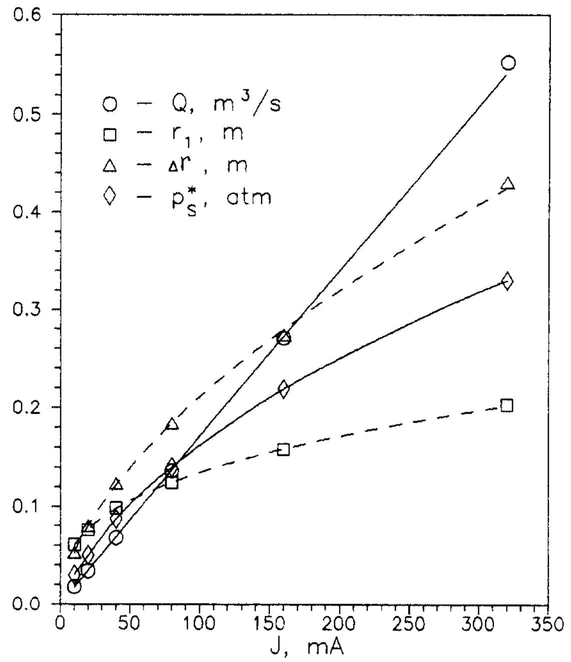

Fig. 6. The eutectic consumption (Q), the lower target's dimension (r_1), the upper target's dimension (Δr) and the minimal pressure over free surface (p_s) for the flowing heavy metal targets within the 10–320 mA proton current (J_b) range.

The calculation results for the liquid eutectic consumption $Q = 0.161$ m^3/s are shown in Fig. 3 on (r, z) plane as the stream lines configuration, the pressure p and the velocity V dependences along some stream lines. We call this situation as a "hole" regime. As it follows from our calculations, with increase of consumption the distance between the free surface and the central body decreases and at consumption value of $Q_{touch} \simeq 0.165$ m^3/s the free surface touches the body. If $Q > Q_{touch}$ the "hole" regime is destroyed.

Now let us consider the regime when the free surface joins the central body. In this regime the eutectic flow fills the target cylindrical part below the joint point z^*. We call this situation as a "whole" regime. As it follows from our calculations, the minimum consumption for the case of "whole" regime existence is equal to $Q_{conj} \simeq 0.158$ m^3/s (see Fig. 4). With increase of consumption over Q_{conj} the joint point z^* goes up and the velocity V near this point decreases (see Fig. 5; for $Q = 0.166$ m^3/s). A considerable increase in consumption may lead in the range $Q > Q_{stagn} > Q_{conj}$ to formation of the "stagnation point" (at z^* point the velocity $V = 0$, but $V_\alpha \neq 0$). We do not give here the accurate value of Q_{stagn} because this is an analytical problem which cannot be solved within the framework of the LQD code only.

One can note that in the conditions under consideration

Table 1
Results of calculations

Proton current J_b [mA]	Consumption Q [m^3/s]	Target's dimensions [m] Upper Δr [m]	Lower r_1 [m]	Min. pressure over free surface P_s^* [atm]	Neutron flux $j_n \times 10^{16}$ [cm^{-2} s^{-1}]
10	0.018	0.073	0.061	0.030	0.2
20	0.034	0.100	0.076	0.050	0.4
40	0.069	0.143	0.098	0.087	0.6
80	0.136	0.204	0.124	0.140	0.9
160	0.271	0.294	0.158	0.219	1.4
320	0.553	0.431	0.203	0.330	2.2

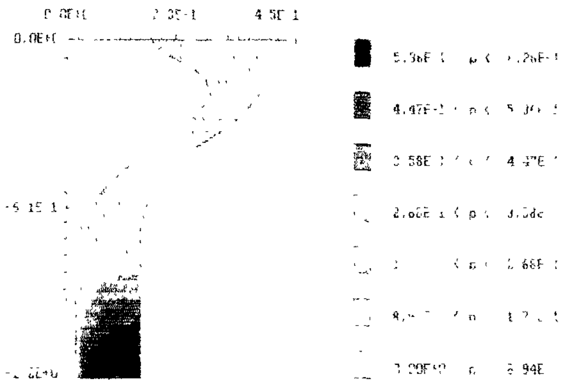

Fig. 7. The scatter plot distribution of the pressure (p) for the target by $J_b = 160$ mA.

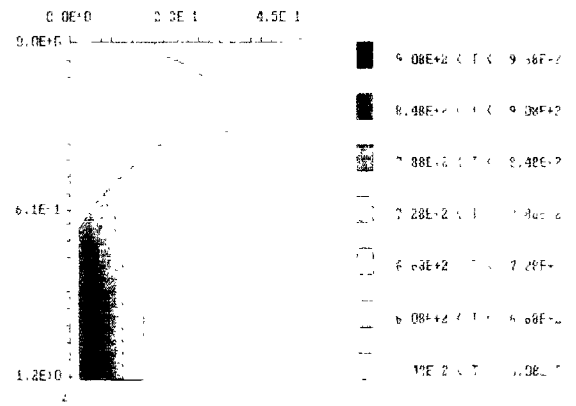

Fig. 9. The scatter plot distribution of the temperature (T) for the target by $J_b = 160$ mA.

the critical consumption Q_{conj} is less than Q_{touch}. It means that within the $Q_{conj} < Q < Q_{touch}$ consumption range both "hole" and "whole" regimes exist formally, but some of these regimes may exist as unstable one. Nobody knows whether "hole" or "whole" regime will be realized practically for some flow within this range. That is why we assume the following consumption requirement: $Q > \max[Q_{touch}, Q_{conj}]$ for the permissible operation regimes of the following heavy metal target.

5. A set of the targets for the 10–320 mA current range

We present here the computed parameters of the lead–bismuth targets for the 10–320 mA current range. All targets meet the conditions and requirements which were described above. In all cases the initial temperature T_0 and velocity V_0 are assumed to be 275°C and 0.5 m/s, respectively. We assume in addition that the average temperature rise

$$\Delta T = \frac{2}{r_1^2 - r_0^2} \int_{r_0}^{r_1} [T_1(r) - T_0] r \, dr$$

is equal to 200°C for all targets ($T_1(r)$ is the temperature distribution in 1-plane). This requirement determines the dimension $\Delta r = r_3 - r_2$ of the upper target's part and the radius r_1 of it's lower part.

The calculation results are represented in Table 1 and in Fig. 6. We point here also that the following approximations take place for the represented set of the targets:

$$Q[\mathrm{m^3/s}] \simeq 1.7 \times 10^{-3} J_b, \quad \Delta r[\mathrm{m}] \simeq 1.3 \times 10^{-2} J_b^{0.6},$$
$$r_1[\mathrm{m}] \simeq 2.7 \times 10^{-2} J_b^{0.35}, \quad p_S^*[\mathrm{atm}] \simeq 6.3 \times 10^{-3} J_b^{0.7}.$$

The values of neutron flux j_n pointed in the Table were calculated roughly with the help of the formula

$$j_n = \xi J_b / (2\pi e r_1 z'_{stop}),$$

where e is the electron charge and $\xi \simeq 55$ is the specific neutron/proton output.

Let us notice, finally, that from the hydrodynamic viewpoint the effectiveness of the liquid target may be characterized by the parameter

$$\eta = eQ/(\xi J_b)$$

which can be called the specific consumption of a liquid metal per one neutron. This parameter depends mainly on the average temperature rise ΔT and decreases with the increase of ΔT. The value of η is approximately 0.5×10^{-20} m^3 within the 10–320 proton current range for the considered set of the targets.

The scatter plot distributions of the pressure p, the energy deposition function U and the temperature T for the target with $J_b = 160$ mA are represented as an example in Figs. 7–9 respectively.

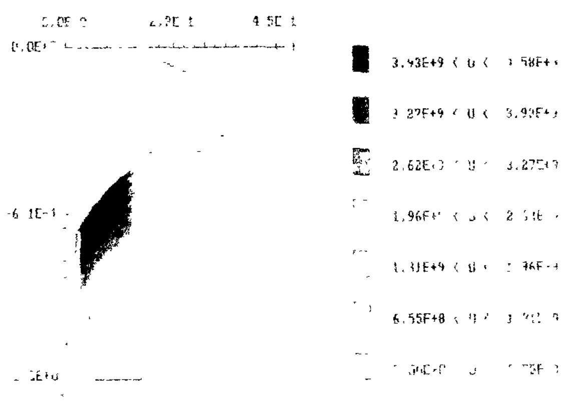

Fig. 8. The scatter plot distribution of the energy deposition (U) for the target by $J_b = 160$ mA.

6. Conclusion

In the present work, we have analyzed the possibility to design the flowing lead–bismuth eutectic target with free surface under the 0.8 GeV proton beam irradiation within the 10–320 mA current range. The scheme of such a target must include the neutron reflector, the central body, the hydrodynamical resistance and pressurized volume above the free liquid surface. This volume must be isolated from the accelerator vacuum line by the supersonic wind-shutter. The proton beam must be circulated around the target axes and the boundaries of the irradiation ring spot are about one centimeter away from the target solid bounders. We have solved the system of hydrodynamic equations for nonviscous and non-heat conductive flow with constant medium density to determine the flow parameters. To simplify the data taking process we have transformed these equations to the natural coordinates. The results of the calculation show that the eutectic consumption, the pressure on a free surface and the initial flow velocity must be in a certain range in order to avoid a destabilization or a destruction of the flow. The liquid metal consumption is a linear function of the beam current. The permissable target size increases when the beam current increases, in particular the radius of the cylindrical target part increases to the power 0.35 by $\Delta T = 200°C$. The neutron flux from the cylindrical target surface increases as the power 0.65 when the beam current increases.

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