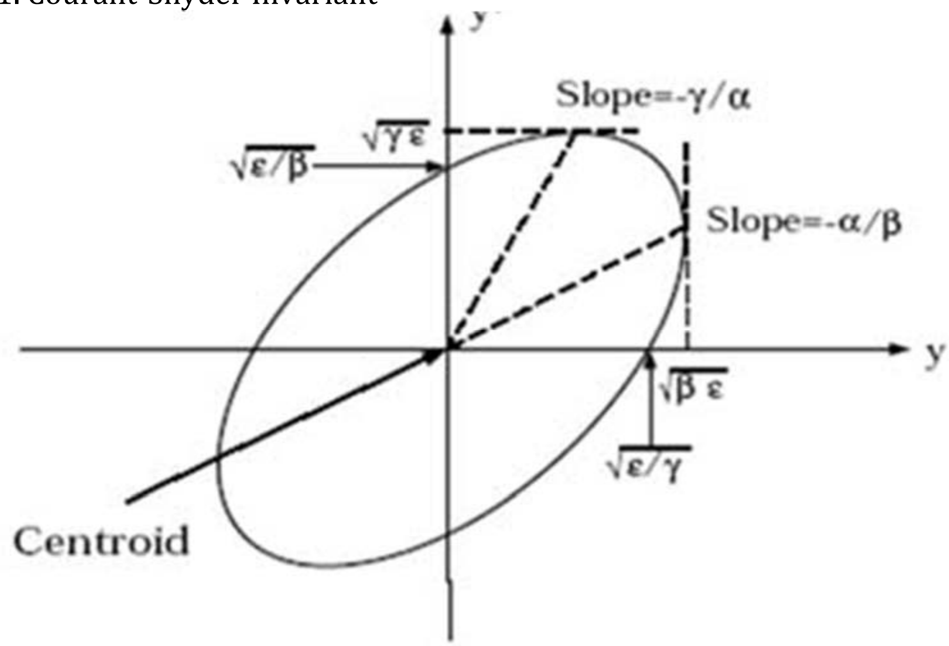
Beam Emittance Characterization and Optimization of Target Parameters

(X. Ding, July 22, 2014; comments by K. McDonald, July 24, 2014)

1. Courant-Snyder Invariant



2. 2-D Transverse Emittance (rms) and Twiss Parameters

$$x, x' = p_x / p_z,$$

$$x_c = \sum x / n,$$

$$x_c' = \sum x' / n$$

$$\langle x^2 \rangle = \sum (x - x_c)^2 / n$$

$$\langle x'^2 \rangle = \sum (x' - x_c')^2 / n$$

$$\langle xx' \rangle = \sum (x - x_c)(x' - x_c') / n$$

$$\varepsilon_{rms,x} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

$$\alpha_x = -\frac{\langle xx' \rangle}{\varepsilon_{rms,x}}$$

$$\beta_x = \frac{\langle x^2 \rangle}{\varepsilon_{rms,x}}$$

$$\gamma_x = \frac{\langle x'^2 \rangle}{\varepsilon_{rms,x}}$$

$$\beta_x \gamma_x - \alpha_x^2 = 1$$

[$\alpha = 0$ at the beam waist.]

The 2-d rms, geometric emittance considered above is only appropriate in zero magnetic field. Inside a magnetic field the x- and y-phase subspaces are mixed and the 4-d transverse emittance should have been considered. And, inside a magnetic field, the validity of Twiss parameters derived from 2-d second moments is doubtful.

3. Gaussian Distribution (Probability Density)

In two dimensional phase space (u,v):

$$w(u,v) = \frac{1}{2\pi\sigma^2}\exp(-\frac{u^2+v^2}{2\sigma^2})$$

where u = transverse coordinate (either x or y), v=αu+βu'.

α, β are the Courant-Synder parameters at the given point along the reference trajectory (central ray).

In polar coordinates (r, θ):
u = rcosθ v = rsinθ
u' = (v-αu)/β = (rsinθ-αu)/β

4. Distribution Function Method

$$\theta = 2\pi\xi_1, \quad \theta \in [0,2\pi], \qquad r = \sqrt{-2\sigma^2 \ln \xi_2}, \quad r \in [0,\infty]$$

Random number generator:
θ = 2π*rndm(-1) r = sqrt(-2*log(rndm(-1))*σ

5. Gaussian Distribution (Fraction of Particles)

The fraction of particles that have their motion contained in a circle of radius "a" (emittance $\varepsilon=\pi a^2/\beta$) is

$$F_{Gauss} = \int_0^a \frac{1}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} r dr = 1 - e^{-\frac{a^2}{2\sigma^2}}$$

6. Fraction of particles

K = a/σ	$\varepsilon_{K\sigma}$ [Normalized emittance: $(\beta\gamma)\varepsilon_{K\sigma}$]	F_{Gauss}
1	$\pi(\sigma)^2/\beta$	39.5%
2	$\pi(2\sigma)^2/\beta$	86.4%
2.5	$\pi(2.5\sigma)^2/\beta$ or $\sim 6\pi\sigma^2/\beta$	95.6%

7. Focused Beam

Intersection point (beam waist) (z = 0 cm): $\alpha^*= 0, \beta^*, \sigma^*$
Beam launch point (z = -100 cm): α, β, σ
L = 100 - 0 = 100 cm = distance from beam waist to the launch point

In zero magnetic field, the Twiss parameters at distance L from the waist would be
$\alpha=L/\beta^*$
$\beta=\beta^*+L^2/\beta^*$
$\sigma=\sigma^*\text{sqrt}(1+L^2/\beta^{*2})$

For a nonzero SC field (peak of 20 T at z = 0), we launched the beam at z = -100 cm based on Twiss parameters calculated as follows (beam emittance characterization).

We first launched a beam (typically 1,000,000 particles) at z = 0 cm with $\alpha^*= 0$, β^*, σ^* at fixed emittance (i.e., 5,10,15,20, ... μm).

We tracked all the particles from z = 0 back to z = -100 cm, inside the magnetic field, but with no target or other material.

We computed α, β, σ from the second moments of x and x' of the beam particles at z = -100 cm, using the expressions given in sec. 2.

Then, we launched a new beam from z = - 100 cm using these Twiss parameters, with the new beam centered in (x,y), and in (x',y'), on their values for the central ray as backtracked from z = 0 to z = - 100 cm.

It would have been more correct simply to use the beam particles tracked to z = - 100 cm in the absence of the target, reverse their momenta, and put back the target and other material. This procedure will be used in the future.

8. Launch Settings with Focused Beam Trajectories in a Magnetic Field

Modeled by the user subroutine BEG1 in m1514.f of MARS code
xv or xh (transverse coordinate: u); xvp or xhp (deflection angle: u')
XINI=x0+xv DXIN=dcx0+xvp
YINI=y0+xh DYIN=dcy0+xhp
ZINI=z0 $\text{DZIN}=\text{sqrt}(1-\text{DXIN}^2-\text{DYIN}^2)$

(x0,y0,z0,dcx0,dcy0,dcz0) are the parameters at z=-100 cm for the central ray (that passes through the point x=0, y=0 and z=0).

9. Optimization of Target (or Beam) Radius

(1) Fixed beam emittance ($\varepsilon_{K\sigma}$) to $\pi(\sigma)^2/\beta$; Fixed target radius to beam radius (TR/BR=4)
(2) Compute (x0,y0,z0,dcx0,dcy0,dcz0) for the central ray at z=-100 cm from single particle (KE=6.75 GeV) having x=0,y=0 and z=0 and specified tilt angle to SC axis at z=0.
(3) Vary beam radius σ^*, while vary the β^* at the same time to fix the beam emittance; Launch a beam (1000,000 particles) at z = -100 cm using the procedures given in secs. 7 and 8.
(4) Let the beam interact (in the MARS simulation) with the target (and other material), and track the secondary particles to z=50m downstream the target. Consider the yield of "good" particles to be those with KE between 40 and 180 MeV, to find the optimized value of the beam radius.