Neutrino Factory and Muon Collider Collaboration
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Targetry Simulation with Front Tracking And Embedded Boundary Method

Jian Du

SUNY at Stony Brook

Collaborators:

Roman Samulyak， Computational Sciences Center, BNL
James Glimm， SBU/CSC

Talk outline

* **MHD System of Equations and Numerical Algorithm Used**

* **Code Validation through Jet Distortion in transverse B field**

* **Target Simulation**
    * **simulation of the interaction of the mercury jet with a proton pulse**
    * **inclusion of bubble collapsing effects**

* **Conclusion**

Full system of MHD equations

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot(\rho\,\mathbf{u})$$

$$\rho\left(\frac{\partial}{\partial t}+\mathbf{u}\cdot\nabla\right)\mathbf{u} = -\nabla P + \mu\Delta\mathbf{u} + \frac{1}{c}(\mathbf{J}\times\mathbf{B})$$

$$\rho\left(\frac{\partial}{\partial t}+\mathbf{u}\cdot\nabla\right)e = -P\nabla\cdot\mathbf{u} + \frac{1}{\sigma}\mathbf{J}^2$$

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) - \nabla\times\left(\frac{c^2}{4\pi\sigma}\nabla\times\mathbf{B}\right)} \Longrightarrow$$

$$P = P(\rho,e), \qquad \nabla\cdot\mathbf{B} = 0$$

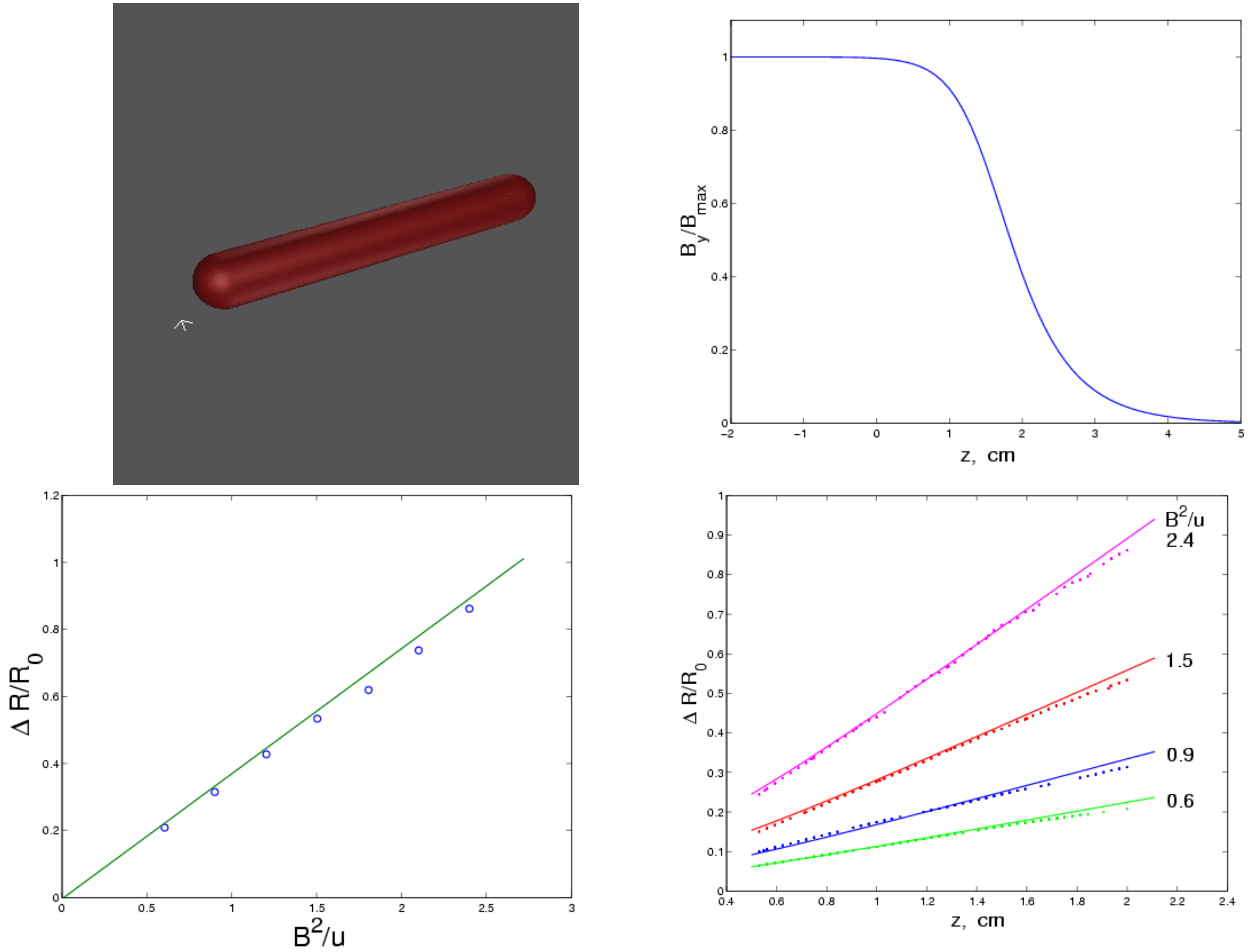
Low magnetic Re approximation & charge neutrality

$$\nabla\cdot J + \frac{\partial\rho_e}{\partial t} = 0$$

$$\boxed{\begin{array}{l}
\mathbf{J} = \sigma\left(-\nabla\phi + \frac{1}{c}\mathbf{u}\times\mathbf{B}\right) \\
\Delta\phi = \frac{1}{c}\nabla\cdot(\mathbf{u}\times\mathbf{B}), \\
\text{with } \left.\frac{\partial\phi}{\partial\mathbf{n}}\right|_{\Gamma} = \frac{1}{c}(\mathbf{u}\times\mathbf{B})\cdot\mathbf{n} \\
\mathbf{B} = \mathbf{B}_{\text{ext}}(x,t), \quad \nabla\cdot\mathbf{B}_{\text{ext}} \equiv 0
\end{array}}$$

- **Implemented in FronTier**
- **Riemann Problem for interface propagation**
- **MUSCL scheme for interior state updating**
- **Different EOS modeling**
- **Embedded Boundary Method for elliptic equation**

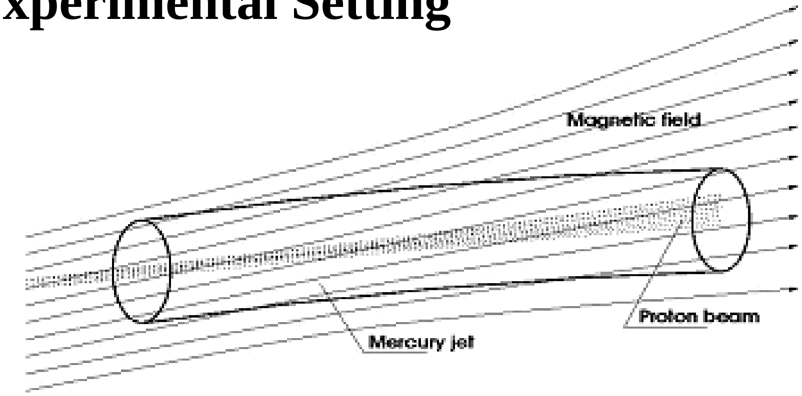
Jet distortion (previous work)

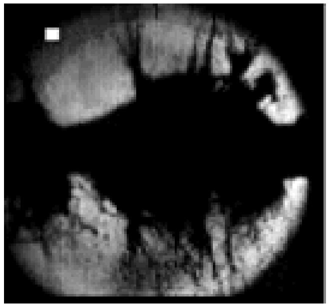


[1] S.Oshima, R. Yamane, Y.Moshimaru, T.Matsuoka, The shape of a liquid metal jet under a non-uniform magnetic field. JCME Int. J., 30(1987),437-448

Simulations of the Muon Collider Target

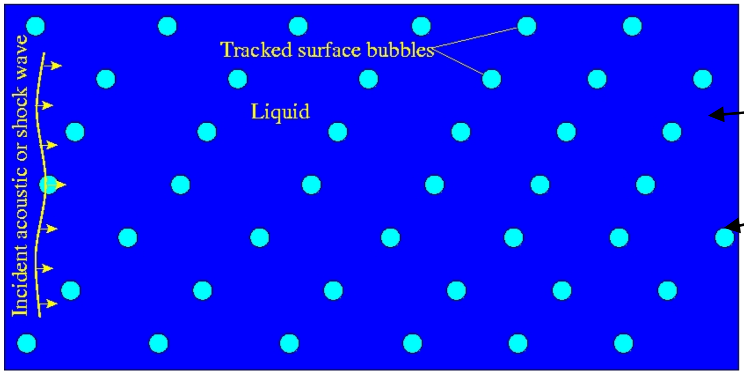
(1) Experimental Setting





(2) Different EOS models used

- Heterogeneous method (Direct Numerical Simulation): Each individual bubble is explicitly resolved using FronTier interface tracking technique.



Stiffened Polytropic EOS for liquid

Polytropic EOS for gas (vapor)

- Homogeneous EOS model. Suitable average properties are determined and the mixture is treated as a pseudofluid that obeys an equation of single-component flow. Need conductivity model.

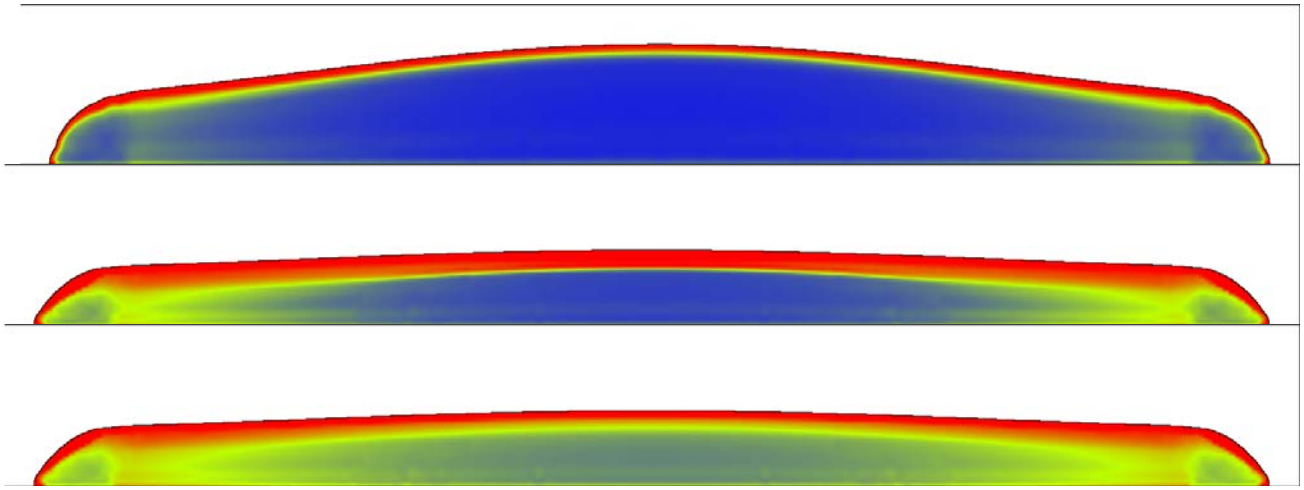
(3) Simulations with Homogeneous Model

Conductivity Model with Phase Transition (Bruggeman Model)

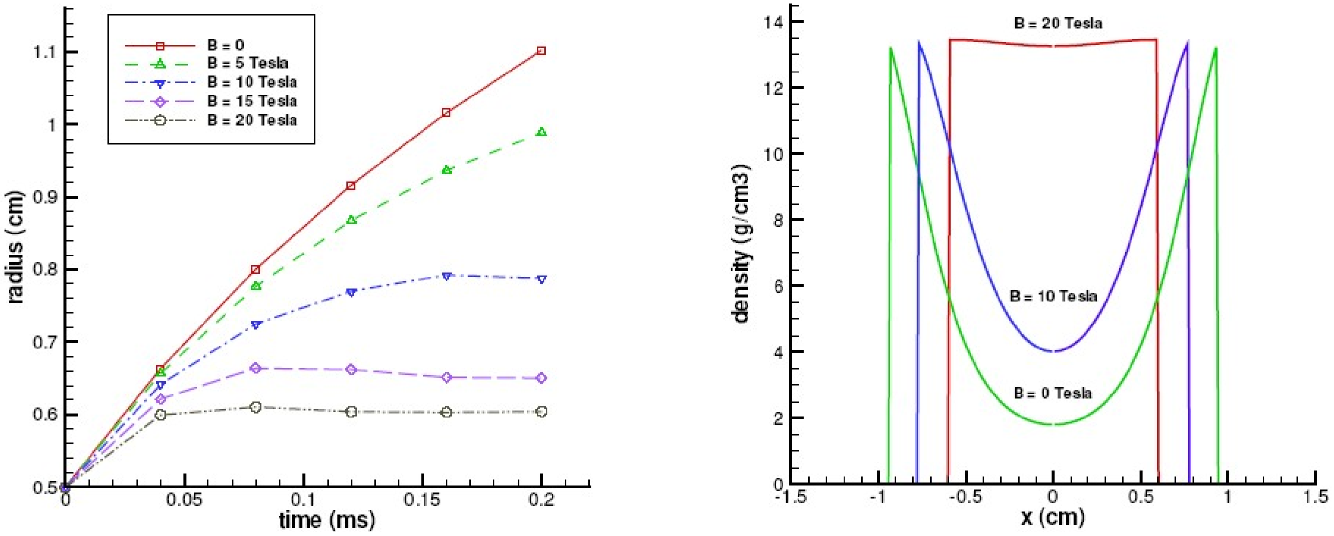
$$\sigma_m = \begin{cases} 0 & (if\ \eta_l \leq \frac{1}{3}) \\ \frac{1}{2}(3\eta_l - 1)\sigma_l & (1 \geq \eta_l \geq \frac{1}{3}) \end{cases}$$

σ_m : the conductivity of the mixed phase
σ_l : the conductivity of the liquid
η_l : volume fraction of the liquid

Simulations with Homogeneous Model

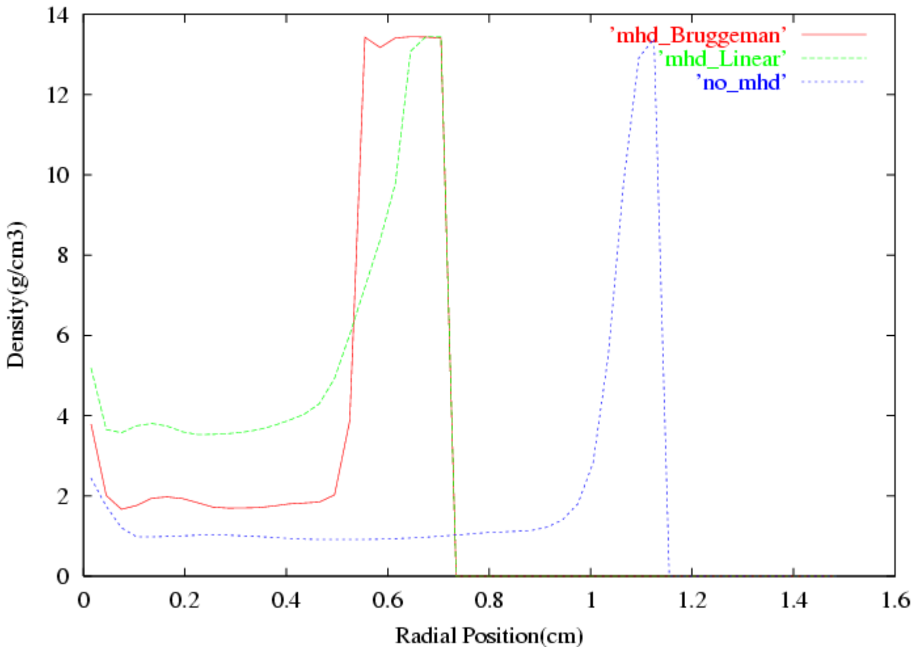


Density Profile for no MHD, MHD with Bruggeman Model, MHD with linear Model, from top to bottom at T = 0.15ms

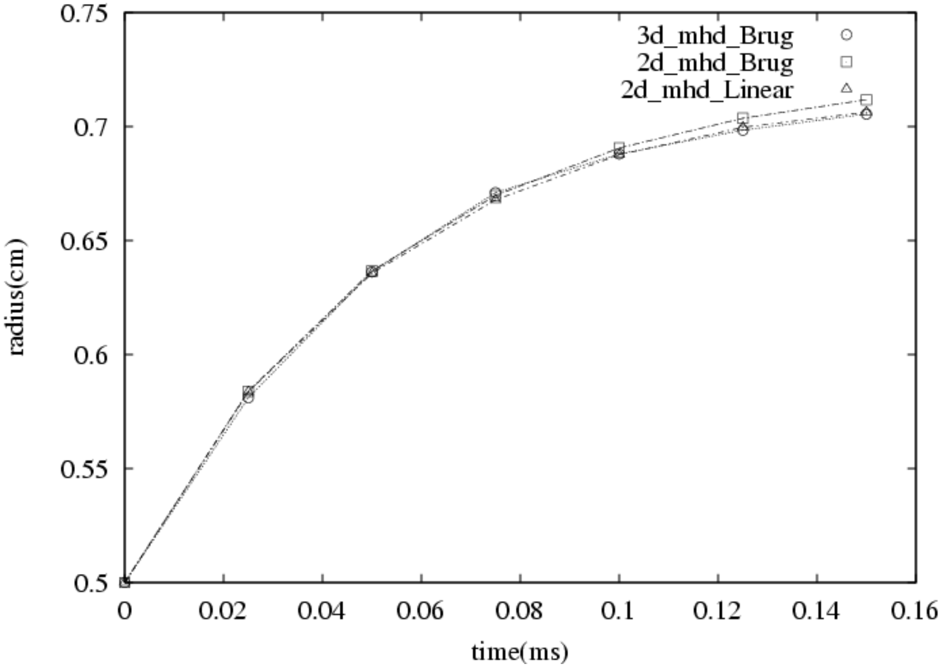


Jet expansion velocity and cross section density profile at t = 0.1 ms for different magnetic field

Density Plot and Expansion Comparison



Density distribution

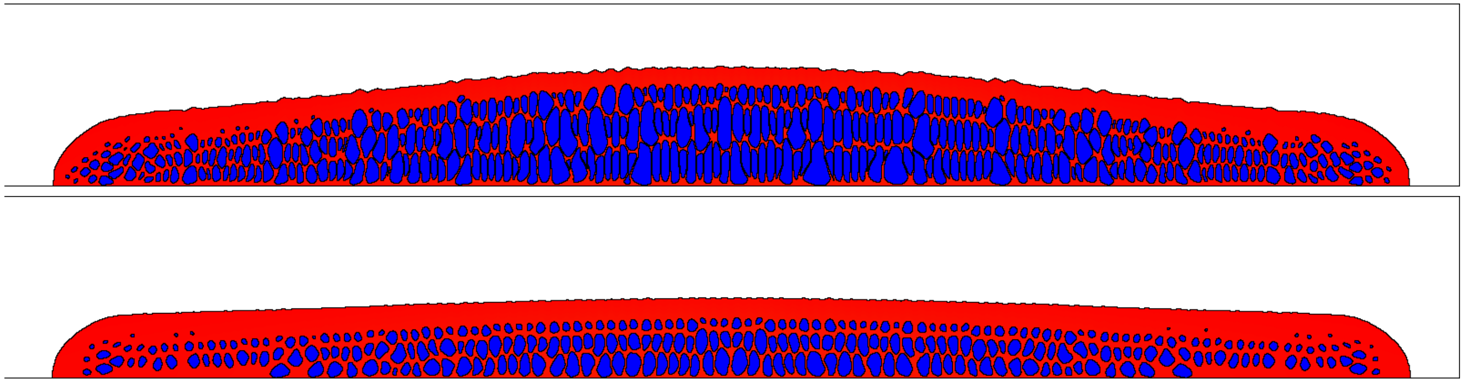


Jet Expansion Velocity

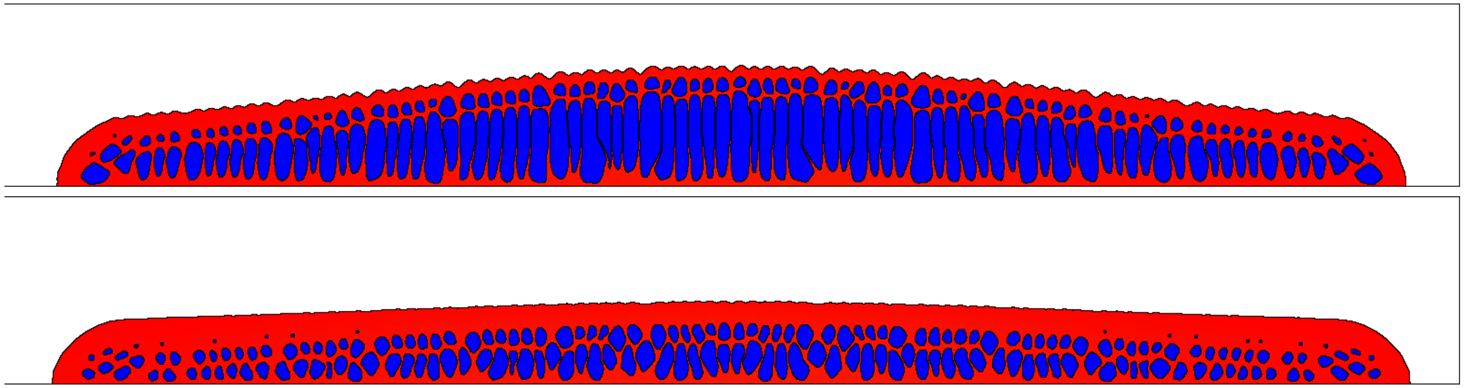
Conclusions:

1. 2D and 3D simulations agree well， both indicate strong restriction of 15 Tesla field on jet expansion and cavitation forming
2. Linear and Bruggeman Models have similar jet expansion
3. Bruggeman model gives larger cavitation region

4. Simulations with Heterogeneous Model



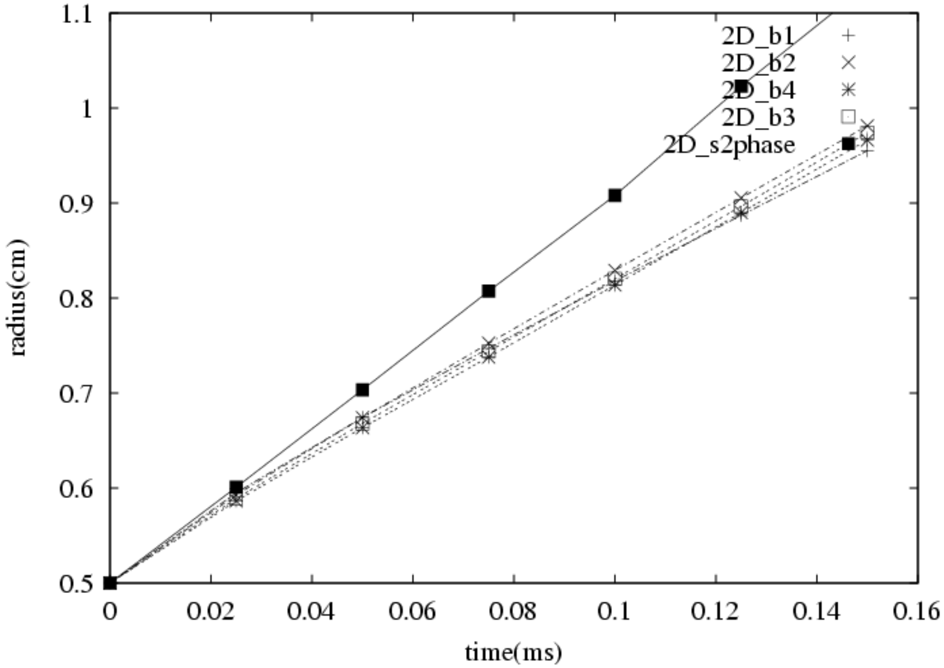
Density Profile for no MHD(top), with MHD (bottom,B=15 Tesla) at T = 0.15ms
Initial R_b=1.5 x (mesh size)，distance 2 x (mesh size)，P_critical = -100 bar



Density Profile for no MHD(top), with MHD (bottom,B=15 Tesla) at T = 0.15ms
Initial R_b=3 x (mesh size)，distance 3.5 x (mesh size)，P_critical = -400 bar

Comparison of hetero- and homogenized EOS models

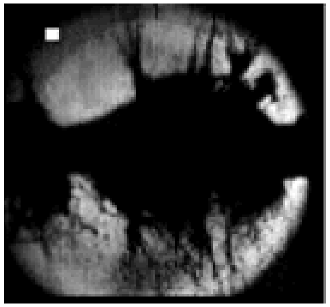
b1 – b4 stand for different bubble radius



- **Conclusion:**

1) **heterogeneous models give uniform jet expansion for different insertion parameters;**
2) **homogeneous model give larger expansion;**
3) **Surface instabilities as in the experiments, have not been obtained in all simulations**

5. Open problems:



- the nature of surface instability
- Is MHD reduction of the jet expansion as strong as in simulations?

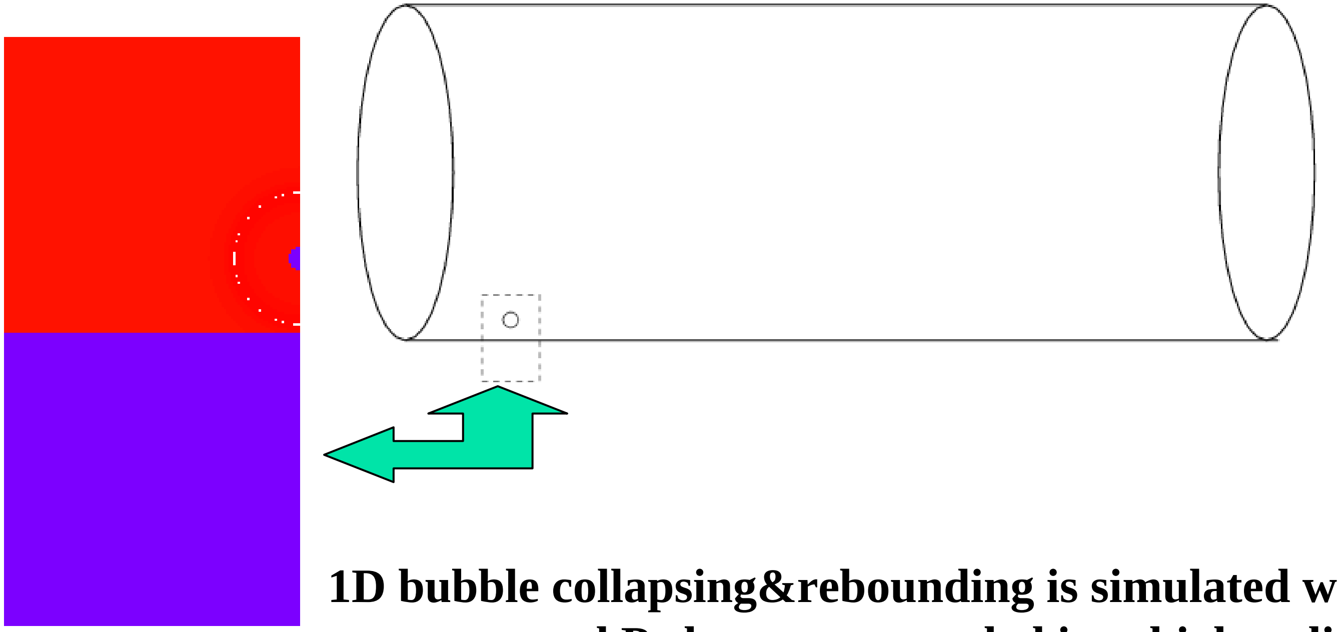
  in the smooth jet, strong azimuthal currents tend to cause strong MHD effects
- If surface instabilities are present, what is the MHD effect on spikes

  or when the topology is significantly different from the smooth jet

6. Surface instability study: problem set-up

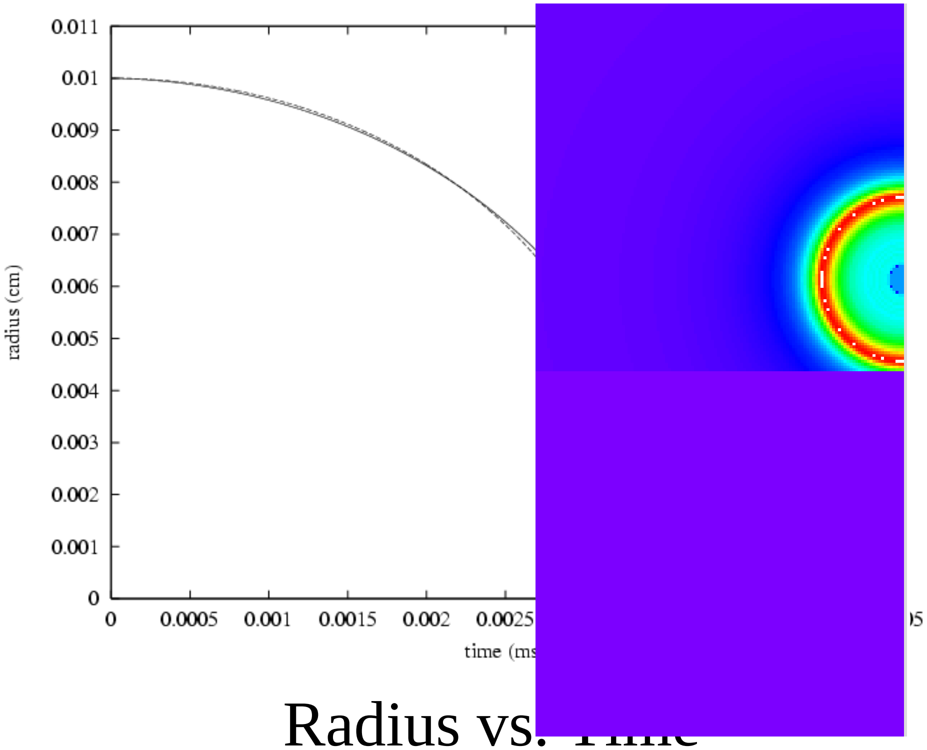
Possible Cause:

- **Turbulence nature of the jet**
- **Incomplete thermodynamics model (homogeneous)**
- **Unresolved bubble evolution (heterogeneous)**

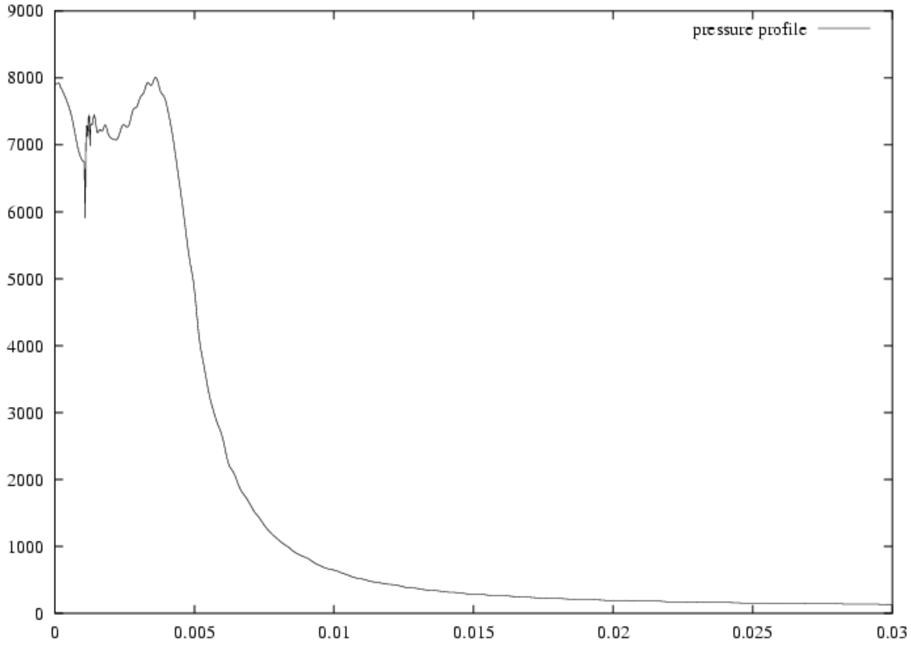


1D bubble collapsing&rebounding is simulated with spherical geometry and P,rho, v are coupled into higher dimension cases

1D bubble collapsing & Keller’s equation



Radius vs. Time

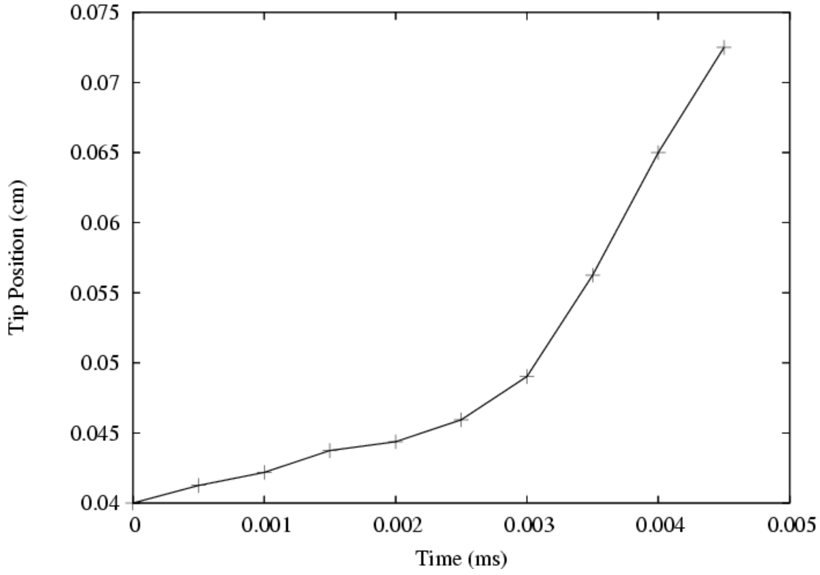


Pressure Profile at t =0.0035 ms

Pressure profile at rebounding stage of 1D simulation is used as input for the 2D simulations (P_{bub}=1.0e-4bar, P_{amb}=100bar)

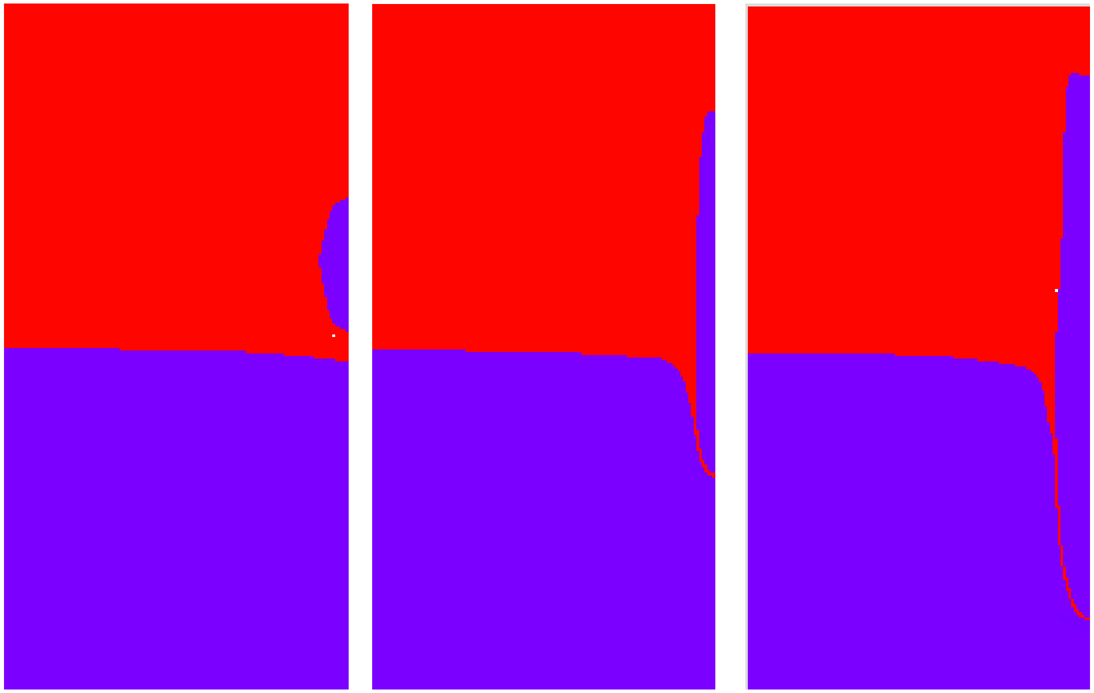
2D Simulations with bubble rebounding

- Surface perturbation quickly develops with bubble rebounding
- Perturbation velocity can reach about 160m/s
- Similar 3D hydro and MHD simulations are underway



Perturbation tip position Vs. Time

Density Profile



T = 0.0005ms T=0.0035ms T=0.0045ms

Conclusions

1) 2D and 3D simulations & different cavitation models give consistent results
2) Using the multi-scale approach, verified the important role of bubble collapsing in jet breakup
3) 3D hydro & MHD simulations with bubble insertion are underway and important to study the MHD effects on jet breakup