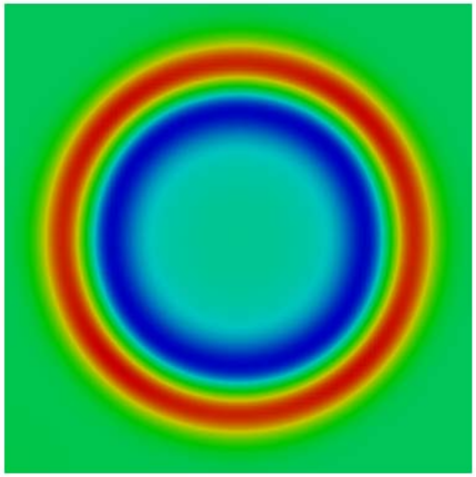
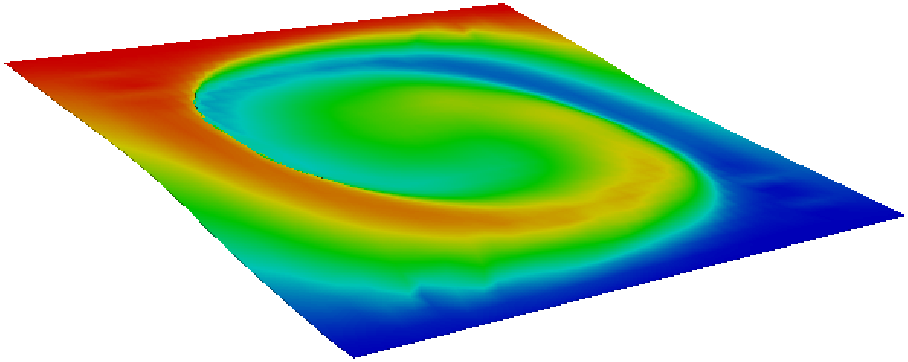
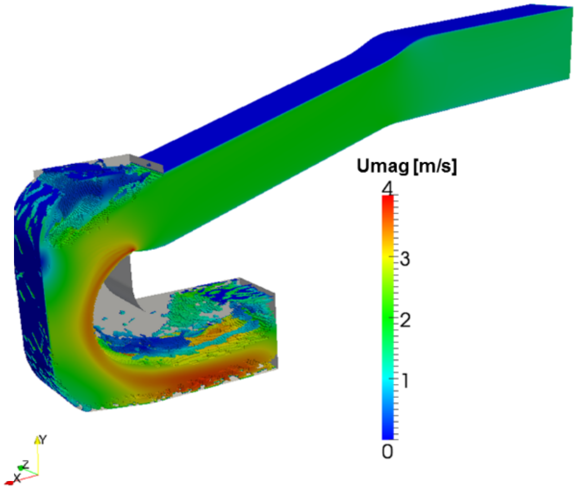
Towards the simulation of proton beam induced pressure waves in liquid metal using the Multiple Pressure Variables (MPV) approach

Jana R. Fetzer, A. Class **(May 21, 2014)**

INSTITUTE FOR NUCEAR AND ENERGY TECHNOLOGIES





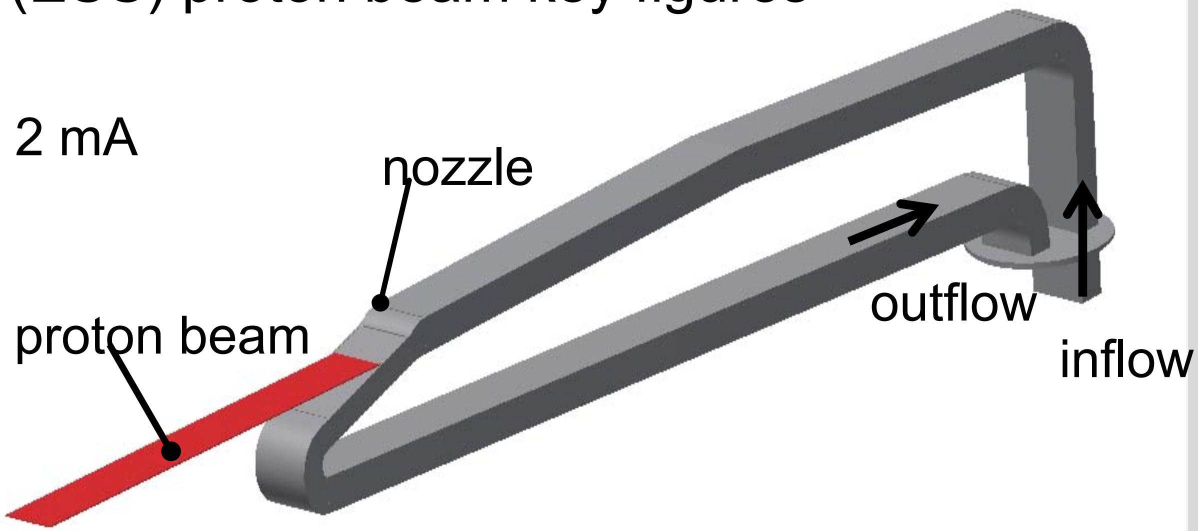


Outline

- Motivation
- Multiple Pressure Variables Approach
- Implementation in OpenFOAM
- Validation
  - Aero acoustic
  - Liquid metal
- Conclusion

Motivation

- European Spallation Source (ESS) proton beam key figures
  - Proton beam power 5 MW
  - Proton beam mean current 2 mA
  - Long-pulse 2.86 ms
  - Repetition rate 14 Hz

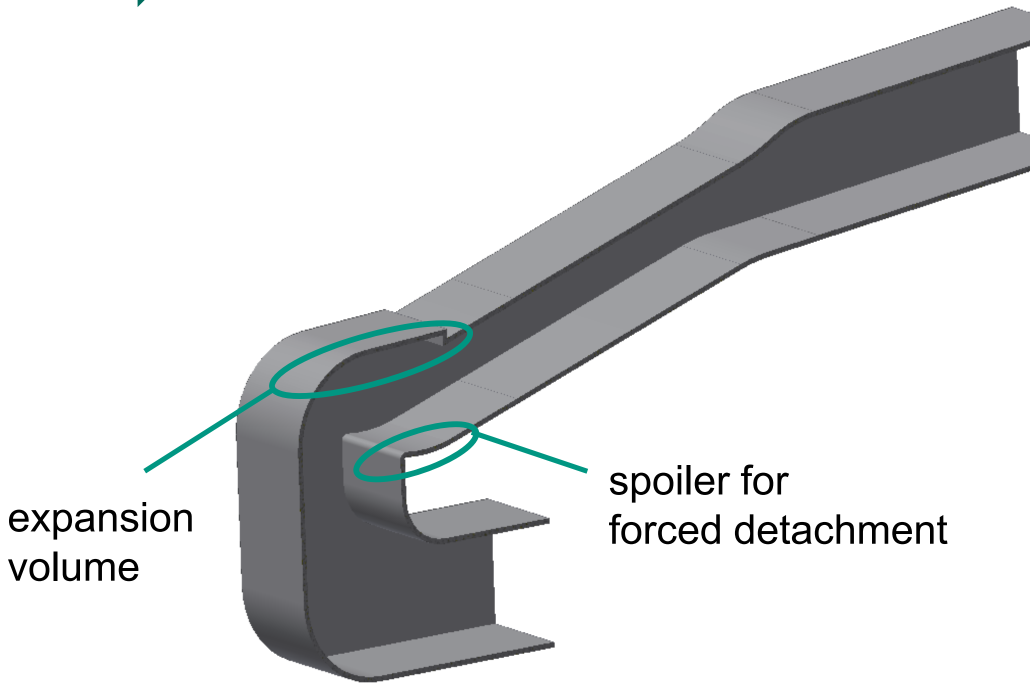


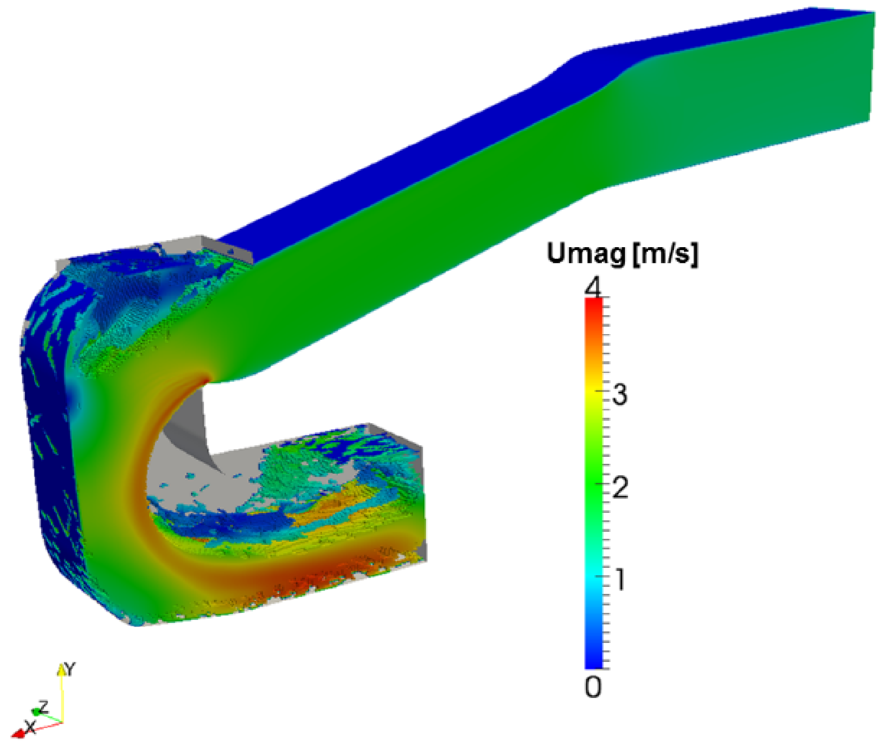
- MEgawatt TArget: Lead bIsmuth Cooled
  - Nozzle for flow conditioning and elimination of cavitation
  - Successive beam pulses interact with fluid not subjected to the beam previously
  - Pressure ~ 1bar
  - Flow velocity 1.5-2 m/s
  - LBE

Motivation

- Dedicated design measures to counteract the effects of pressure waves
  - Expansion chamber in Ubend
  - Spoiler enforcing flow detacement

➡ **Two internal free surfaces**





Motivation

- Transient incompressible flow simulation
  - Time step approx.: $\Delta t_{flow} = \frac{\Delta x}{U_{flow}} = 10^{-4} s$

Δx- grid size
U_{flow}- flow velocity

- Transient simulation resolving flow phenomena and acoustic phenomena
  - Time step approx.: $\Delta t_{ac} = \frac{U_{flow} \Delta t_{flow}}{a} = 10^{-8} s$

a- speed of sound
~ 1700 m/s (LBE)

Multiple Pressure Variables Approach

- Multiple Pressure Variables Approach
  - Klein (1995) performed an asymptotic analysis using two spatial scales and one time scale to capture long wavelength phenomena

- Basic non- dimensionalized variables

$$\rho = \frac{\bar{\rho}}{\bar{\rho}_{ref}}, \boldsymbol{v} = \frac{\bar{\boldsymbol{v}}}{\bar{v}_{ref}}, x = \frac{\bar{x}}{\bar{x}_{ref}}, p = \frac{\bar{p}}{\bar{p}_{ref}}, t = \frac{\bar{t}}{\bar{x}_{ref}/\bar{v}_{ref}}$$

$$M = \frac{\bar{v}_{ref}}{\sqrt{\bar{p}_{ref}/\bar{\rho}_{ref}}}$$ - global Mach number

- Non-dimensionalized compressible Navier Stokes equations in primitive variables for the equation of state of a perfect gas $p = (\gamma - 1)\rho\varepsilon$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\, v) = 0$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (v \cdot \nabla)v + \frac{1}{\rho M^2}\nabla p = \frac{1}{Re}\nabla \cdot \tau$$

$$\frac{\partial e}{\partial t} + \nabla \cdot \big(v(e + p)\big) = \frac{M^2}{Re}\nabla \cdot (\tau v) + \frac{1}{PrRe}\frac{\gamma}{\gamma - 1}\nabla \cdot q$$

Multiple Pressure Variables Approach

- Asymptotic Considerations

  - Vector of the primitive variables $u^i = u^i(x, \xi, t)$
    - x – local variable associated with convective phenomena
    - $\xi = Mx$ – large scale coordinate associated with acoustic wave propagation

  - Asymptotic expansion $u = u^{(0)} + Mu^{(1)} + M^2 u^{(2)} + \ ...$

Asymptotic Limit Equations

- Leading order continuity equation:

$$M^0: \qquad \frac{\partial \rho^{(0)}}{\partial t} + \nabla_x \cdot (\rho v)^{(0)} = 0$$

- Leading, first and second order velocity equation:

$$M^0: \qquad \nabla_x p^{(0)} = 0$$

$$M^1: \qquad \nabla_x p^{(1)} + \nabla_\xi p^{(0)} = 0$$

$$M^2: \qquad \frac{\partial v^{(0)}}{\partial t} + \left(v^{(0)} \cdot \nabla_x\right) v^{(0)} + \frac{1}{\rho^{(0)}}\left(\nabla_x p^{(2)} + \nabla_\xi p^{(1)}\right) = \frac{1}{\rho^{(0)} Re} \nabla_x \tau^{(0)}$$

- Leading order pressure equation

$$M^0: \qquad \frac{\partial p^{(0)}}{\partial t} + (v \cdot \nabla_x p)^{(0)} + (\gamma p \nabla_x \cdot v)^{(0)} = \frac{\gamma}{PrRe} \nabla_x \cdot q^{(0)}$$

Interpretation of asymptotic equations

- From the leading and first order velocity equation follows

$$p^{(0)} = p^{(0)}(t)$$
$$p^{(1)} = p^{(1)}(\xi, t)$$

$$\Rightarrow p(x,t;M) = p^{(0)}(t) + Mp^{(1)}(\xi,t) + M^2 p^{(2)}(x,\xi,t) + O(M^3)$$

- Temporal evolution of $p^{(0)}$ from leading order pressure equation
  - Integration with respect to x over the domain V
  - Applying Gauß-Green theorem with $\boldsymbol{n}$ – outward directed unit normal on the boundary ∂V of V

$$\frac{\partial p^{(0)}}{\partial t} = -\underbrace{\frac{\gamma p^{(0)}}{|V|}\int_{\partial V} \boldsymbol{v}^{(0)} \cdot \boldsymbol{n}\, ds}_{\text{compression}} + \underbrace{\frac{\gamma}{PrRe|V|}\int_{\partial V} q^{(0)} \cdot \boldsymbol{n}\, ds}_{\text{heat transfer}}$$

Interpretation of asymptotic equations

- Large scale average of second order velocity equation
- First order pressure equation

$$\overline{\frac{\partial (\boldsymbol{v})^{(0)}}{\partial t}} + \frac{1}{\rho^{(0)}} \nabla_{\xi} p^{(1)} = 0$$

$$\frac{\partial p^{(1)}}{\partial t} + \gamma p^{(0)} \nabla_{\xi} \cdot \overline{v^{(0)}} = 0$$

" - " average over local structures

$$\overline{v^{(0)}} = \frac{1}{|V_{ac}|} \int_{V_{ac}} v^{(0)}\, dx$$

➡ Evolution equations for the large wavelength acoustics

Interpretation of asymptotic equations

- Formal limit of asymptotic equations for $M = 0$ in a bounded domain without heat conduction and global compression from the boundary

$$\frac{\partial \rho^{(0)}}{\partial t} + \nabla \cdot (\rho v)^{(0)} = 0$$

$$\frac{\partial v^{(0)}}{\partial t} + \left(v^{(0)} \cdot \nabla\right) v^{(0)} + \frac{1}{\rho^{(0)}} \nabla p^{(2)} = \frac{1}{\rho^{(0)} Re} \nabla \cdot \tau$$

$$\nabla \cdot v^{(0)} = 0$$

➡ mathematical model of an incompressible flow with variable density

Implementation in OpenFOAM

Karlsruhe Institute of Technology

multi region solver

pisoMPVFoam

decompose pressure
thermodynamic pressure evolution

caclulate new pressure estimate p

velocity predictor
→ intermediate velocity field

solve

$$Av^{(0)} + \frac{1}{\rho^{(0)}}\nabla_x p^{(2)} = f^n + \frac{1}{\rho^{(0)}\Delta t}(mom)$$

solve energy equation
calculate material properties f(T)

pressure-velocity corrector
→ new $p^{(2)}$

solve

$$M^2\frac{\partial p^{(2)}}{\partial t} - p\nabla\cdot\left[A^{-1}\frac{1}{\rho^{(0)}}\nabla p^{(2)}\right] + v^{(0)}\cdot\nabla p + p\nabla\cdot v^{(0)} = -\frac{\partial p^{(0)}}{\partial t} - \frac{\partial p^{(1)}}{\partial t}$$

correct velocity

inner Iteration

outer Iteration

map variables →

← Interpolate $p^{(1)}$ & mom

acoustic solver

solve acoustic system

$$p = p^{(0)} + Mp^{(1)} + M^2 p^{(2)}$$

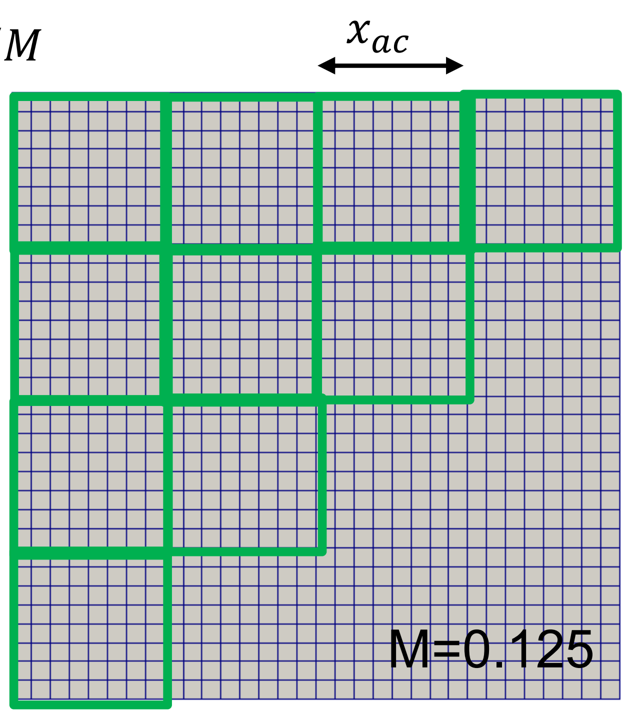
Implementation of the MPV Method in OpenFOAM

- Implementation is based on Munz et al. (2003)
  - Pressure decomposition
    - Thermodynamic pressure: $p^{(0)} = \frac{1}{|V|}\int_V p(x)dx$
    - Acoustic pressure $p^{(1)}$
      - $p^{(1)}$ constant on small spatial scale and varies on large scale
      - Reference length hydrodynamic scale: x_{ref}
      - Reference length acoustic scale: $x_{ac} = x_{ref}/M$

      $$p^{(1)} = \frac{1}{M}(\bar{p} - p^{(0)})$$

      $$\bar{p} = \frac{1}{|V_{ac}|}\int_{V_{ac}} p(x)dx$$
    - Hydrodynamic pressure

    $$p^{(2)} = \frac{1}{M^2}(p(x) - p^{(0)} - Mp^{(1)})$$



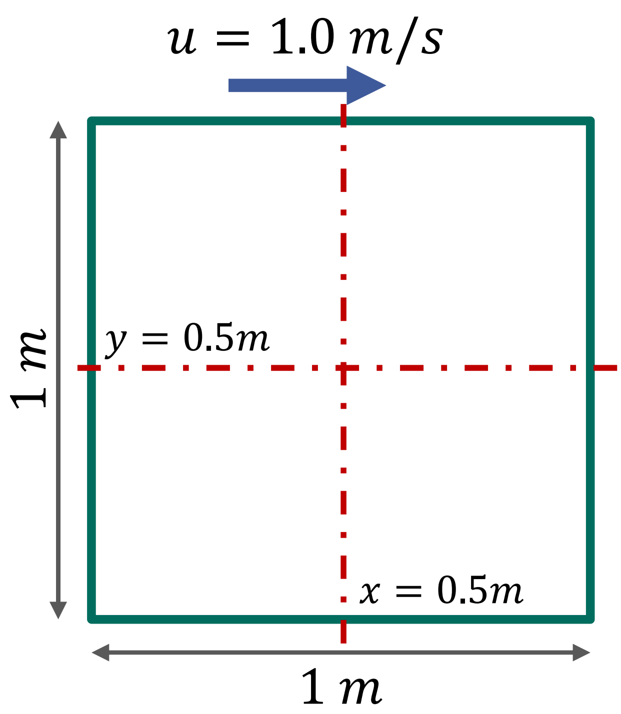
Validation

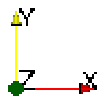
- Validation Matrix

		M→0 Limit	Heat Transfer & material properties	Mach number independent behavior	Interaction flow - acoustic	turbulence
aero acoustic	cavity	x				
	cavity heated	x	x			
	pressure pulse propagation			x		
	vortex box			x	x	
	co-rotating vortex pair			x	x	
liquid metal	backwards facing step	x				x
	heated rod	x	x			x
	…					
	target	x	x		x	x

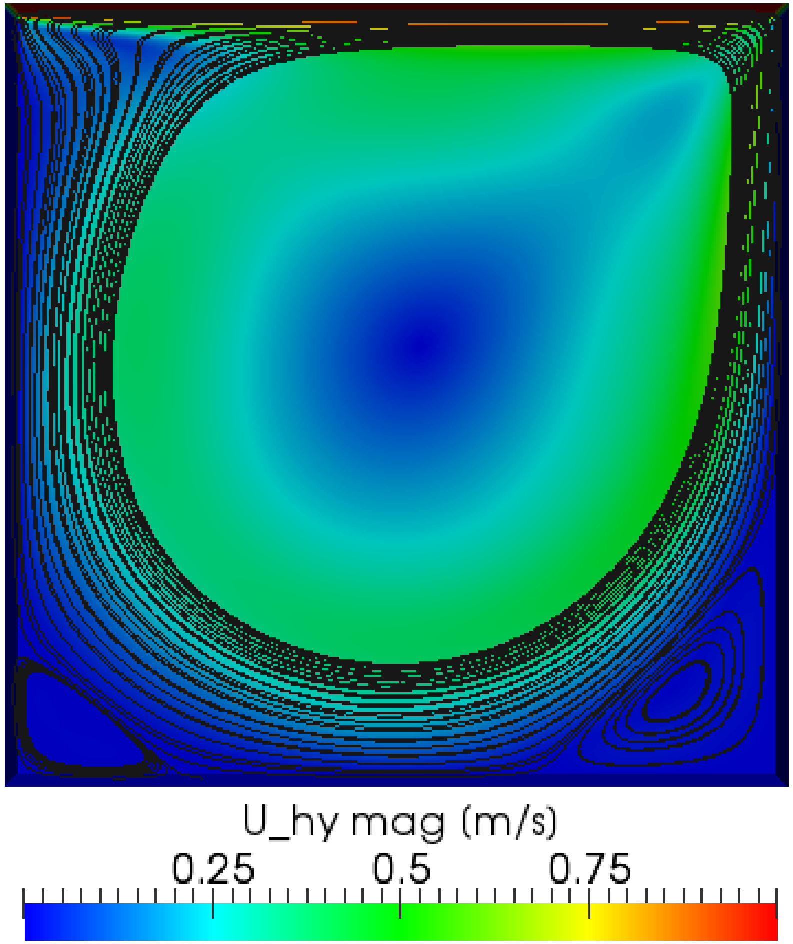
Test Cases – Aero acoustic

- Lid Driven Cavity
  - Laminar, Re=1000
  - M = 0
  - Mesh: 129 x 129 nodes



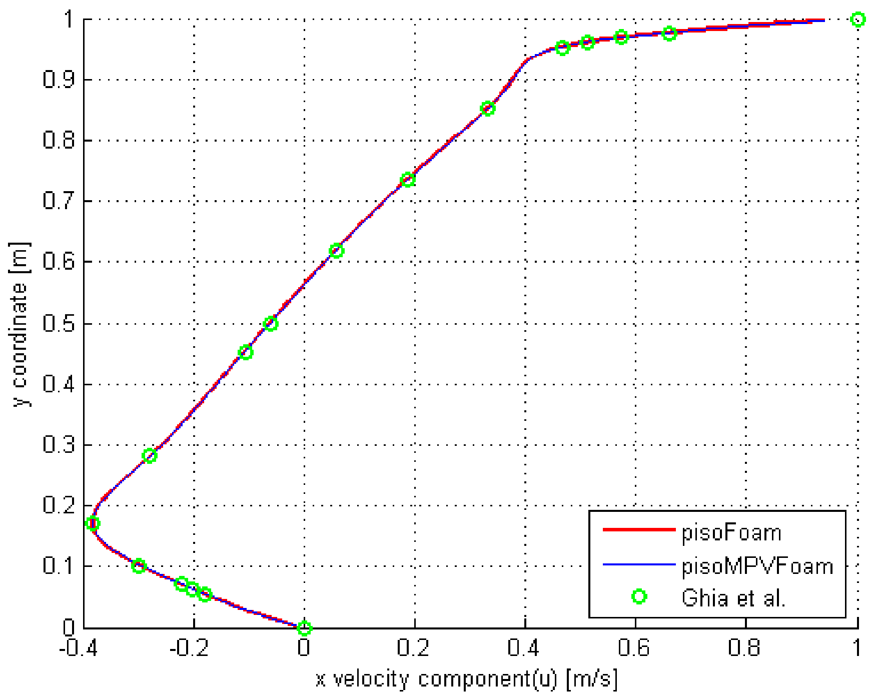


Stream tracer visualization

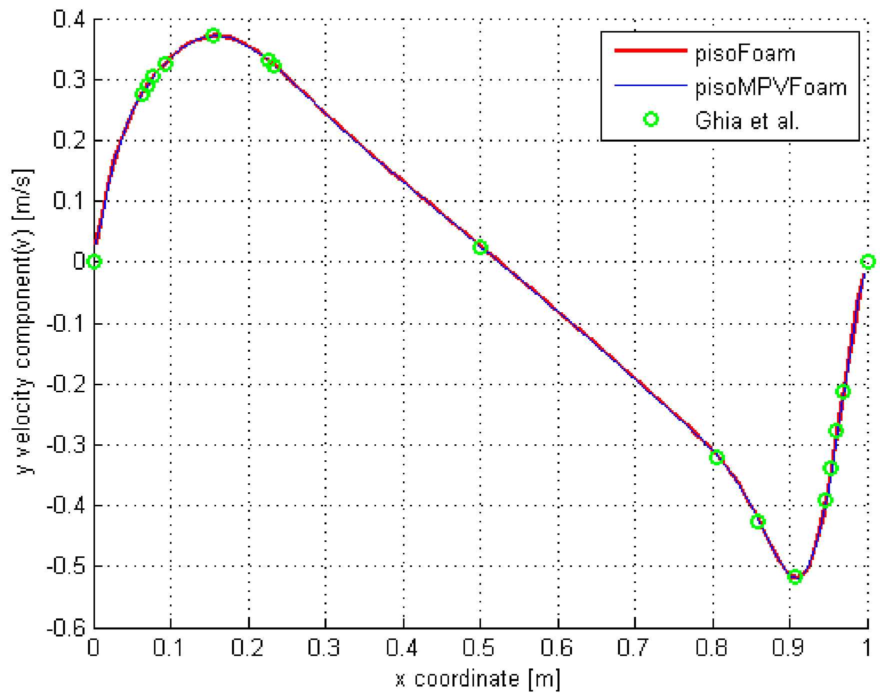


Test Cases – Aero acoustic

- Horizontal and vertical velocity along vertical/horizontal cross-section
  - pisoFoam and benchmark (Ghia et al.) comparison



Horizontal velocity at vertical cross-section
$x = 0.5m$

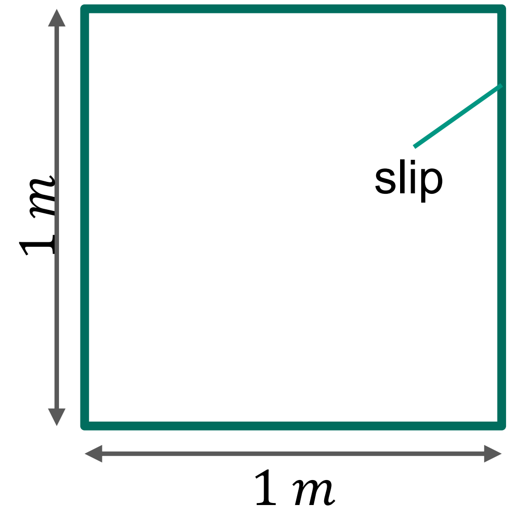


Vertical velocity at horizontal cross-section
$y = 0.5m$

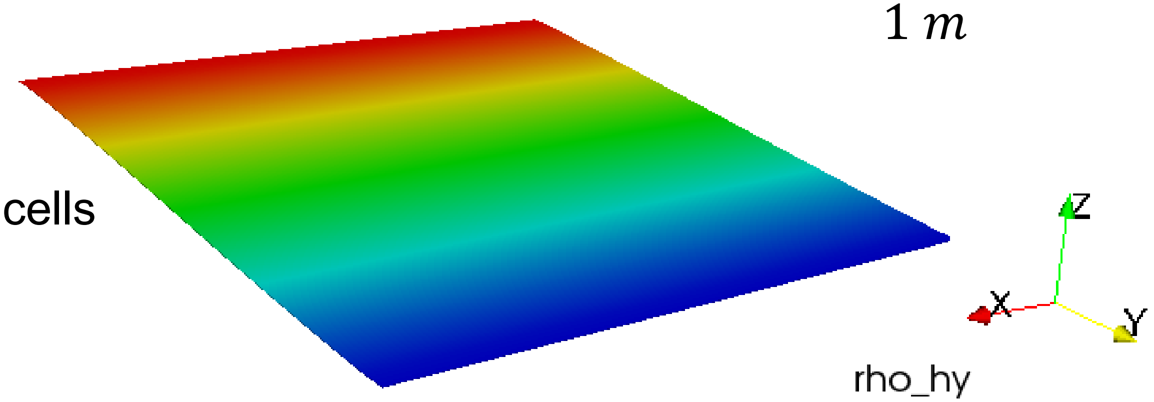
Test Cases – Aero acoustic

- „Vortex in a box“ – generation of acoustic waves due to a rotating vortex
  - Initial Conditions

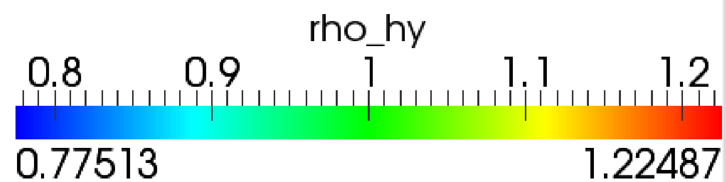
$$u(x,y,0) = 2sin^2(\pi x)\sin(\pi y)\cos(\pi y)$$
$$v(x,y,0) = -2sin(\pi x)\cos(\pi x)\,sin^2(\pi y)$$
$$\rho(x,y,0) = 1 - \frac{1}{2}\tanh\left(y - \frac{1}{2}\right)$$
$$p(x,y,0) = 1.0$$
$$M = 0.125$$



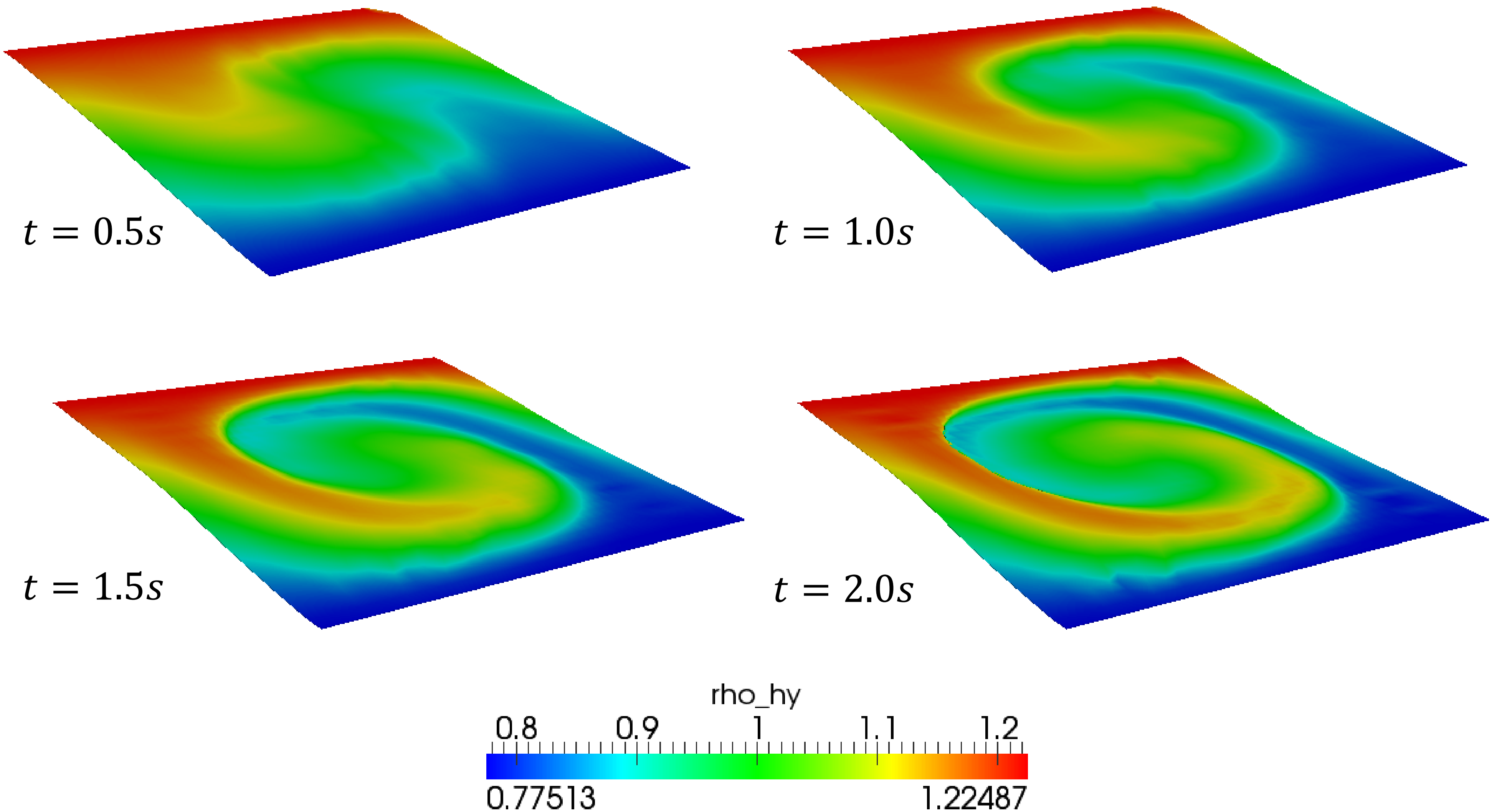
  - Mesh:
    - Hydrodynamic 32x32 cells
    - Acoustic 4x4 cells



Initial density distribution

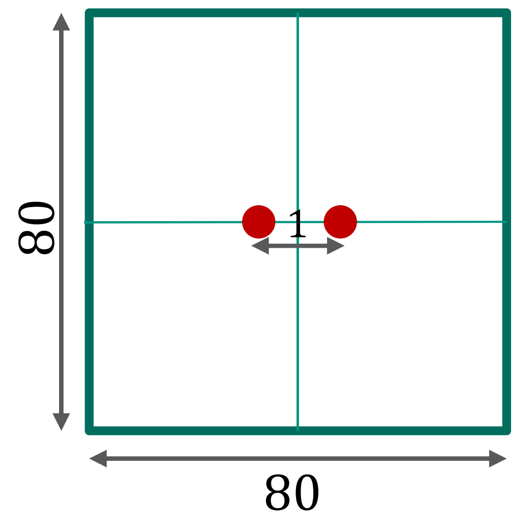


Test Cases – Aero acoustic



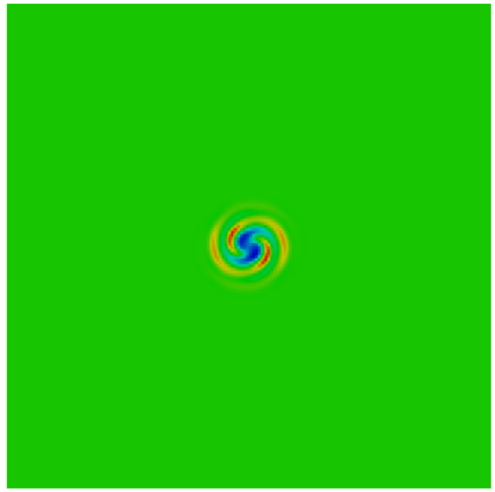
Test Cases – Aero acoustic

- Generation of Acoustic waves by „Co-rotating vortex pair"
  - Initial conditions
    - $u(r) = \frac{\Gamma}{2\pi r^2}\left(1 - e^{-\alpha (r/r_c)^2}\right)$ $\Gamma = \frac{4\pi}{1 - e^{-4\alpha/r_c^2}}$
    - p=1 bar
    - $\rho = p^{\frac{1}{\gamma}}$ $\rho_0 = 1\ [kg/m^3]$
    - M = 0.095
  - Mesh
    - Hydrodynamic: 100 000 cells
    - Acoustic: 5184 cells

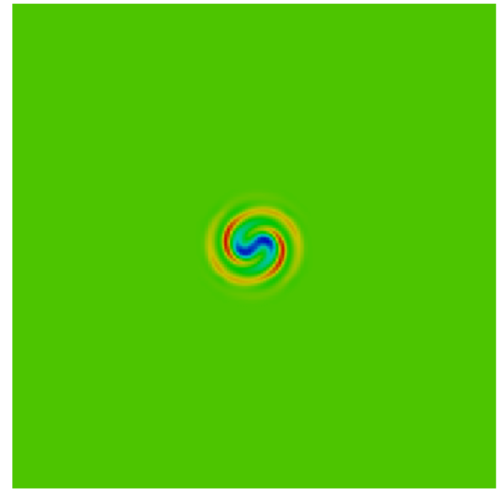


Test Cases – Aero acoustic

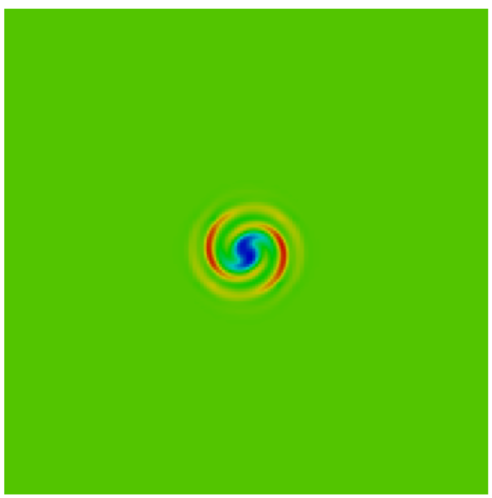
t =0.9 s



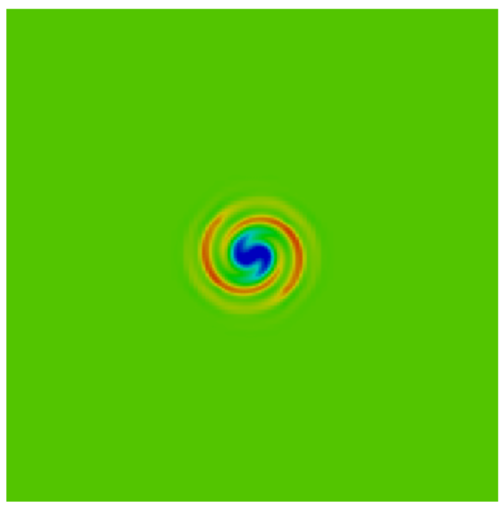
t = 1.2 s



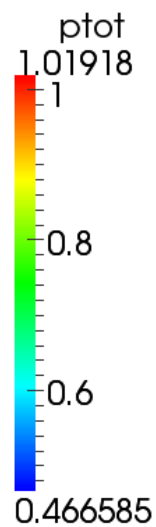
t = 1.5 s



t = 1.8 s



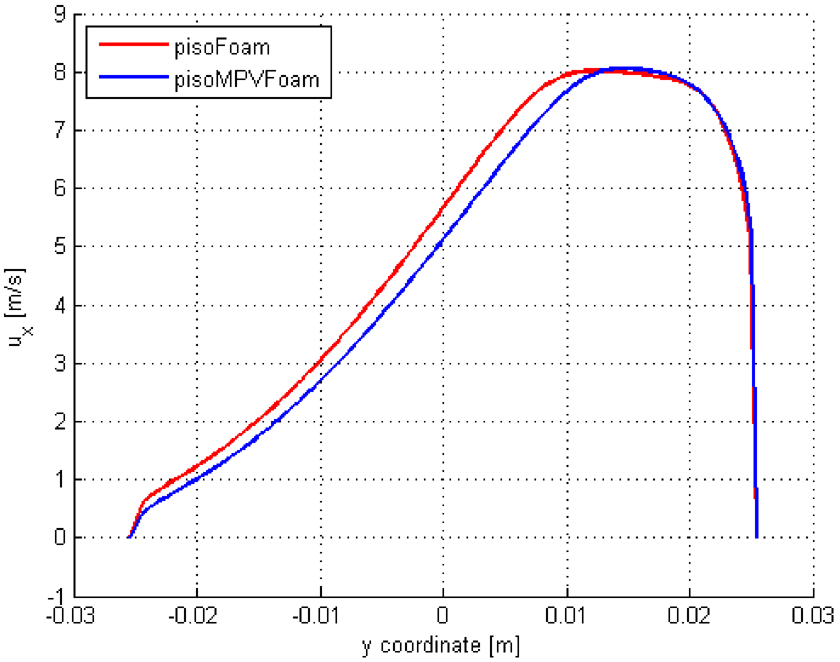
t = 2.1 s



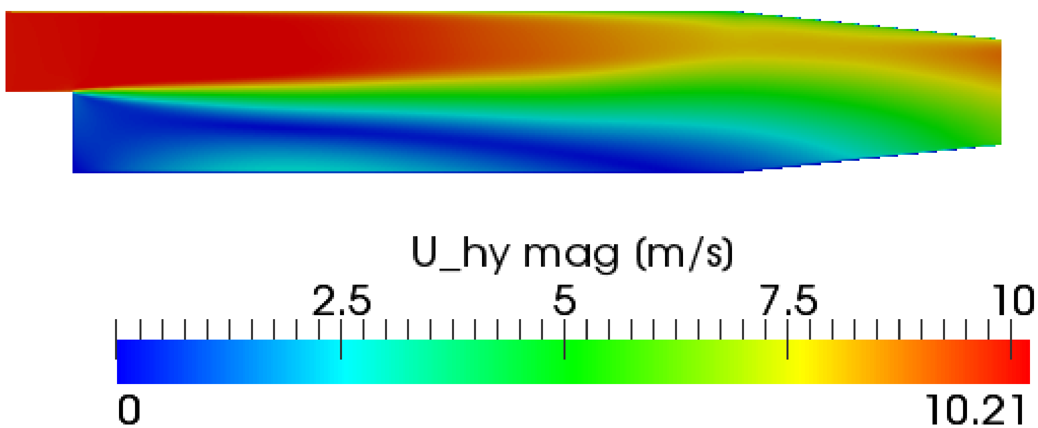
Vortex merging

Test cases liquid metal

- Backwards facing step
  - Inlet velocity 10 m/s
  - k-ε turbulence model
  - LBE @ T=450K



x-velocity component at x = 0.2m vertical cut



Conclusion & Outlook

- MPV approach proposed for simulation of pulse-induced pressure wave/ hydrodynamics interaction
- Low extra numerical effort for solving acoustics
- Suitable for design optimization

- Implementation based on OpenFOAM architecture (pisoFoam)
- Extensive ongoing validation

- Design measures to limit effects of pressure waves in META:LIC target will be analyzed

- Input on experimental data on pressure waves is appreciated for validation

References

1. R. Klein, „Semi-implicit extension of a Godunov-type scheme based on low Mach number asymptotics I: One dimensional flow“ Journal of Computational Physics, 121, p 213-237, 1995
2. C.-D. Munz, S. Roller, R. Klein, K.J. Geratz, „The extension of incompressible flow solvers to the weakly compressible regime“ Journal of Computers and Fluids, 32 (2003) 173-196