Note on the Calculation of Meson (π, K) production distributions for different proton bunch length

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Abstract

The meson production distribution created by an extremely short proton bunch can be used as a Green function to calculate the distribution for arbitrary variance σ_p^2.

I. INTRODUCTION

Recent studies on the efficiency *vs.* proton bunch length of the ST2A front end of a Neutrino Factory [1] have shown that the number of muons per proton on target (*i.e.* efficiency) within the canonical acceptance ($A_T = 30$ mm and $A_L = 150$ mm) is nearly linear with the width of the proton bunch distribution represented by the standard variation σ_p.

II. CALCULATIONS

There are two distinct ways to perform the calculations:

- we let MARS-14 [2] to generate Gaussian proton bunch of given σ_p and then we collect the generated π's and K's at a z=constant plane set at the end of the target.
- we collect, at the same plane, π's and K's generated by an extremely short proton bunch and assume, this distribution, to be a *kernel* to be used in a Green function method for any given σ_p.

In other words, in the first case we use a Gaussian proton beam; in the second it is a meson Gaussian beam which is used. We show next that both methods lead to identical results.

In very general terms, the π and K distribution is given by

$$\rho_\pi\left(\{x\},t\right) = \int d\{y\}dt'\mathcal{G}\left(\{x\}-\{y\},t-t'\right)\rho_P(\{y\},t') \tag{1}$$

where the argument $\{x\}$ represents all the position and momentum variables and t is the time of flight; $\mathcal{G}\left(\{x\},t\right)$ stands for the physics of production processes.

We assume that we can write for the proton distribution

$$\rho_P^\delta\left(\{y\},t\right) = \Lambda_P\left(\{y\}\right)\delta(t) \quad , \quad \rho_P^G\left(\{y\},t\right) = \Lambda_P\left(\{y\}\right)\exp(-\frac{t^2}{2\sigma_P^2})/\sqrt{2\pi}\sigma_P \tag{2}$$

for a delta function and a Gaussian proton distribution of variance σ_P^2, respectively.

The relation between mesons distributions, with ρ_π^δ as a kernel is given by

$$\rho_\pi^G\left(\{x\},t\right) = \int \frac{dt'}{\sqrt{2\pi}\sigma_P}e^{-\frac{(t-t')^2}{2\sigma_P^2}}\rho_\pi^\delta\left(\{x\},t'\right) \tag{3}$$

using Eqs. 1 and 2, we substitute $\rho_\pi^\delta\left(\{x\},t'\right)$ to get

$$\rho_\pi^G\left(\{x\},t\right) = \int d\{y\}dt'\mathcal{G}\left(\{x\}-\{y\},t-t'\right)\rho_P^G(\{y\},t') \tag{4}$$

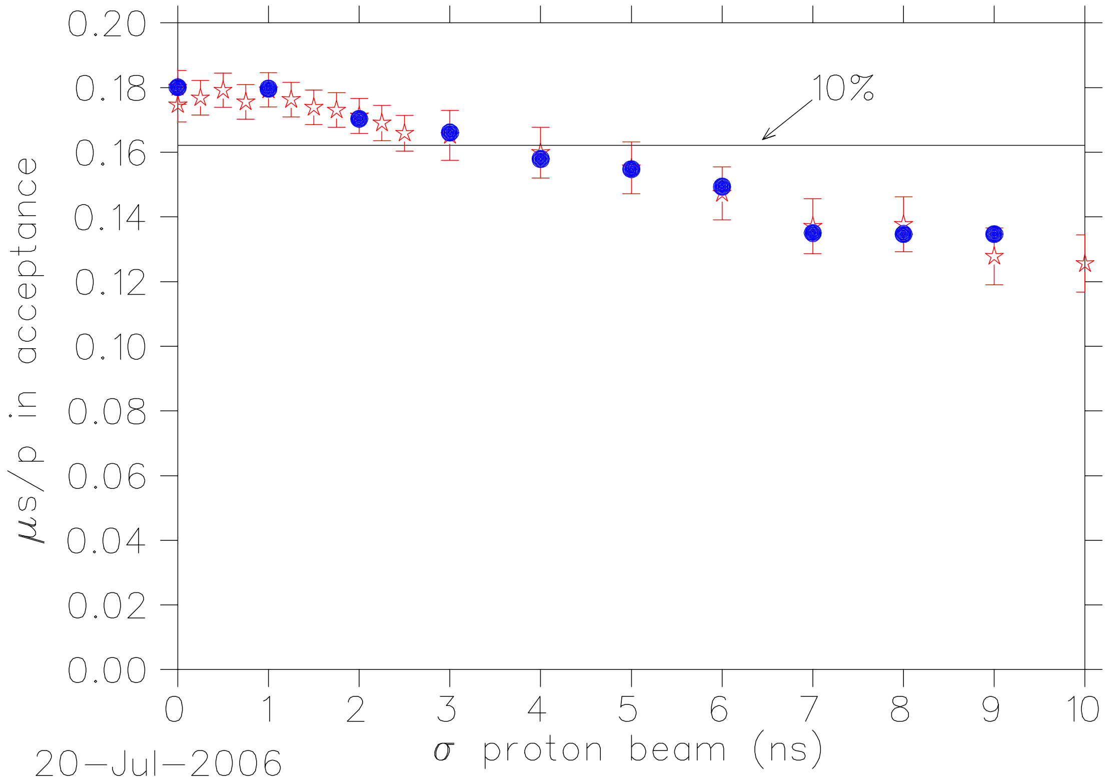

FIG. 1: (Color) Efficiency *vs.* proton bunch length; blue circle symbol is for the first approach and the red star is for the second one. The horizontal line represent the 10% reduction in efficiency.

This proves the assertion. Consequently, we expect that simulations with both alternative procedures will yield similar results. In Fig.1 we plot the efficiency *vs.* proton bunch length. Clearly, both results are essentially the same within the $\frac{1}{\sqrt{N}}$ statistical error.

We now discuss the implementation of the second approach; the discrete meson distribution produced by a very short proton bunch is represented by the function

$$\rho_\pi^\delta\left(\{x\}, t\right) = \frac{1}{\sum_i w_i} \sum_{i=1}^{N} A(\{x\}, t_i) w_i \delta(t - t_i) \tag{5}$$

where t_i is the time of flight assigned to each particle i with weight w_i; N is the total number of mesons sample particles created by MARS, assuming a fixed number of initial protons and $A(\{x\}, t_i) = \int d\{y\} \mathcal{G}(\{x\} - \{y\}, t_i) \Delta_P(\{y\})$.

To generate a Gaussian of variance σ_P^2 we perform the transformation $u_i = t_i + \sigma_P G_i$ where $G_i = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right)$ is a Gaussian random function of variance $V[t] \equiv \sigma_P^2 = 1$ and average value $E[t] = 0$.

The meson distribution $\rho_\pi(u)$, with u the sum of two independent random variables (t_i and G) is constructed as a convolution product

$$\begin{aligned} \rho_\pi^G(\{x\}, u) &= \int dt \frac{1}{\sigma_P} G(\frac{(u-t)}{\sigma_P}) \rho_\pi^\delta(\{x\}, t) \\ &= \frac{1}{\sum_i w_i} \sum_{i=1}^{N} A(\{x\}, t_i) w_i \frac{\exp\left(-(u-t_i)^2/2\sigma_P^2\right)}{\sqrt{2\pi}\sigma_P}. \end{aligned} \tag{6}$$

Using this last expression and assuming that $A(\{x\}, t_i)$ is time independent, it is not difficult to show that

$$E[u] = E[t] \quad \text{and} \quad V[u] \equiv E[(u - E[u])^2] = \sigma_P^2 + V[t]$$

$i.e$ the mean value is unchanged and the variance of the sum distribution is equal to the sum of the variances.

III. SUMMARY

We have shown that the meson production distribution for any variance σ_P^2 can be obtained by convolution of a meson distribution produced by a delta function proton beam with a Gaussian distribution of mean value $E[t] = 0$ and variance σ_P^2. In addition we gave the expression of the meson distribution as a discrete function (see Eq. 6).

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[1] J.S.Berg *et al.*, Phys. Rev. ST Accel. Beams **9**, 011001 (2006).

[2] N. Mokhov, `http://www-ap.fnal.gov/MARS/`, nucl-th/9812038.