The dynamics of mercury flow in a curved pipe

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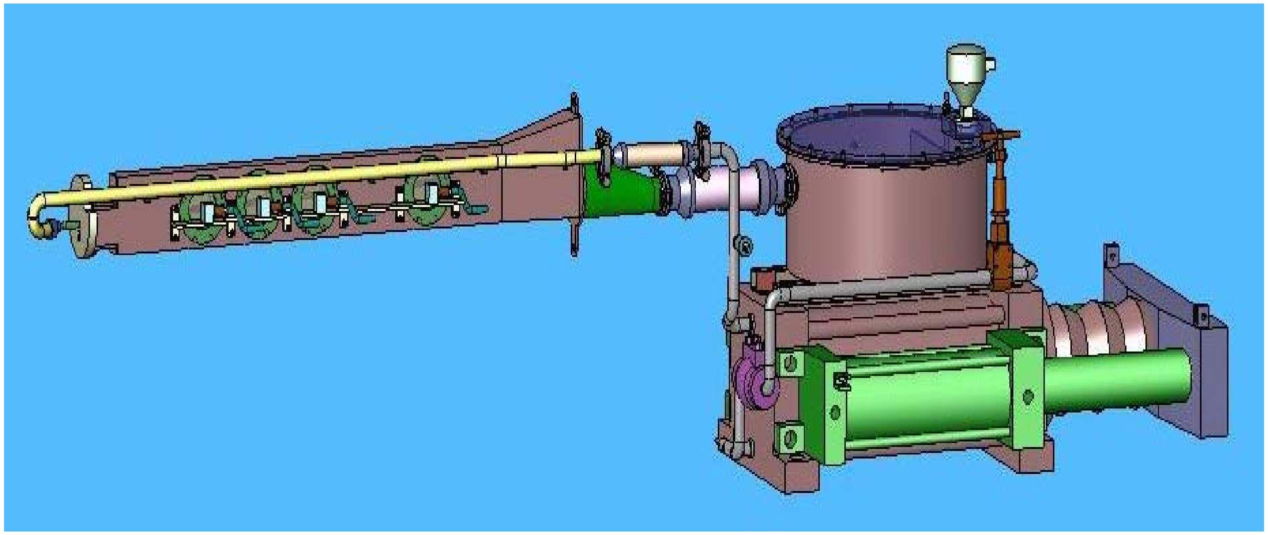
Outline

- Motivation
- Objective
- Previous work
- Scheme of the problem
- Pipe curvature effect
- Laminar flow in the mercury supply pipe
- Conclusion

Motivation

- Liquid **<u>mercury</u>** as a potential high-Z target for Moun Collider Accelerator Project.
- Target delivery systems involves pipe curvature, axially-dependent radius, nozzle diameter and nozzle length etc.
- Proper nozzle design to achieve a less turbulent jet at the nozzle outlet.



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Objective

- Study the dynamics of mercury flow in the target delivery system
- Obtain a basic physical understanding of this internal flow problem for achieving a proper nozzle design
- Start with laminar mercury flow in curved pipe first

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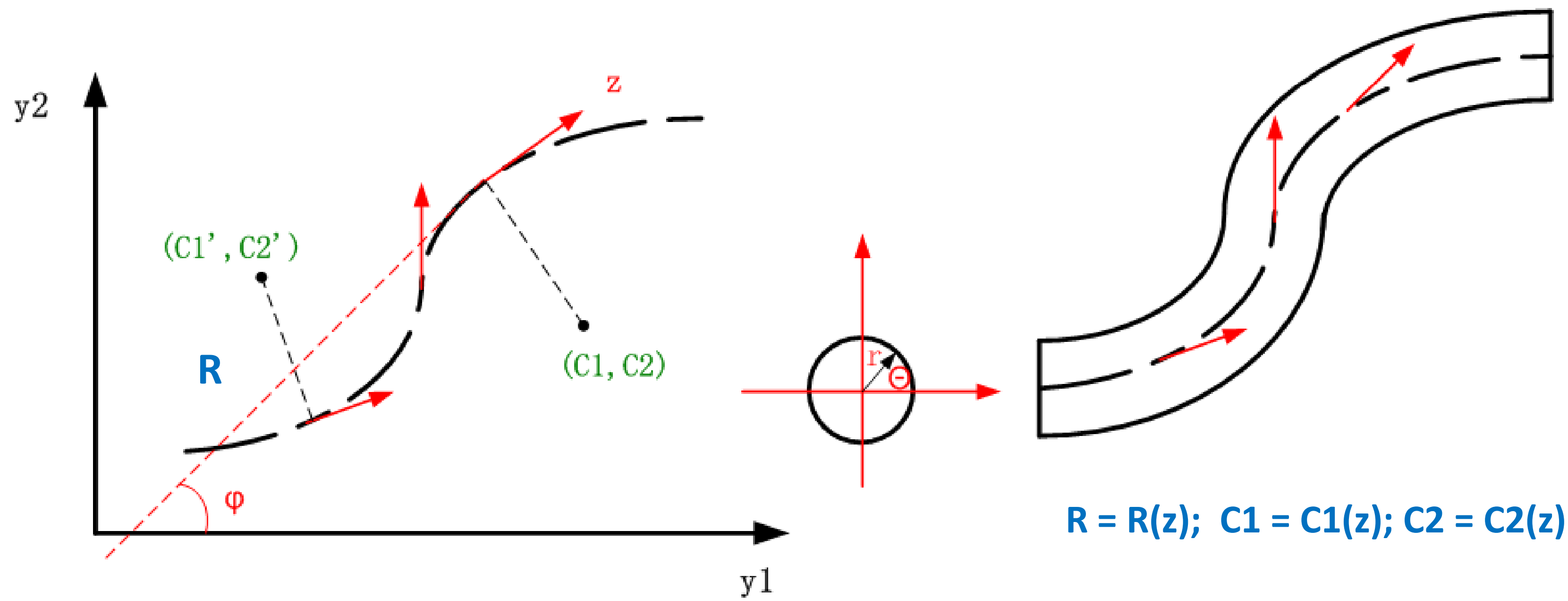
Scheme of the problem

— Equations applied to pipe of arbitrary curvature (1)



R = R(z); C1 = C1(z); C2 = C2(z)

Cartesian (y1, y2, y3) ⟹ Curvilinear (z, r, θ)

Continuity equation

$$\frac{R}{R+r\sin\theta}\frac{\partial u_z}{\partial z}+\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\sin\theta}{R+r\sin\theta}u_r+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{\cos\theta}{R+r\sin\theta}u_\theta=0$$

Scheme of the problem

— Equations applied to pipe of arbitrary curvature (2)

z-momentum equation

$$\frac{\partial u_z}{\partial t} + \frac{R}{R+r\sin\theta}u_z\frac{\partial u_z}{\partial z} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + \frac{\sin\theta}{R+r\sin\theta}u_r u_z + \frac{\cos\theta}{R+r\sin\theta}u_\theta u_z = -\frac{1}{\rho}\frac{R}{R-r\sin\theta}\frac{\partial P}{\partial z}$$

$$-\nu\left[\left(\frac{1}{r}+\frac{\partial}{\partial r}\right)\left(\frac{\partial u_z}{\partial r}+\frac{\sin\theta}{R+r\sin\theta}u_z\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\cos\theta}{R+r\sin\theta}u_z\right) - \left(\frac{1}{r}+\frac{\partial}{\partial r}\right)\frac{R}{R+r\sin\theta}\frac{\partial u_r}{\partial z}\right.$$

$$\left.-\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{R}{R+r\sin\theta}\frac{\partial u_\theta}{\partial z}\right)\right] - \nu\frac{\sin\theta u_r + \cos\theta u_\theta - r\sin\theta(\partial u_z/\partial z)}{(R+r\sin\theta)^3}R\frac{dR}{dz}$$

r-momentum equation

$$\frac{\partial u_r}{\partial t} + \frac{R}{R+r\sin\theta}u_z\frac{\partial u_r}{\partial z} + u_r\frac{\partial u_r}{\partial r} + \frac{1}{r}u_\theta\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} - \frac{\sin\theta}{R+r\sin\theta}u_z^2 = -\frac{1}{\rho}\frac{\partial P}{\partial r}$$

$$+\nu\left[\left(\frac{1}{r}\frac{\partial}{\partial \theta}+\frac{\cos\theta}{R+r\sin\theta}\right)\left(\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}-\frac{1}{r}\frac{\partial u_r}{\partial \theta}\right) - \frac{R^2}{(R+r\sin\theta)^2}\frac{\partial^2 u_r}{\partial z^2}\right.$$

$$\left.+\frac{R}{R+r\sin\theta}\left(\frac{\partial^2 u_z}{\partial z\partial r}+\frac{\sin\theta}{R+r\sin\theta}\frac{\partial u_z}{\partial z}\right)\right] + \nu\frac{\sin\theta u_z + r\sin\theta(\partial u_r/\partial z)}{(R+r\sin\theta)^3}R\frac{dR}{dz}$$

θ-momentum equation

$$\frac{\partial u_\theta}{\partial t} + \frac{R}{R+r\sin\theta}u_z\frac{\partial u_\theta}{\partial z} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} - \frac{u_z^2\cos\theta}{R+r\sin\theta} = -\frac{1}{\rho}\frac{1}{r}\frac{\partial P}{\partial \theta} + \nu\left[\frac{R^2}{(R-r\sin\theta)^2}\frac{\partial^2 u_\theta}{\partial z^2}\right.$$

$$\left.\frac{1}{r(R+r\sin\theta)}\frac{\partial^2 u_z}{\partial z\partial \theta} - \frac{R\cos\theta}{(R+r\sin\theta)^2}\frac{\partial u_z}{\partial z} + \left(\frac{\partial}{\partial r}+\frac{\sin\theta}{R+r\sin\theta}\right)\left(\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}-\frac{1}{r}\frac{\partial u_r}{\partial \theta}\right)\right]$$

$$+\nu\frac{r\sin\theta(\partial u_\theta/\partial z) + \cos\theta u_z}{(R-r\sin\theta)^3}R\frac{dR}{dz}$$

Scheme of the problem

— Analytic solution for fully developed flow (1)

To get the Analytical solutions, assumptions are needed as follows:

a. Isothermal newtonian laminar flow

b. Incompressible (dose not depend on the pressure)

c. Fully developed (d()/dz=0, , except P ; d()/dt=0)

d. Constant small curvature (dR/dz=0, a/R << 1)

Scheme of the problem

— Analytic solution for fully developed flow (1)

- W.R.Dean's solution*

$$u_r / u_0 = na \sin\theta (1 - r'^2)^2 (4 - r'^2) / 288R$$

$$u_\theta / u_0 = na \cos\theta (1 - r'^2)(4 - 23r'^2 + 7r'^4) / 288R$$

$$u_z / u_0 = (1 - r'^2)[1 - \frac{3r\sin\theta}{4R} + \frac{n^2 r\sin\theta}{11520\ R}(19 - 21r'^2 + 9r'^4 - r'^6)]$$

Where $u_0 = Aa^2, n = Aa^3/\nu, r' = r/a,$

A is a constant referring to the pressure gradient.

Further th e stream function in the pipe cross - section is

$$\sec\theta = kr'(1 - r'^2)^2(1 - r'^2/4)$$

Where k is an arbitray constant.

* W.R. Dean, Note on the motion of fluid in a curved pipe, Imperial College of Science, 1927

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- Pipe curvature effect
  - Straight pipe
  - Curved pipes (δ=0.5;δ=0.013)
  - Comparisions
- Laminar flow in the mercury supply pipe
- Conclusion

Pipe curvature effect

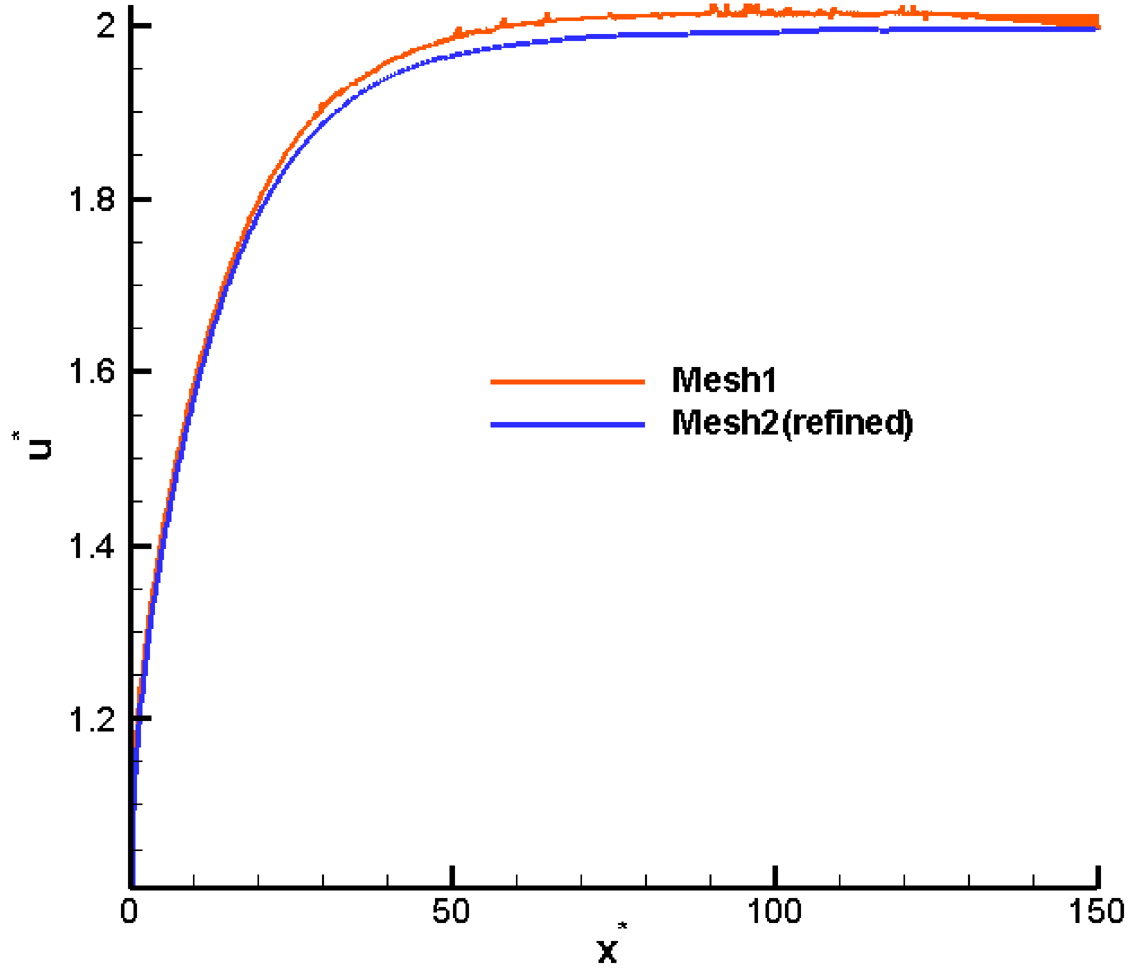
— Straight pipe (1)

Reynolds number	**1000**	
Pipe diameter	1.127 mm	
Pipe length	150a	
Inlet condition	Uniform inlet velocity 0.1m/s and static pressure of 18.5bar	
Mesh1 ($N_z \times N_r \times N_\theta$)	Axial direction Radial direction Circumferential direction Total	500 (uniform) 48 (Δ=0.01) 24 576000
Mesh2 ($N_z \times N_r \times N_\theta$)	Axial direction Radial direction Circumferential direction Total	1000 (uniform) 56 (Δ=0.005) 24 1344000

Pipe curvature effect

— Straight pipe (2)

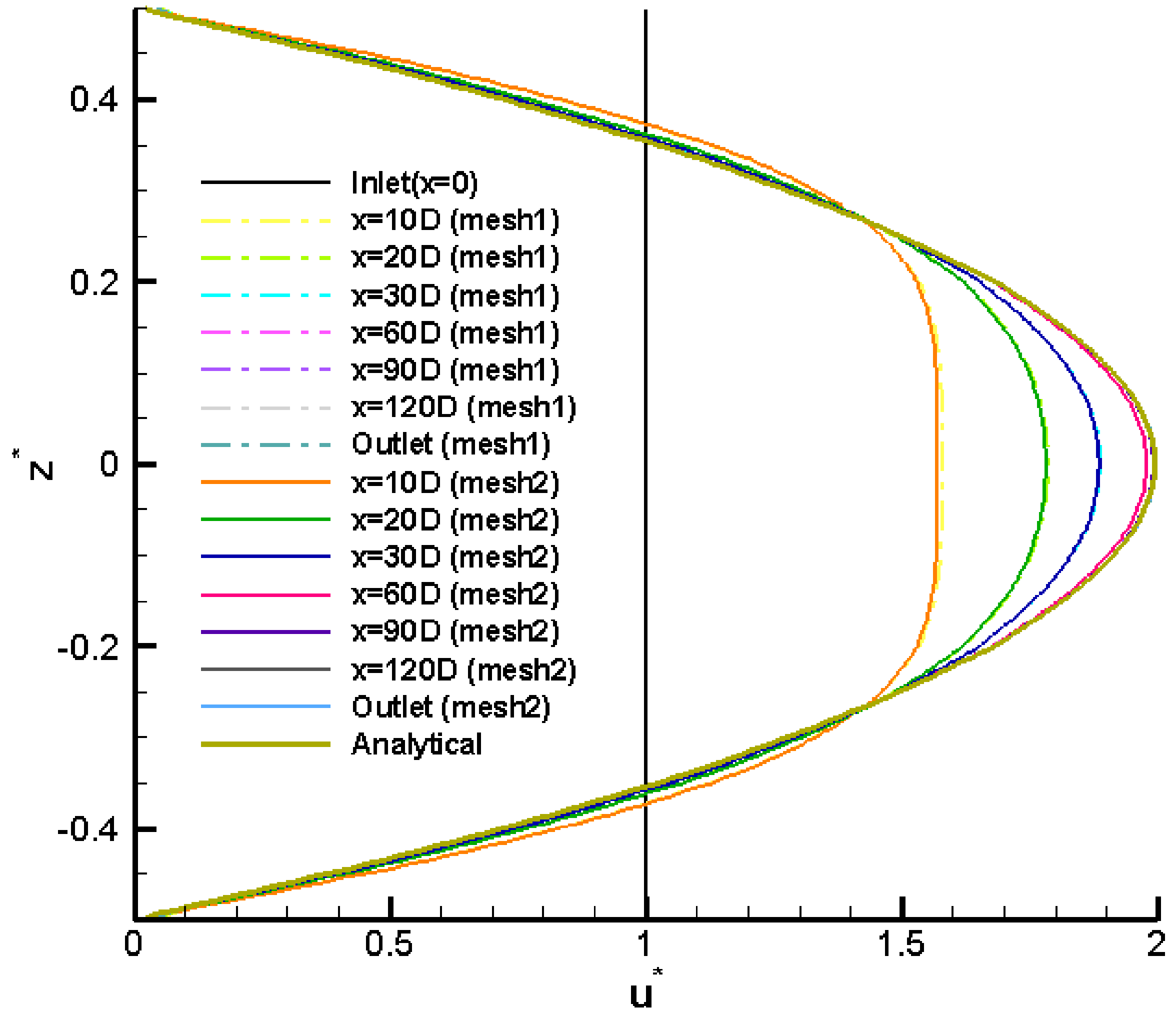
Fig.1 Mesh comparison for the axial velocity along the center line



Pipe curvature effect

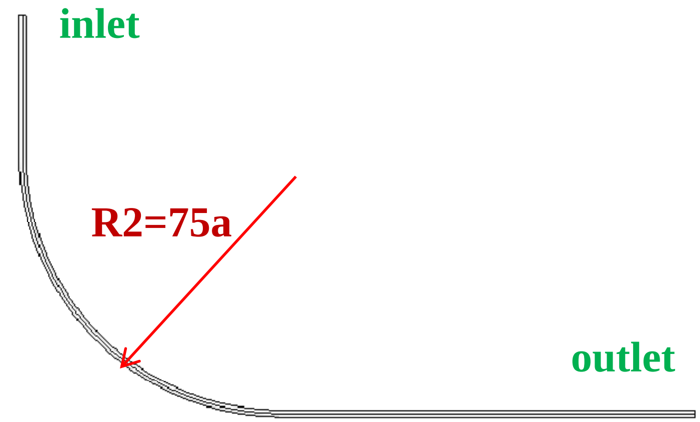
— Straight pipe (3)

Fig.2 Axial velocity profile comparison for different mesh



Pipe curvature effect

— Curved pipe (1)

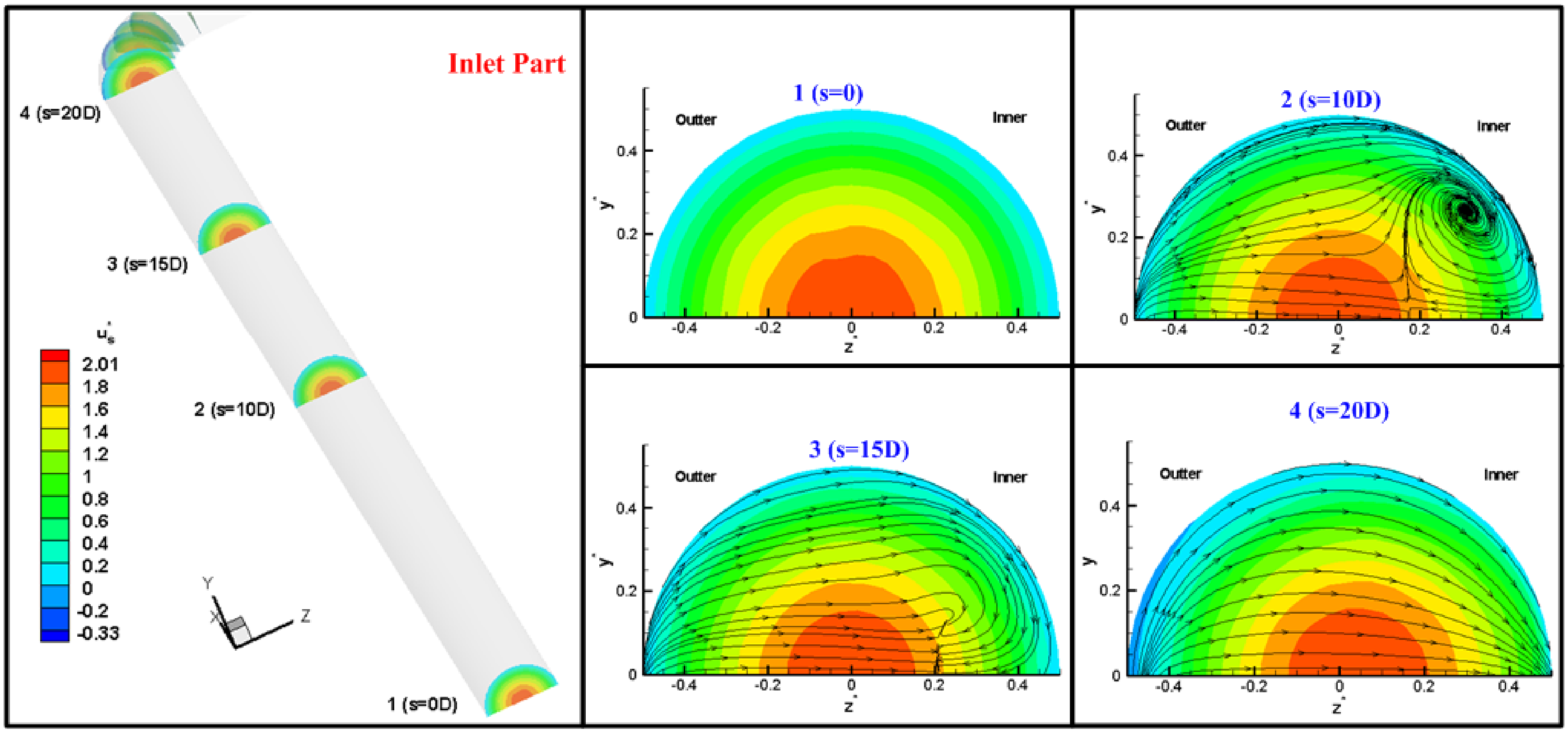


Reynolds number	1000	
Pipe diameter	1.127 mm	
Pipe Length	20 diameter before bend and 60 diameter after bend	
Inlet condition	Fully developed velocity profile and static pressure of 18.5bar	
Mesh for Curvature1 ($\delta 1=0.5$)	Axial direction Radial direction Circumferential direction Total	586 56 ($\Delta=0.01$) 24 787584
Mesh for Curvature2 ($\delta 2=0.013$)	Axial direction Radial direction Circumferential direction Total	1560 56 ($\Delta=0.005$) 24 2096640

Pipe curvature effect

— Curved pipe (2)

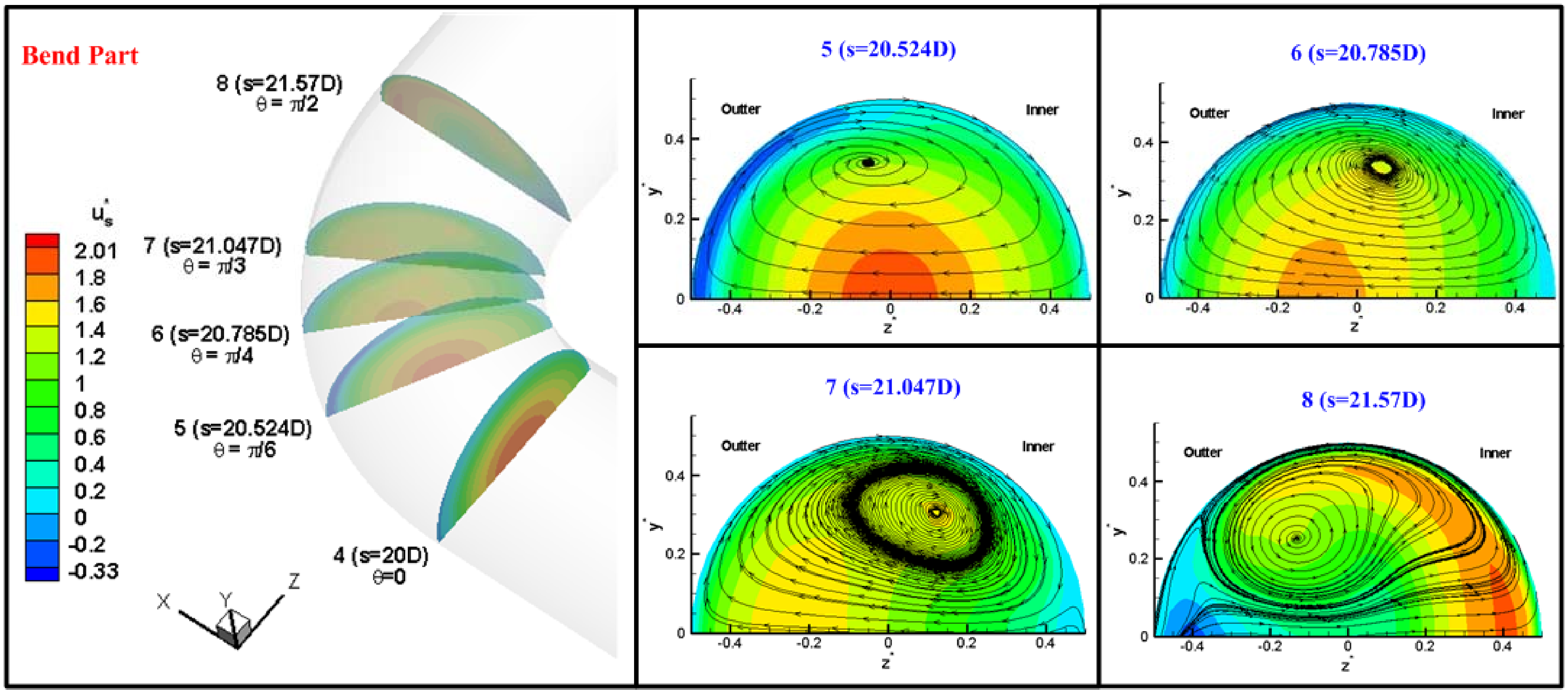
Fig.3 Numerical results for pipe of curvature of 0.5 at the inlet part



Pipe curvature effect

— Curved pipe (3)

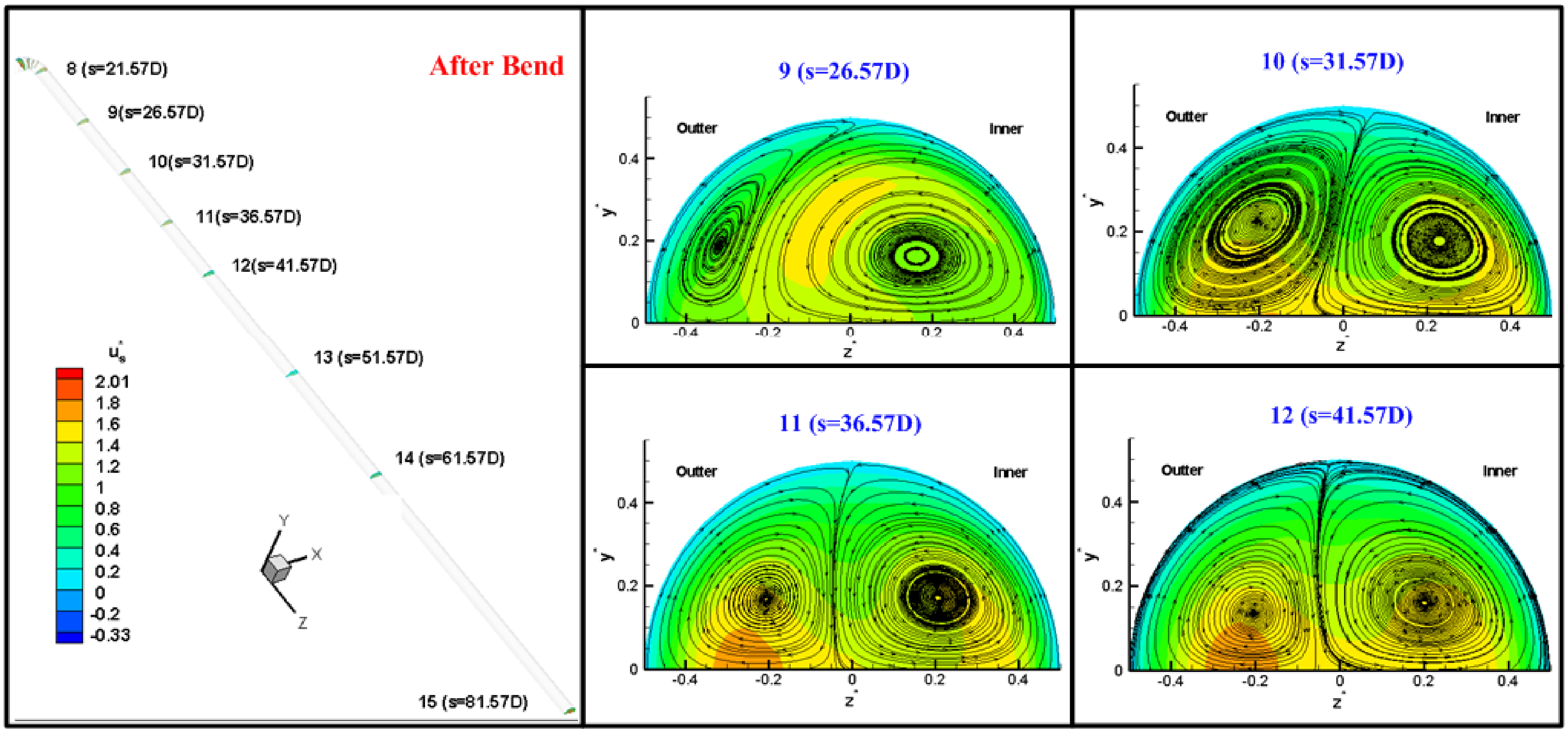
Fig.4 Numerical results for pipe of curvature of 0.5 at the bend part



Pipe curvature effect

— Curved pipe (4)

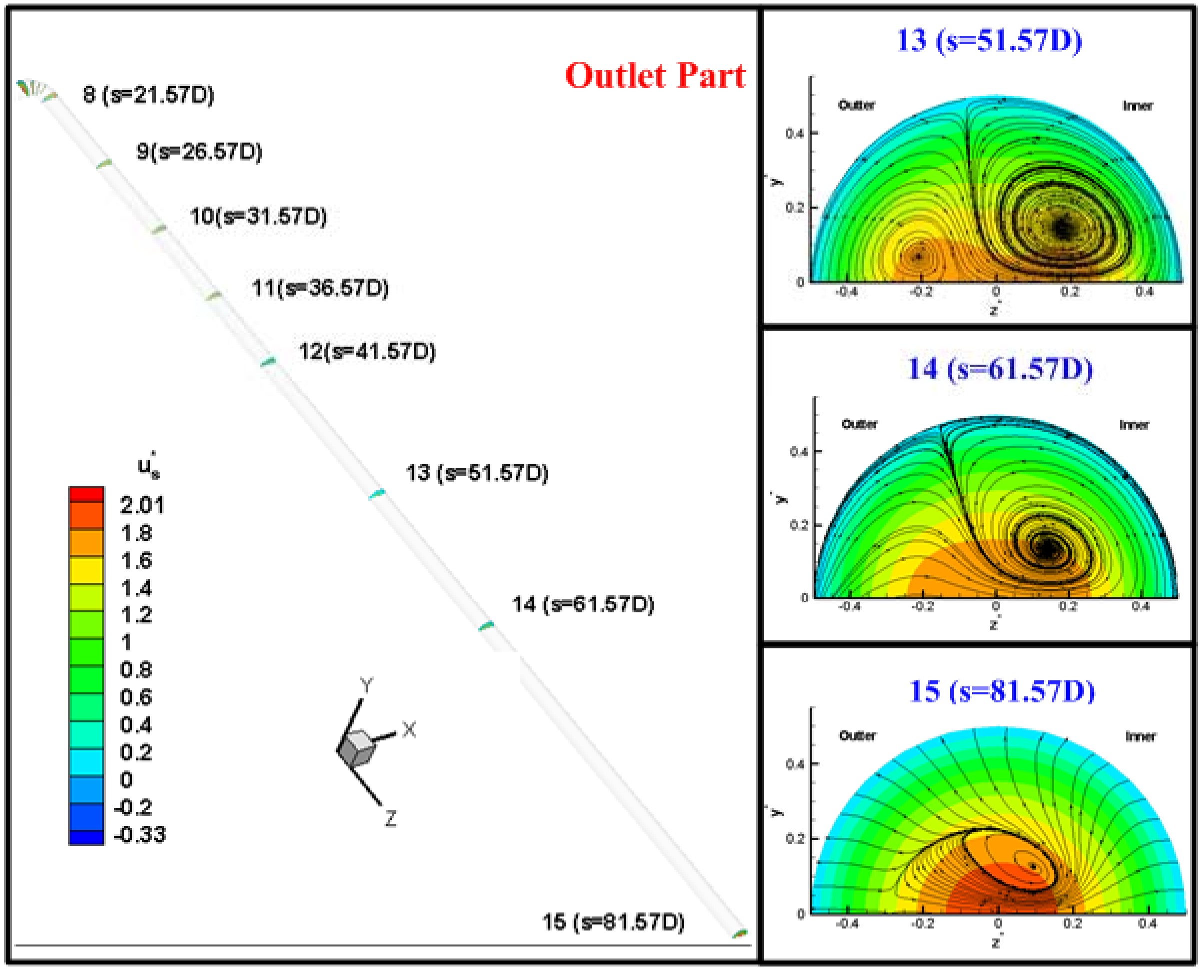
Fig.5 Numerical results for pipe of curvature of 0.5 after the bend part



Pipe curvature effect

— Curved pipe (5)

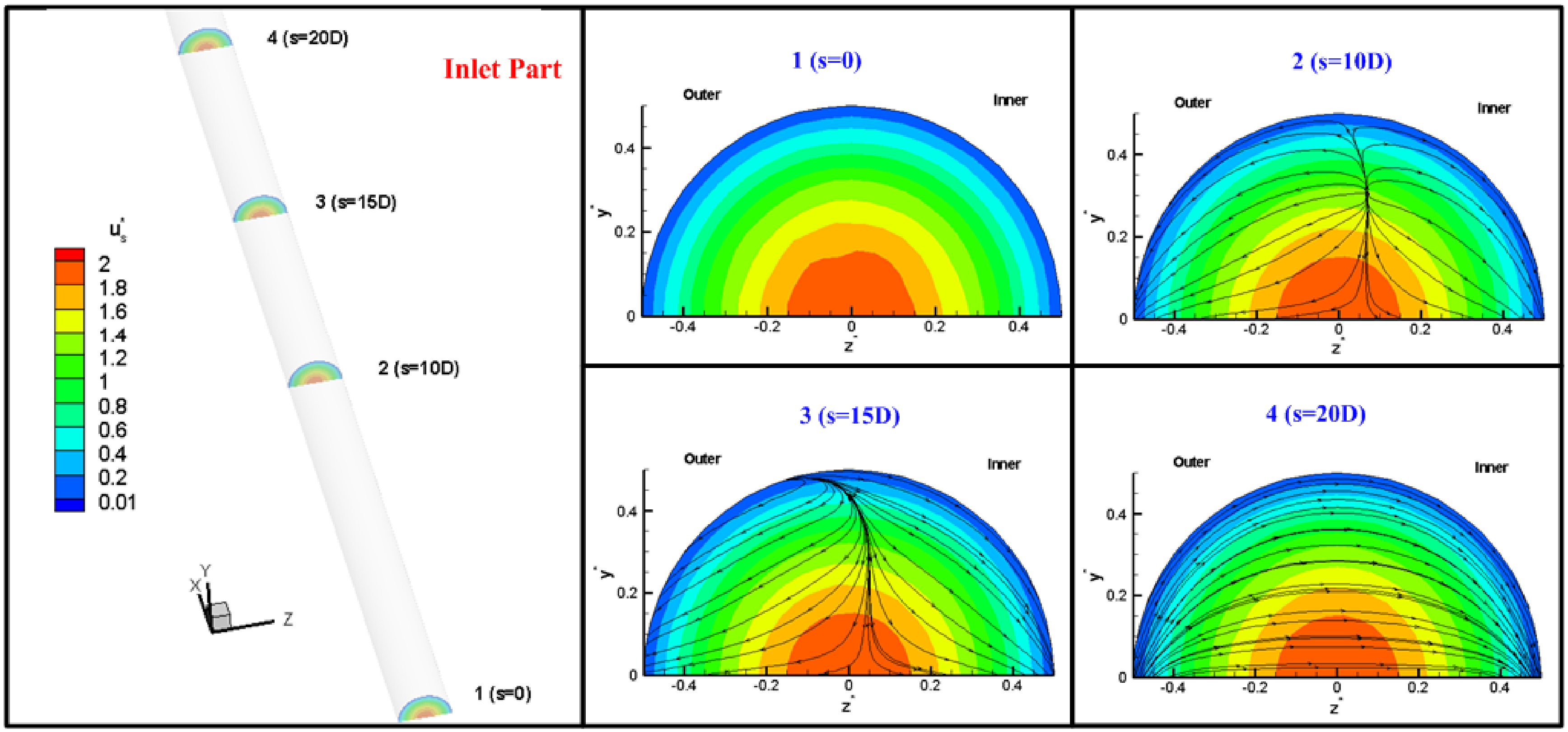
Fig.6 Numerical results for pipe of curvature of 0.5 at the outlet part



Pipe curvature effect

— Curved pipe (6)

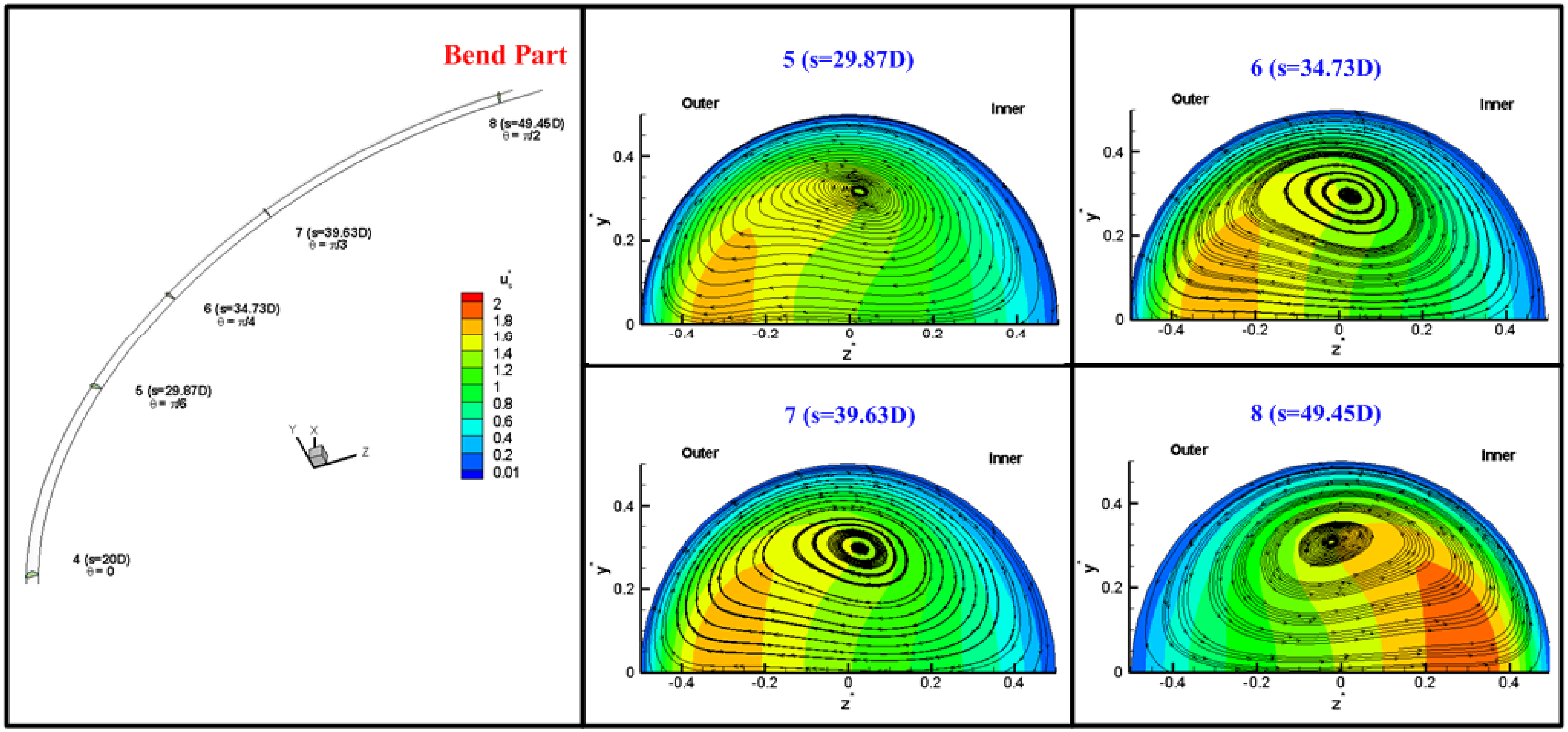
Fig.7 Numerical results for pipe of curvature of 0.013 at the inlet part



Pipe curvature effect

— Curved pipe (7)

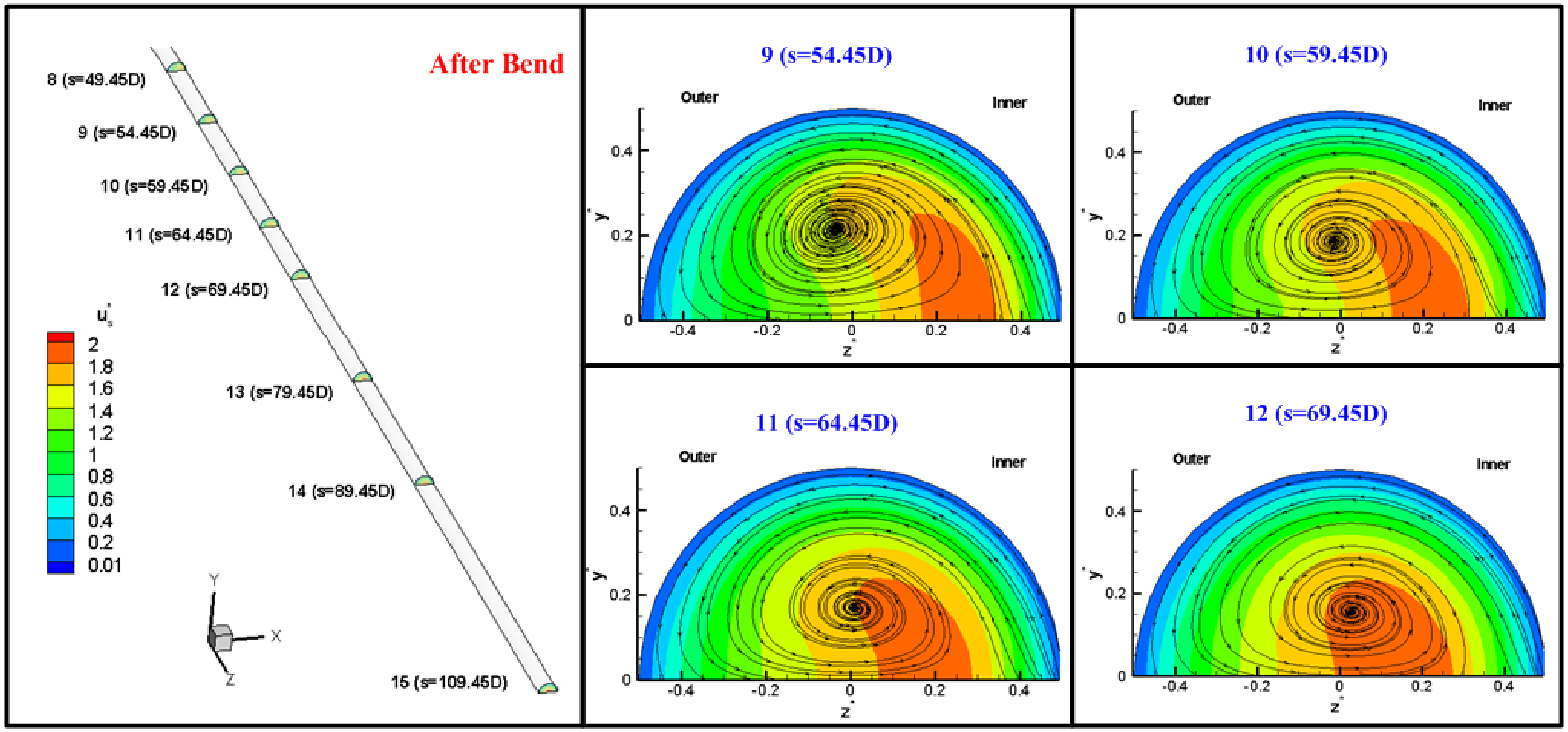
Fig.8 Numerical results for pipe of curvature of 0.013 at the bend part



Pipe curvature effect

— Curved pipe (8)

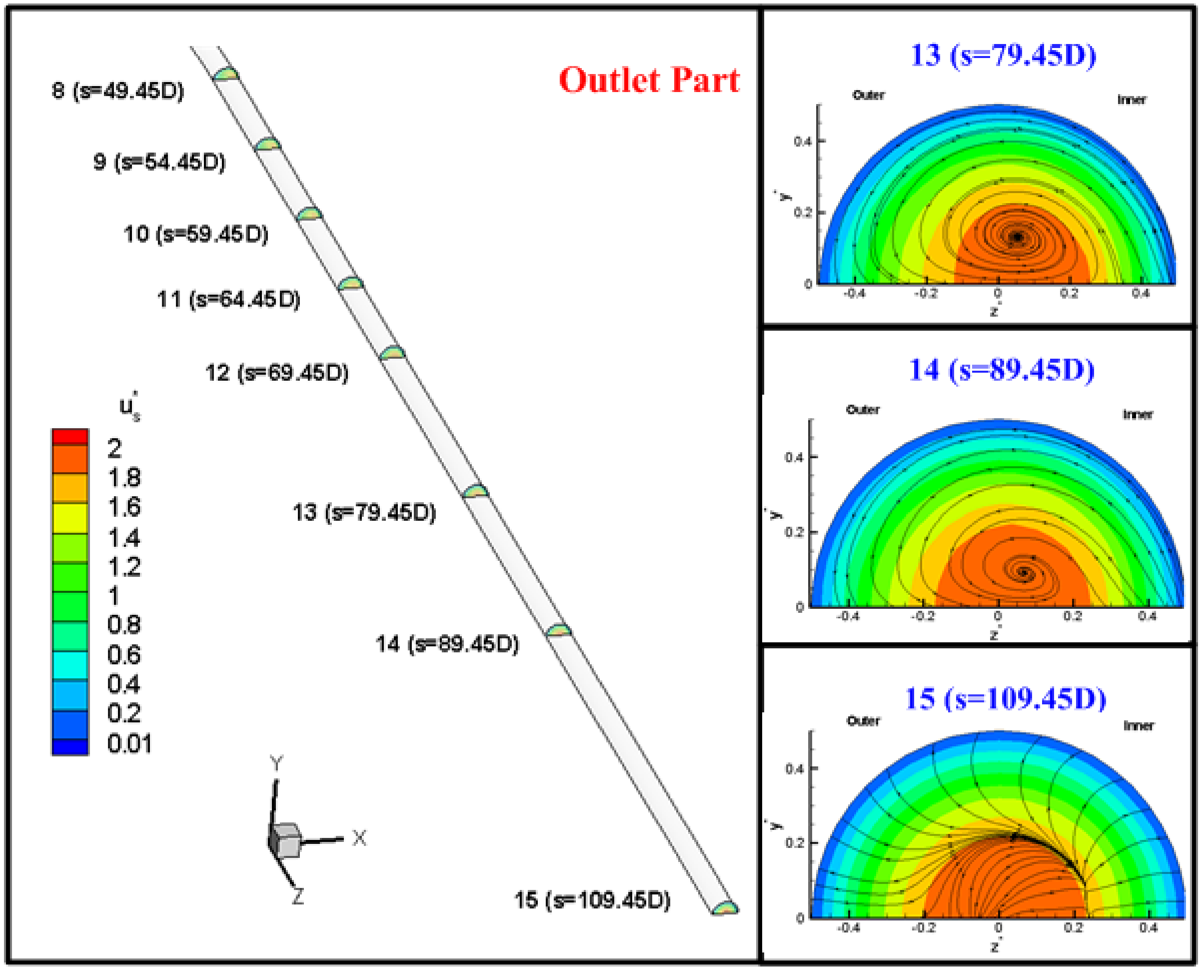
Fig.9 Numerical results for pipe of curvature of 0.013 after the bend part



Pipe curvature effect

— Curved pipe (9)

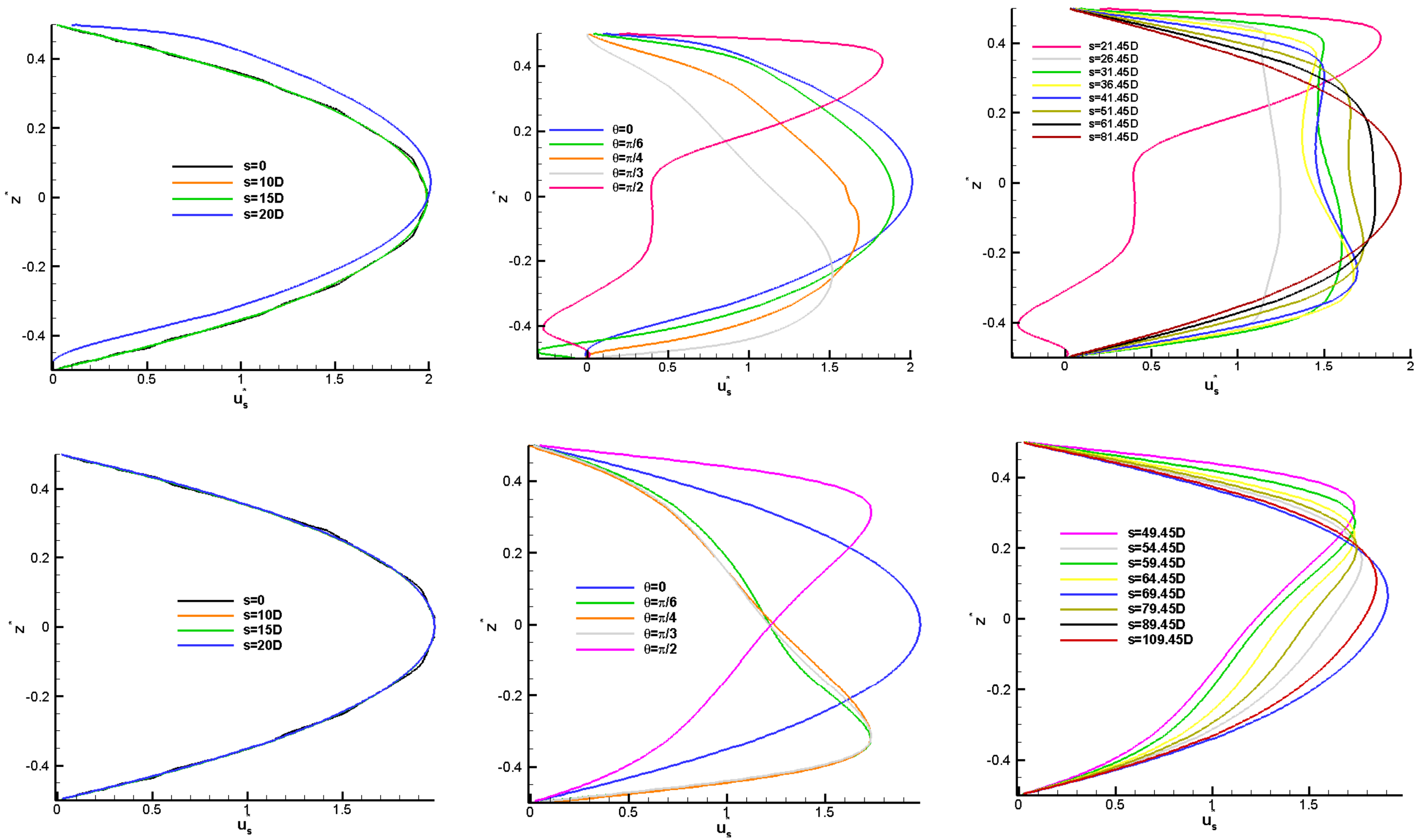
Fig.10 Numerical results for pipe of curvature of 0.013 at the outlet part



Pipe curvature effect

— Comparison

Fig.11 Axial velocity profile compared at different position s of these two pipes



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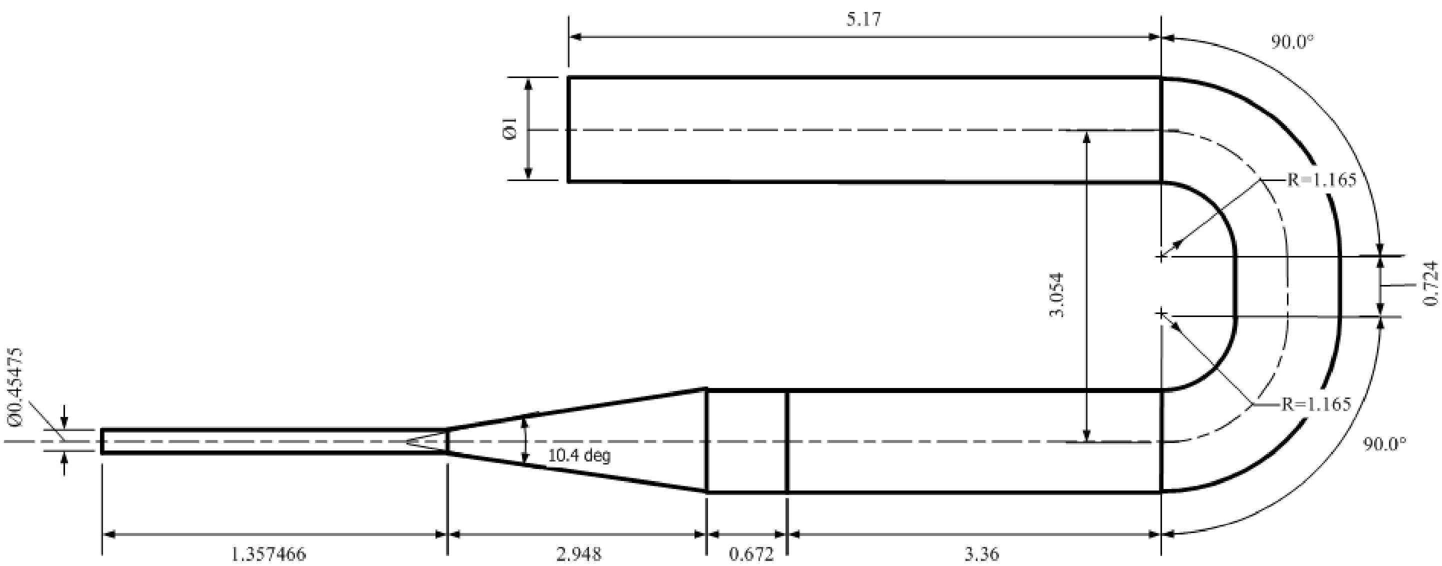
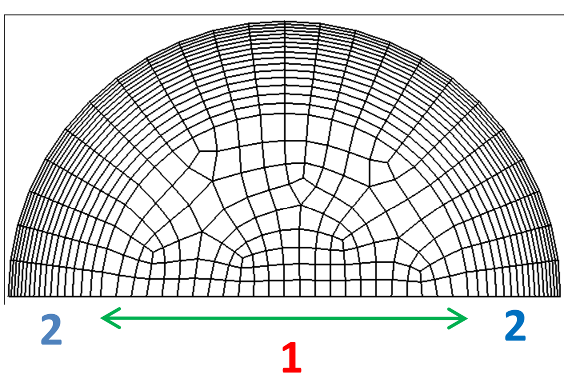
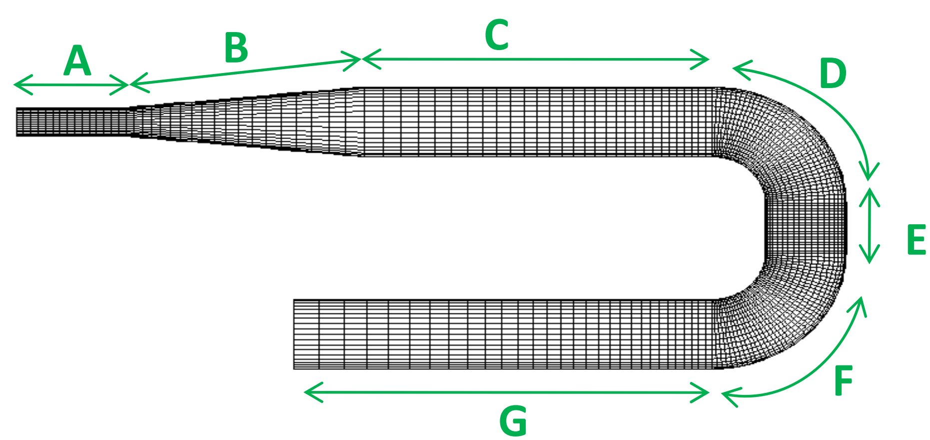
Laminar flow in mercury supply pipe (1)



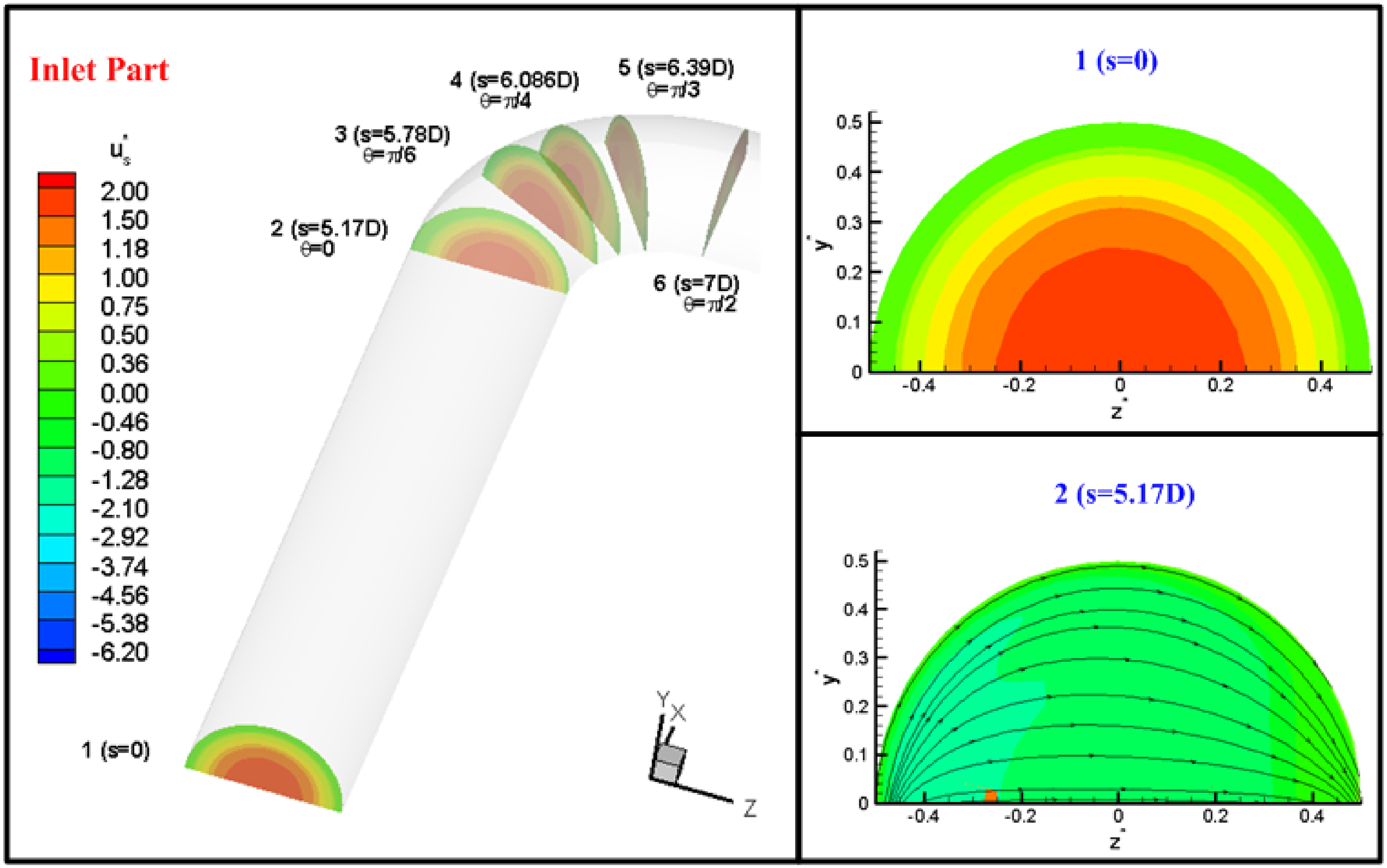




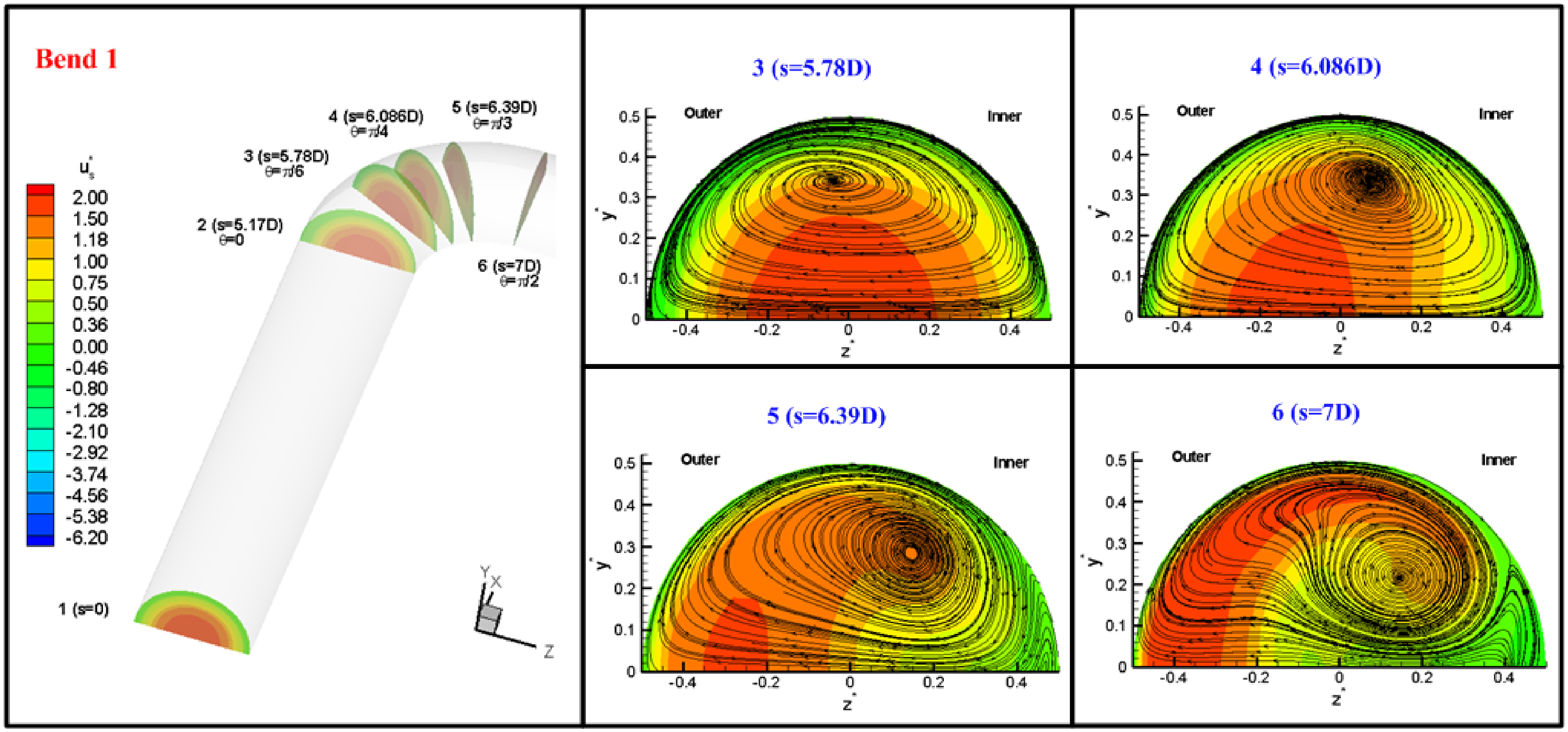
Laminar flow in mercury supply pipe (2)

Reynolds number	**1000**
Pipe diameter	1.127 mm
Curvature radius	1.165a
Inlet condition	Fully developed velocity profile (0.1m/s) and static pressure of 18.5bar
Mesh ($N_z \times N_r \times N_\theta$)	Axial direction 258 Zone A 30 Zone B 30 Zone C 50 Zone D 40 Zone E 18 Zone F 40 Zone G 50 Radial direction 56 Zone 1 24 Zone 2 16 (Δ=0.005a) Circumferential direction 24 Total 346752

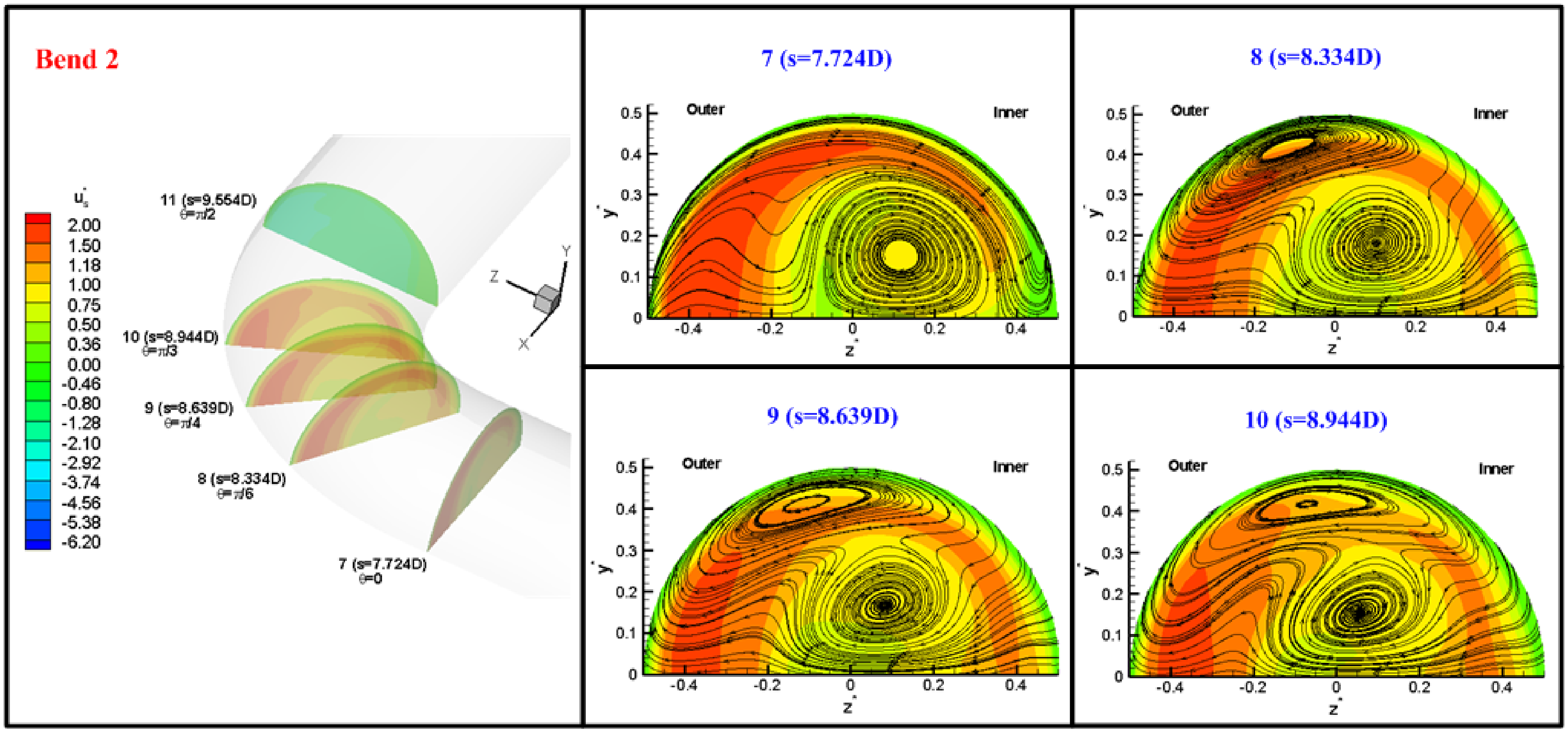
Laminar flow in mercury supply pipe (3)



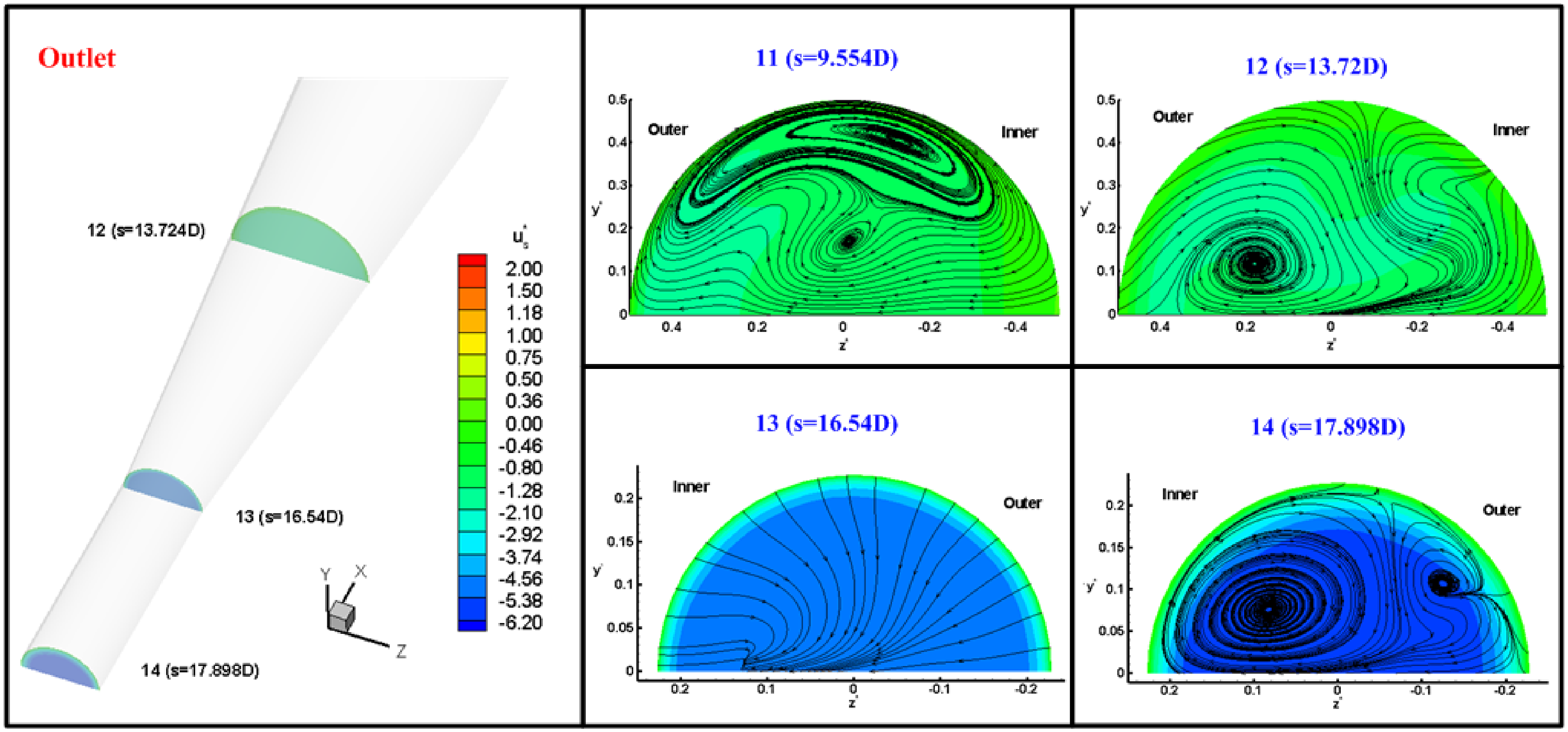
Laminar flow in mercury supply pipe (3)



Laminar flow in mercury supply pipe (3)



Laminar flow in mercury supply pipe (3)



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Simple conclusions

- Larger curvature pipe affects further upstream and downstream.
- Four vortices show in the large curvature pipe.