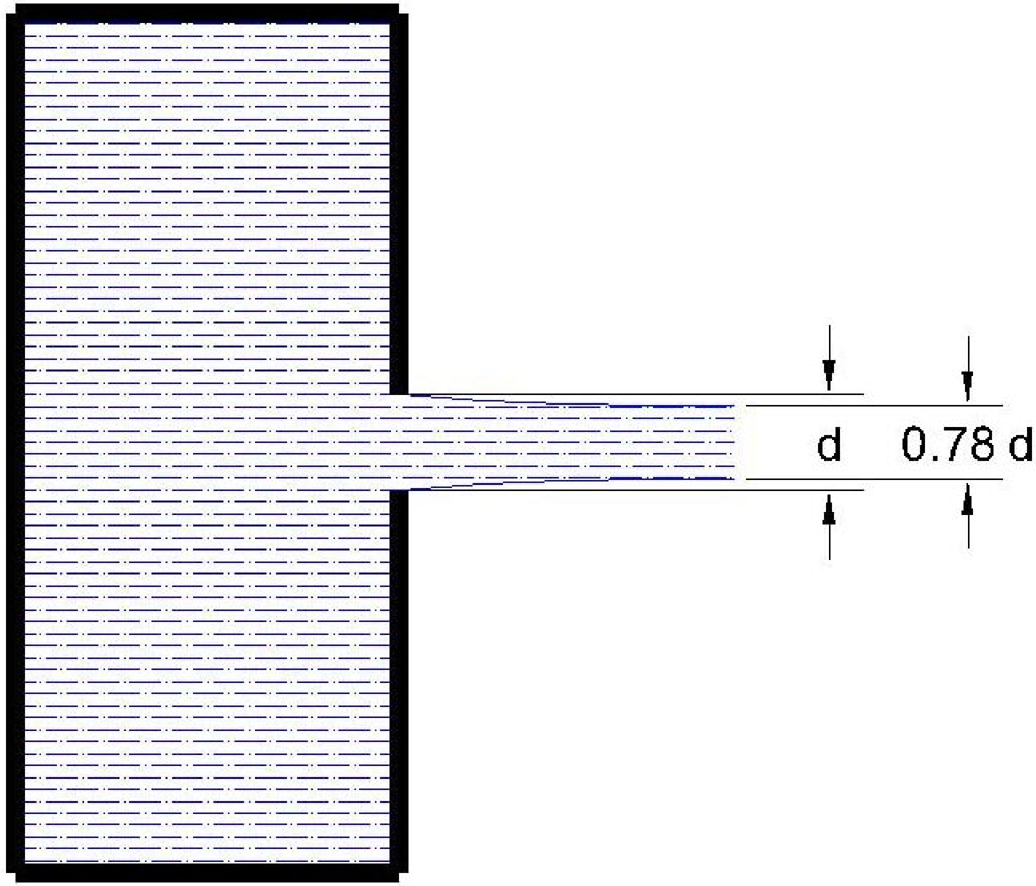
Vena Contracta



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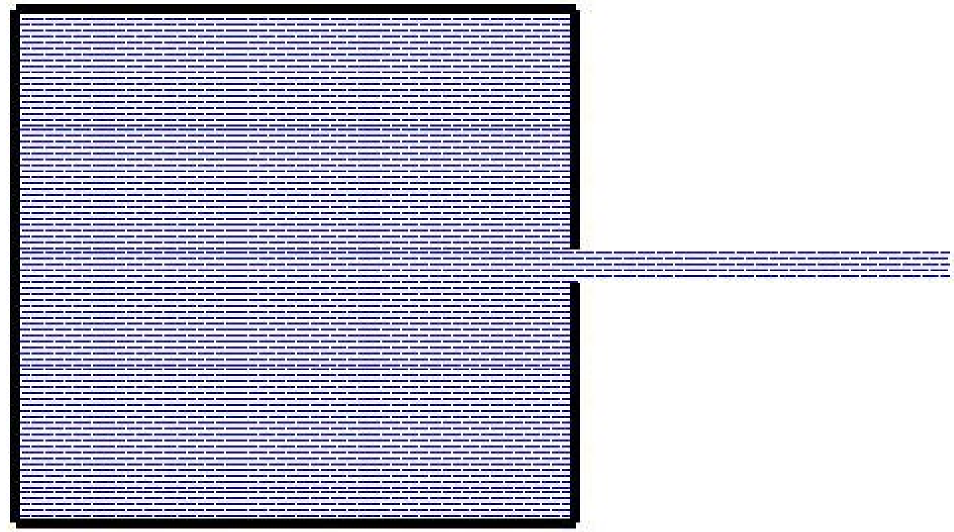
February 16, 2005

http://puhep1.princeton.edu/mumu/target/

The Best Nozzle is No Nozzle(?)

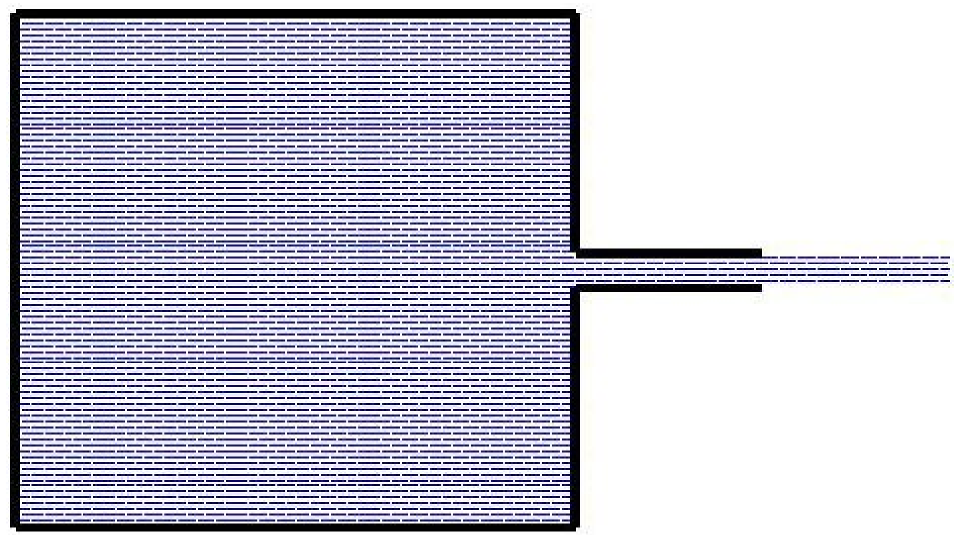
Reservoir at pressure P with small aperture:



$$v_{\rm reservoir} \approx 0, \qquad v_{\rm jet} \approx \sqrt{\frac{P}{\rho}}.$$

Jet emerges perpendicular to the plane of the aperture.

Reservoir + short nozzle:



No reservoir, just a straight tube. $v_{\rm jet} = v_{\rm tube}$:

Most nozzle R&D is concerned with making a jet break up quickly and uniformly (atomizing), rather than with preserving the jet.

Conservation of Energy *vs.* $\mathbf{F} = d\mathbf{P}/dt$ at a Contraction? (Borda, 1766)

Incompressible fluid $\Rightarrow V_1 A_1 = V_2 A_2$.

$$A_2 \ll A_1 \Rightarrow V_1 \ll V_2.$$

Conservation of Energy $\Rightarrow$ Bernoulli's Law:

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2.$$

$$V_1 \ll V_2 \Rightarrow \qquad V_2^2 \approx 2\frac{P_1 - P_2}{\rho}.$$

Argument does not depend on the area.

$\mathbf{F} = d\mathbf{P}/dt$:

Mass flux $= \rho V A$.

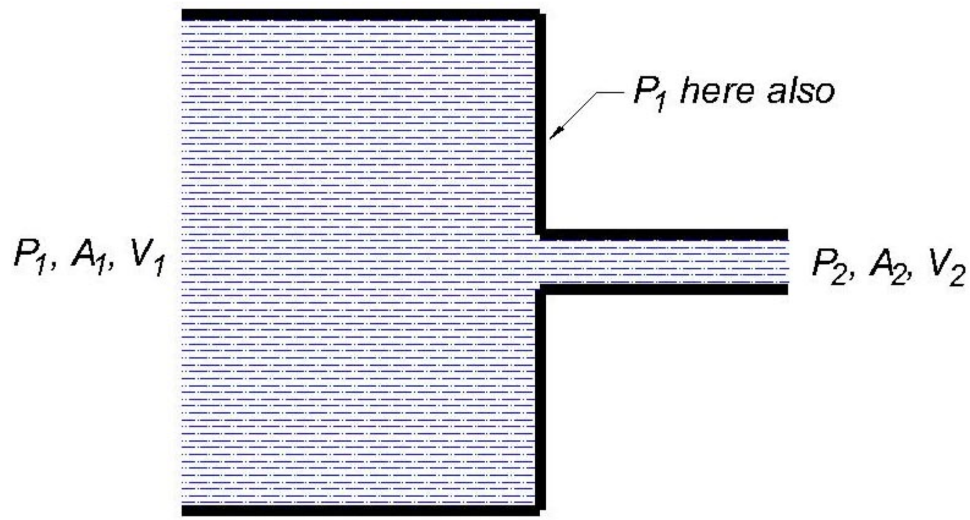
Momentum flux $= \rho V^2 A$.

$$\text{Net momentum flux} = \rho(V_2^2 A_2 - V_1^2 A_1) = \rho V_2 A_2 (V_2 - V_1) \approx \rho V_2^2 A_2.$$

Force $\approx (P_1 - P_2) A_2$.

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} \Rightarrow \qquad V_2^2 \approx \frac{P_1 - P_2}{\rho}.$$

Consistency $\Rightarrow$ dissipative loss of energy,
OR jet pulls away from the wall and contracts.



Vena Contracta

Cavitation can be induced by a sharp-edged aperture.

A jet emerging from a small aperture in a reservoir contracts in area:

$$A_{\rm jet} = \frac{\pi}{\pi + 2} A_{\rm aperture} = 0.62 A_{\rm aperture}$$

$$d_{\rm jet} = 0.78\ d_{\rm aperture}$$

2-d potential flow (conservation of energy) $\Rightarrow$ analytic form:

$$x = \frac{2d}{\pi + 2}(\tanh^{-1}\cos\theta - \cos\theta), \qquad y = d - \frac{2d}{\pi + 2}(1 + \sin\theta),$$

$$\theta = \text{angle of streamline}, \qquad -\frac{\pi}{2} < \theta < 0.$$

90% of contraction occurs for $x < 0.8d$.

Good agreement between theory and experiment.