

Peak ΔT in Edge-Cooled Beryllium Window at $z = 3$ m in Magnet IDS120h

Bob Weggel, M.O.R.E., LLC
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This report extends the analysis of file “Water-cooled2Be.docx” (e-mailed 11/22/’11 at 11:16 PM) to incorporate Nick’s latest predictions of the power density in a beryllium window at $z = 3$ m in Magnet IDS120h. The previous analysis, its graphs reproduced here as Figs. 1 and 2, predicted a peak temperature rise ΔT of 281°C with a uniform power density of 10 W/cm³. The total integrated power is $\pi(10\text{W/cm}^3)(15\text{ cm})^2 = 7.1\text{ kW/cm}$ in a window of 15-cm radius.

As shown in Fig. 1, one can reduce the temperature rise by flaring the window thickness linearly as $t(r) \sim [1+\beta(r/15)]$. If $\beta = 75$ — *i.e.*, the window is 76 times thicker at $r = 15$ cm than at its center — the peak temperature rise is 190°C. For $\beta = 0.3, 0.75, 1.5, 3.0, 7.5$ and 15 the respective temperature rises are 266, 251, 236, 221, 205 & 198°C. As shown in Fig. 2, for parabolic flaring $t(r) \sim [1+\beta(r/15)^2]$, the respective temperature rises for $\beta = [0.3, 0.75, 1.5, 3.0, 7.5, 15 \text{ \& } 75]$ are [274, 246, 226, 206, 181, 167 & 149] °C.

Nick’s file “IDS120hm_BeWind_TDP_NO_SH1_NP100000_nx20_ny20_nz1_a.txt” (e-mail of 12/1/’11, 1:37 AM) reports a peak power density (at $-1, +0.7$) of 103 W/g—190 W/cm³ for beryllium (1.85 g/cm³). [Note: File “IDS120hm_BeWind_TDP_NO_SH1_NP100000_nx40_ny40_nz1_a.txt” reports an even-higher peak power density (at $-0.50, 1.75$) of 201 W/g, or 372 W/cm³.]

In hopes of generating analytic predictions of the temperature rise as a function of radius, I approximated Nick’s predictions of power density with Fig. 3’s red dashed curve, a simple inverse polynomial— $q_0 a^2/(a^2+r^2)$. The parameter q_0 is the maximum power density—190 W/cm³—noted in the first file above, and the parameter “ a ”—1.68 cm—matches the total deposited power (TDP) to the 7.4 kW predicted by Nick for a beryllium window 1-cm thick.

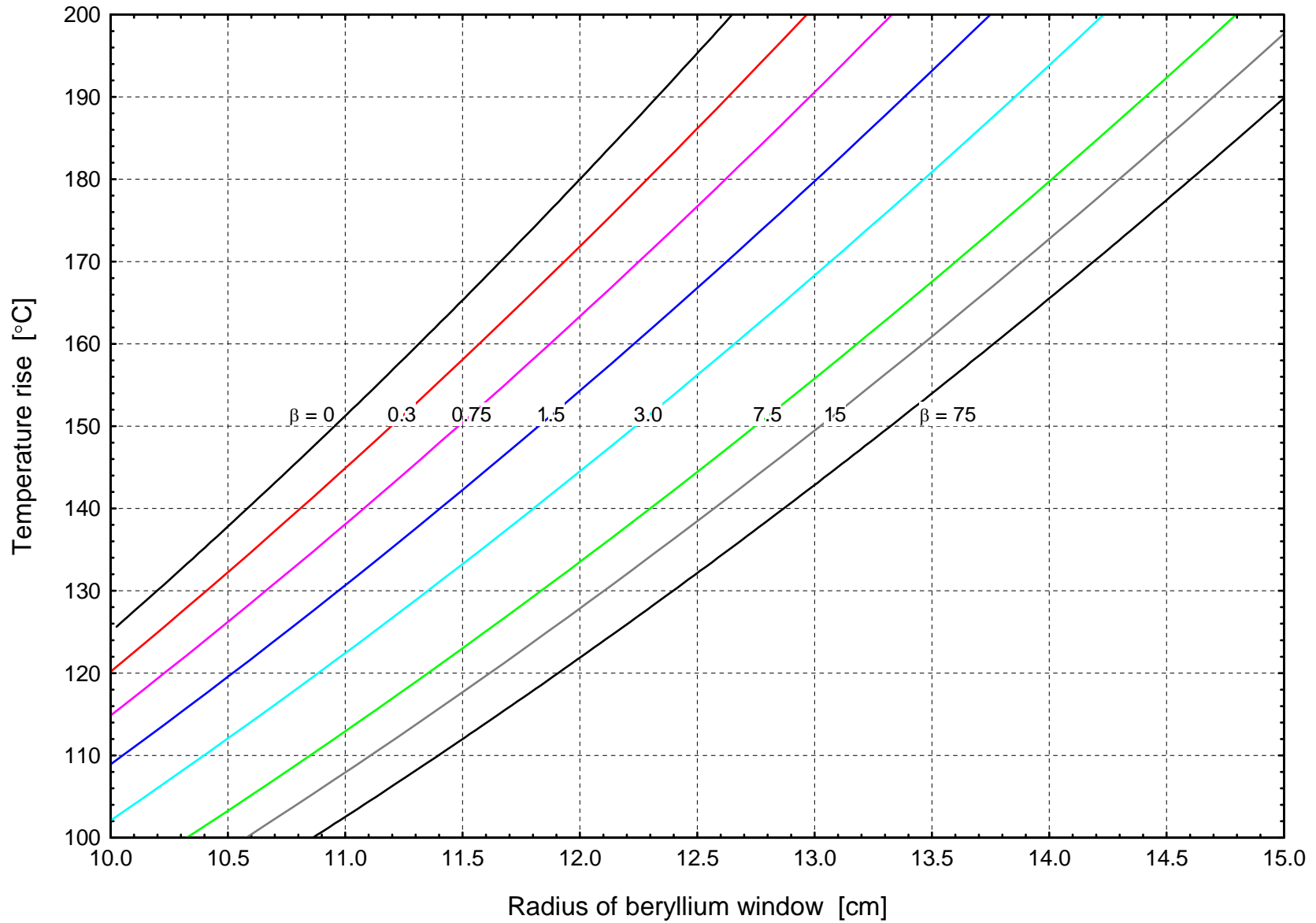
$q_0 a^2/(a^2+r^2)$ integrates to give a total deposited power $Q(r) = \pi q_0 a^2 \ln(1+r^2/a^2)$ —the dashed magenta curve of Fig. 3. However, analytic integration to predict the temperature rise $\Delta T(r)$ was possible only with a window thickness proportional to r^n , with $n \geq 1$, and such windows would have zero thickness at $r = 0$. Likewise, only the first stage of analytical integration was possible for the solid red line of Fig. 3, $q(r) = q_0 (1-r/a) \exp^{-r/a}$, with $a = 1.44$ cm, for which $Q(r) = 2\pi q_0 [3a^2 - (3a^2 + 3ar + r^2) e^{-r/a}]$.

Numerical integration of each $r^{-1} Q(r)$ predicted the temperature rise $\Delta T(r)$ as the dashed curve of Fig. 4 for the power density $q(r) = q_0 a^2/(a^2+r^2)$ and the solid curve for $q(r) = q_0 (1-r/a) \exp^{-r/a}$. Note that the nineteen-fold increase in peak power density — even though quite localized — greatly increases the temperature rise in the window: 753°C if $q(r) = q_0 a^2/(a^2+r^2)$ and 938°C if $q(r) = q_0 (1-r/a) \exp^{-r/a}$.

A window of optimized thickness $t(r)$ can reduce the temperature rise greatly. For the power density $q(r) = q_0 (1-r/a) \exp^{-r/a}$ I subdivided the window into annuli 1 mm to 3 mm in radial depth and sought the axial thickness of each — constrained to a minimum thickness of 1 mm — that minimized ΔT while penalizing, to six different extents, the volume of beryllium used. Fig. 5 plots the optimum-thickness functions, $t(r)$; Fig. 6 plots the integrated power, which ranges from 2.9 kW for a window of 0.87 liters to 10 kW for one of 4.6 liters.

Fig. 7 plots $\Delta T(r)$ for each of the six cases. The peak ΔT at the center of a window of 15-cm radius ranges from 309°C for a window of 4.6 liters to 414°C for a window of 0.87 liters.

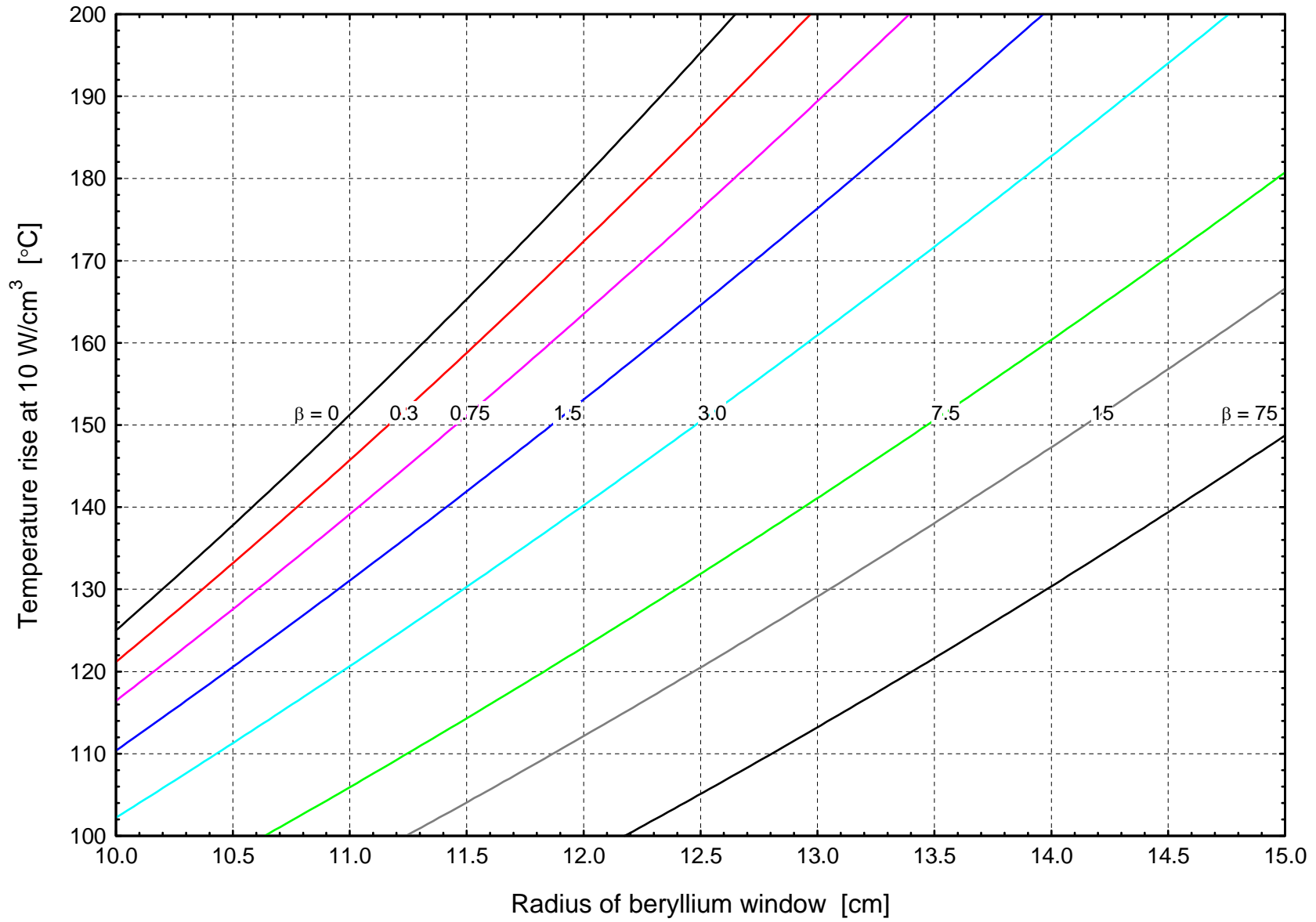
Temperature Rise at 10 W/cm^3 in Beryllium Window of Thickness $t = (1 + \beta r/15) t_0$



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Fig. 1: Temperature rise at 10 W/cm^3 at center of beryllium window of radius 10 cm to 15 cm and thickness $t(r) = [1 + \beta(r/15)] t_0$.

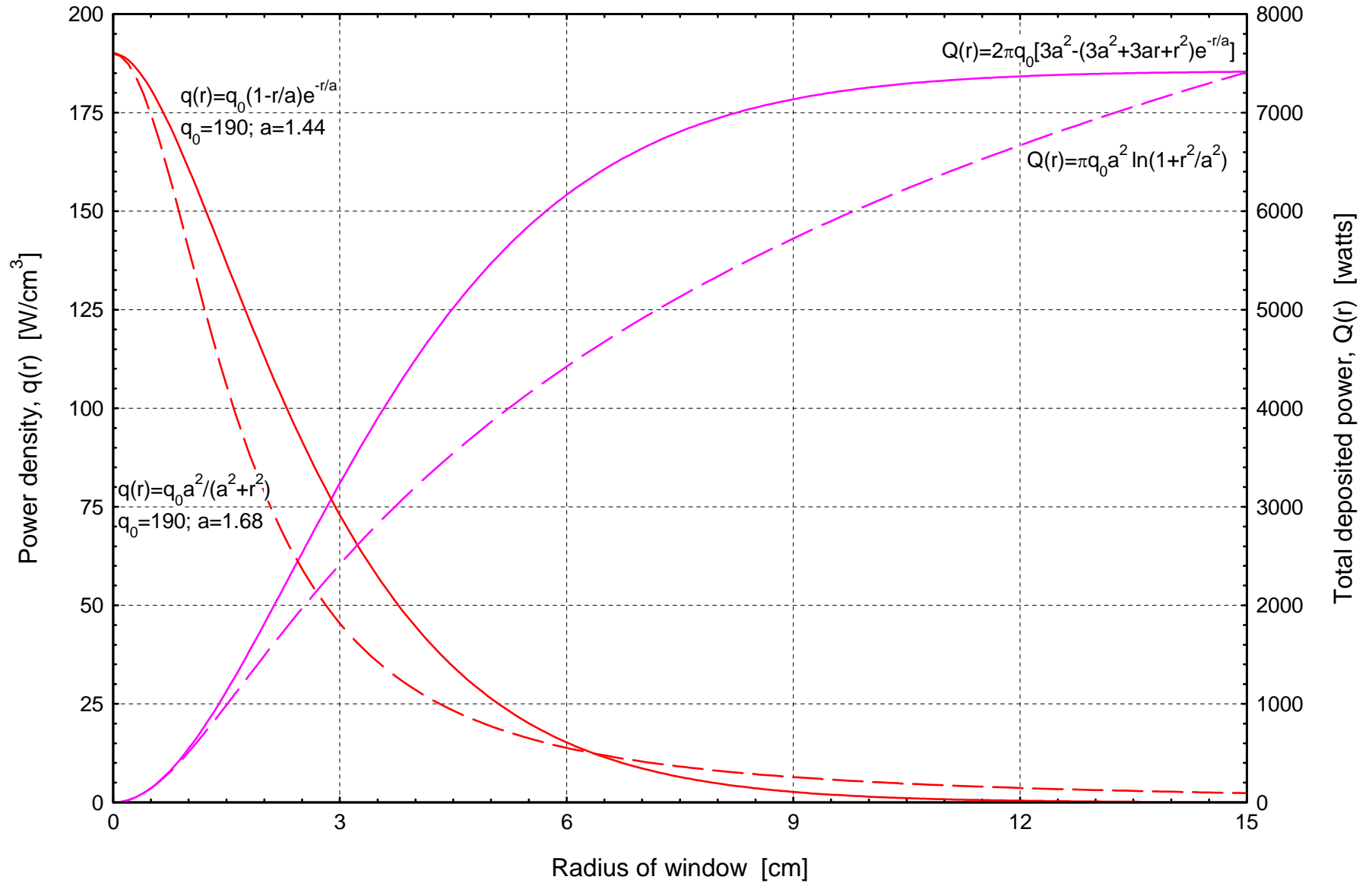
Temperature Rise at Center of Edge-Cooled Beryllium Window of Thickness $t = [1 + \beta (r/15)^2] t_0$



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Fig. 2: Temperature rise at 10 W/cm³ at center of beryllium window of radius 10 cm to 15 cm and thickness $t(r) = [1 + \beta (r/15)^2] t_0$.

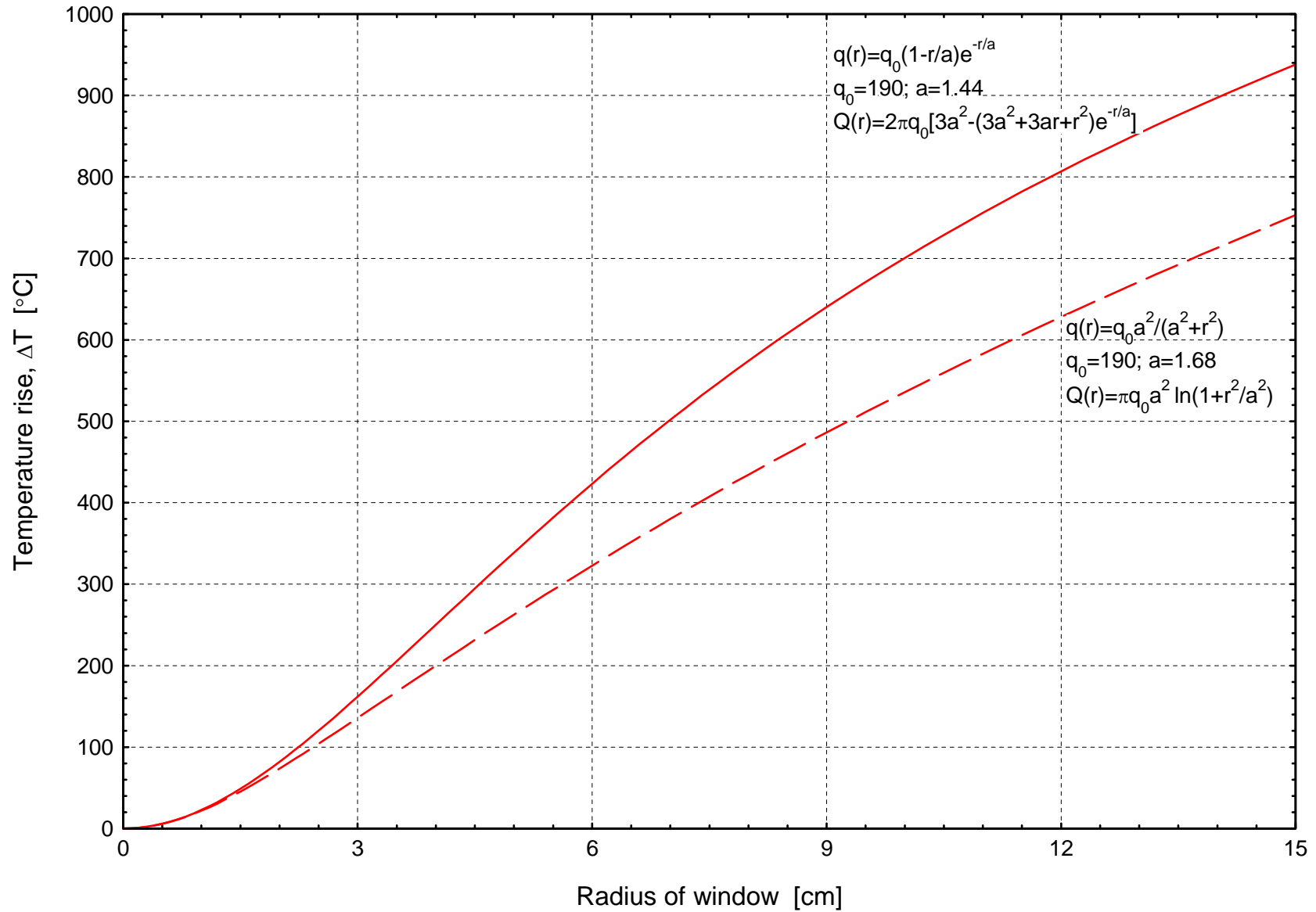
Curve Fits to Power Density and Total Power in IDS120h Beryllium Window 1-cm Thick at z = 3 m



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Fig. 3: Exponential (solid-line) & inverse-polynomial (dashed-line) curve fits to power density, $q(r)$, & total deposited power, TDP, in 1-cm-thick beryllium window at z = 3 meters in Magnet IDS120h. For each curve, $q_{\max} = q(r=0) \equiv q_0 = 190 \text{ W/cm}^3$, and $\text{TDP} (r=15 \text{ cm}) = 7.4 \text{ kW}$.

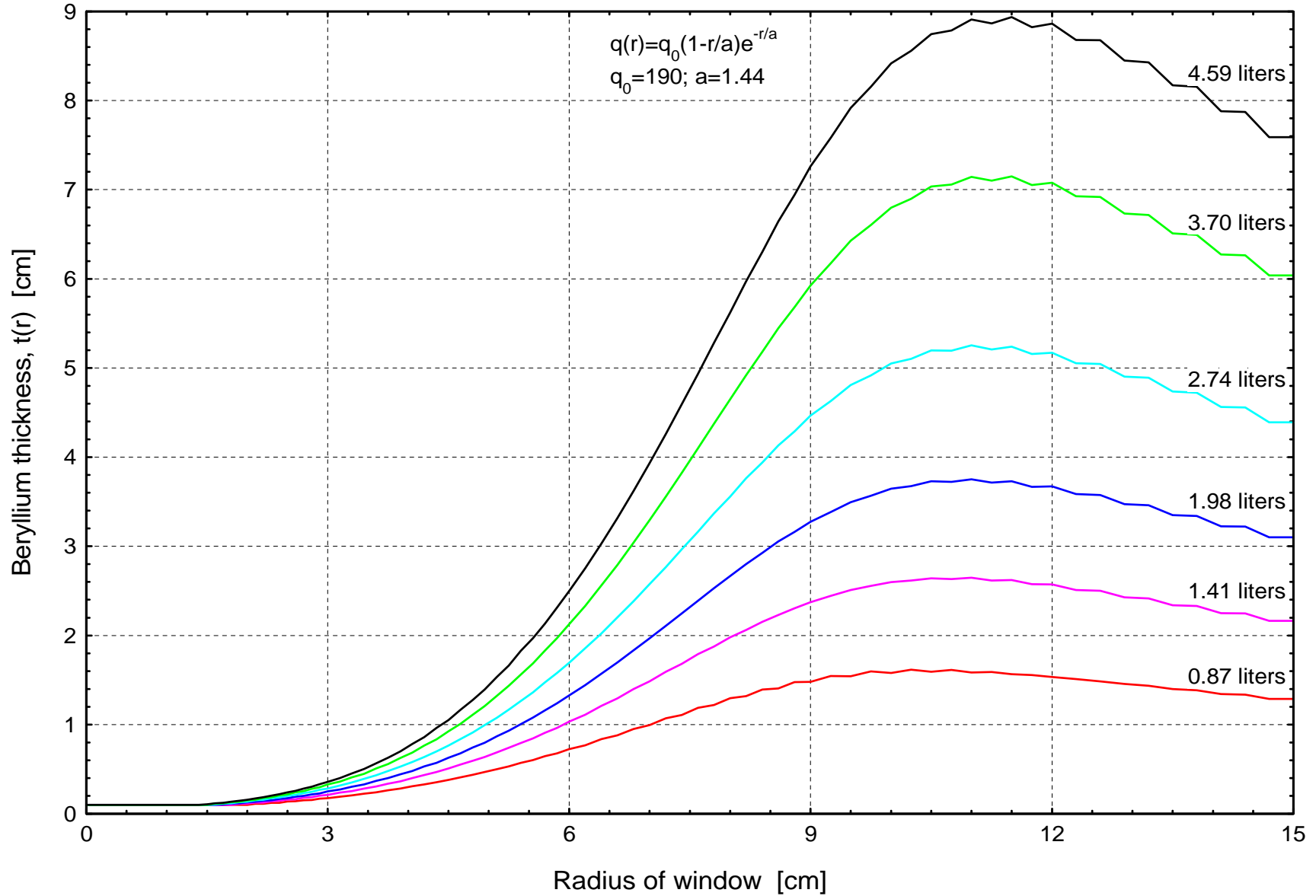
Temperature Rise vs. Radius of 1-cm-Thick Beryllium Window at z = 3 m in IDS120h



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Fig. 4: Temperature rise vs. radius in Be window 1 cm thick with exponential or inverse-polynomial power density $q(r)$ of Magnet IDS120h at z = 3 m.

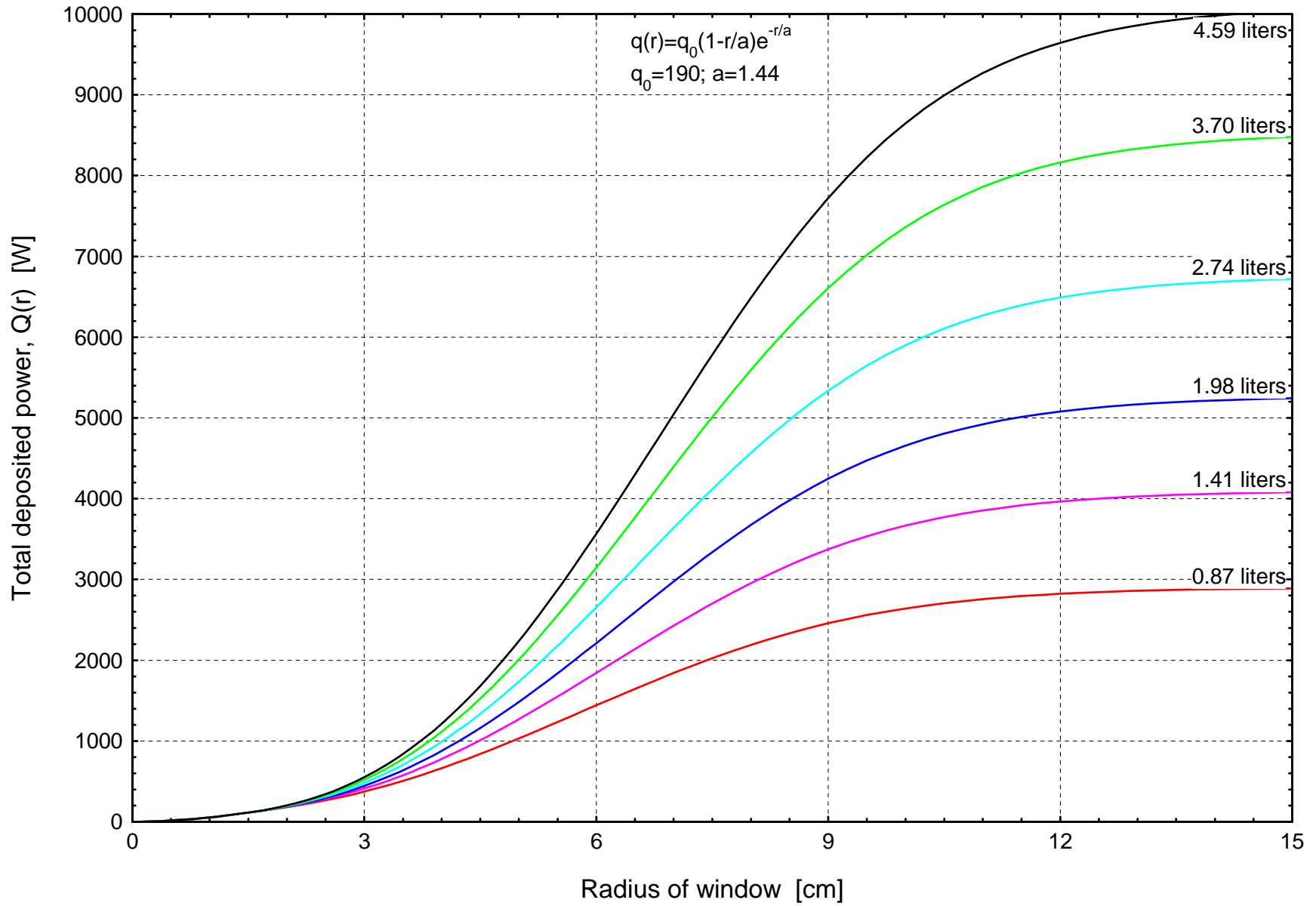
ΔT -Minimizing Thickness vs. Radius of Beryllium Window at $z = 3$ m in IDS120h



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Fig. 5: Thickness $t(r)$ of six beryllium windows that minimize $\Delta T_{r=15\text{cm}}$ with exponential power density of Magnet IDS120h at $z = 3$ meters.

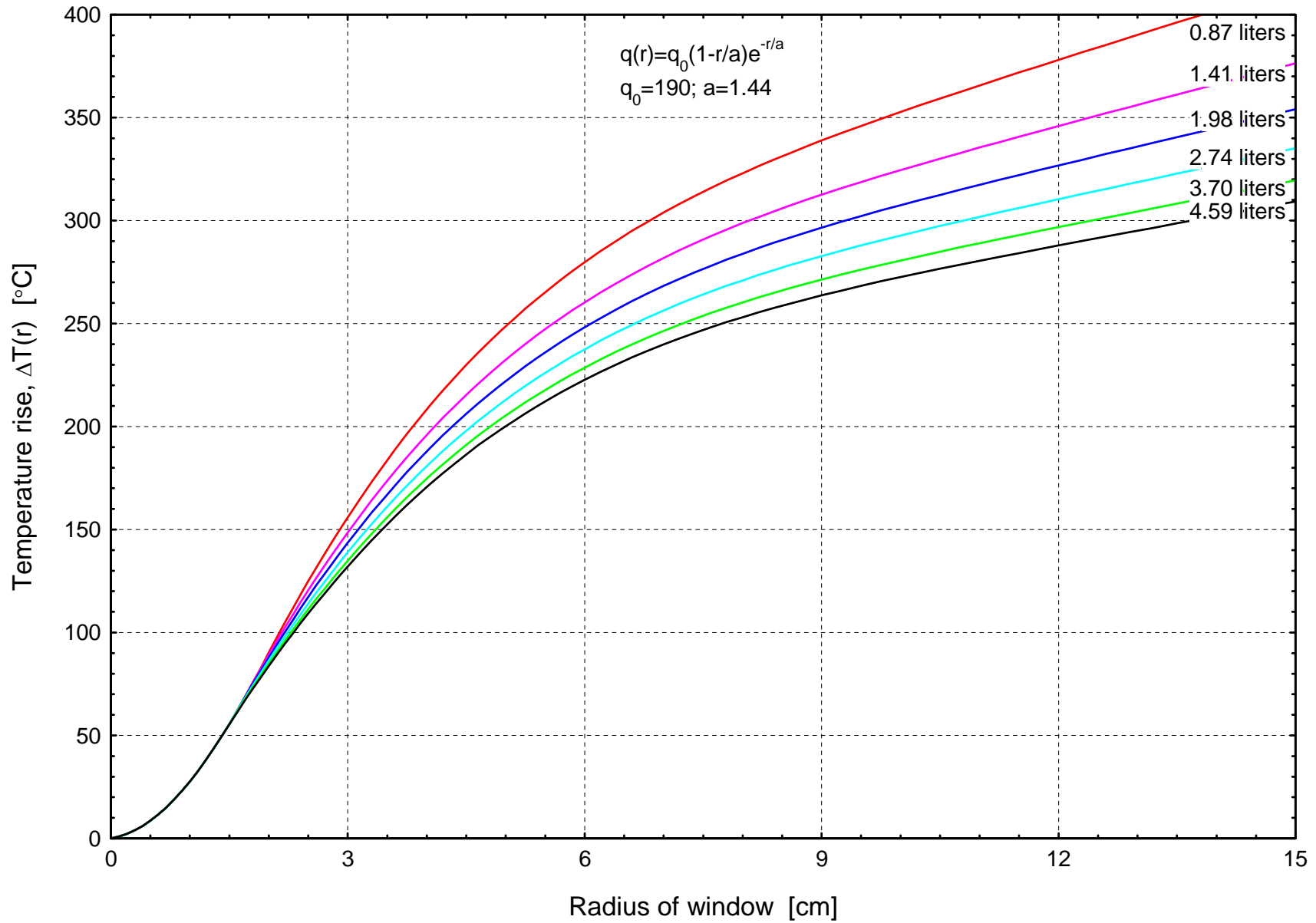
Total Deposited Power, $Q(r)$, in Beryllium Window with ΔT -Minimizing Thickness



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Fig. 6: Total deposited power in six beryllium windows of ΔT -minimizing thickness with exponential power density of Magnet IDS120h at $z = 3$ m.

Temperature Rise, $\Delta T(r)$, in Beryllium Window with ΔT -Minimizing Thickness



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Fig. 7: ΔT at center of six beryllium windows of ΔT -minimizing thickness with exponential power density of Magnet IDS120h at $z = 3$ meters.