Target-Magnet Field Profile that Ramps from 20 T to 1.5 T at 7 m: IDS120L20to1.5T7m

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Table I: Parameters of Target Magnet "IDS120′20to1p5T7m.xlsx" of 4/8/2013

Current density $j\lambda_{coil}$	kA/cm^2	2.078	1.665	1.513	1.393	1.294	1.911	3.451	4.131	4.343
Coil length	cm	129.1	165.4	165.4	165.4	165.4	397.53	49.33	46.80	32.99
Gap between coils	cm							122.29	88.80	90.98
Upstream end	cm	-86.7	-123.0	-123.0	-123.0	-123.0	-238.2	281.6	418.8	556.5
Downstream end	cm	42.4	42.4	42.4	42.4	42.4	159.3	331.0	465.6	589.5
Inner radius [cm]	0.50	18.28	23.75	29.78	36.09	42.66	120.0	120.0	120.0	120.0
Radial depth of conductor	cm	4.760	5.318	5.579	5.815	6.032	79.63	17.19	8.386	10.16
Outer radius [cm]	50.0	23.04	29.07	35.35	41.90	48.69	199.63	137.19	128.39	130.16
Maximum on-axis field	T	20.00	18.83	17.77	16.78	15.86	15.00	4.86	2.76	1.84
Current density $j\lambda_{coil}$	kA/cm^2	4.570	4.588	4.548	4.673	4.715	4.715	4.715	4.715	4.715
Coil length	cm	122.90	323.49	30.00	20.00	479.49	20.00	20.00	479.49	20.00
Gap between coils	cm	70.00	10.00	24.08	50.00	20.25	20.25	40.00	20.25	20.25
Upstream end	cm	659.5	792.4	1140.0	1220.0	1260.3	1760.0	1820.0	1860.3	2360.0
Downstream end	cm	782.4	1115.9	1170.0	1240.0	1739.7	1780.0	1840.0	2339.7	2380.0
Inner radius	cm	90.0	90.0	90.0	60.0	60.0	60.0	60.0	60.0	60.0
Radial depth of conductor	cm	2.252	2.434	8.713	5.775	2.434	7.623	7.623	2.434	7.623
Outer radius	cm	92.25	92.43	98.71	65.77	62.43	67.62	67.62	62.43	67.62
Maximum on-axis field	T	1.51	1.48	1.52	1.51	1.51	1.52	1.51	1.51	1.52

The "desired field" is the inverse-polynomial $B(u) = 180/[9+37u^2(4-u^6)]$, where $u \equiv x/L$, $x \equiv z+37.5$ cm, and $L \equiv 737.5$ cm. B(u) involves only even powers of u, and therefore is symmetric about u=0—i.e., z = −37.5 cm. The derivative, dB/du is $53280u(u^2-1)[(u^2+1)^2-u^2]/(37u^8-148u^2-9)^2$, which is zero at u = 0 and u = 1—i.e., x = 737.5 cm, or z = 700 cm.

A more general expression, likewise with zero slope at x = 0 and x = L, is $B(u) = nB_0/[n+bu^2(n+2-2u^n)]$, where $B_0 \equiv B(u{=}0)$ and $b \equiv [B_0/B(L)]-1$. Its first derivative is $n[2bu(2u^n-n-2)+2bnu^{n+1}]/[bu^2(2u^2-n+2)-n]^2$. The equation for the 2nd derivative is two lines long; that for the 3rd derivative takes five lines—rather inconvenient for analytic prediction of the paraxial field by a power-series expansion in Legendre polynomials.

A form more amenable to analytic differentiation is $B(u) = B_0-\Delta B(au^2+bu^p+cu^q)$, where the parameters q and p need not be integers but neither should be less than 2.0, if the field near u=0 is to be dominated by the quadratic term. The three parameters a, b & c enable the expression to achieve at the end of the ramp not only the desired field and slope (zero), but also **zero curvature**. The figure plots two illustrative field profiles.

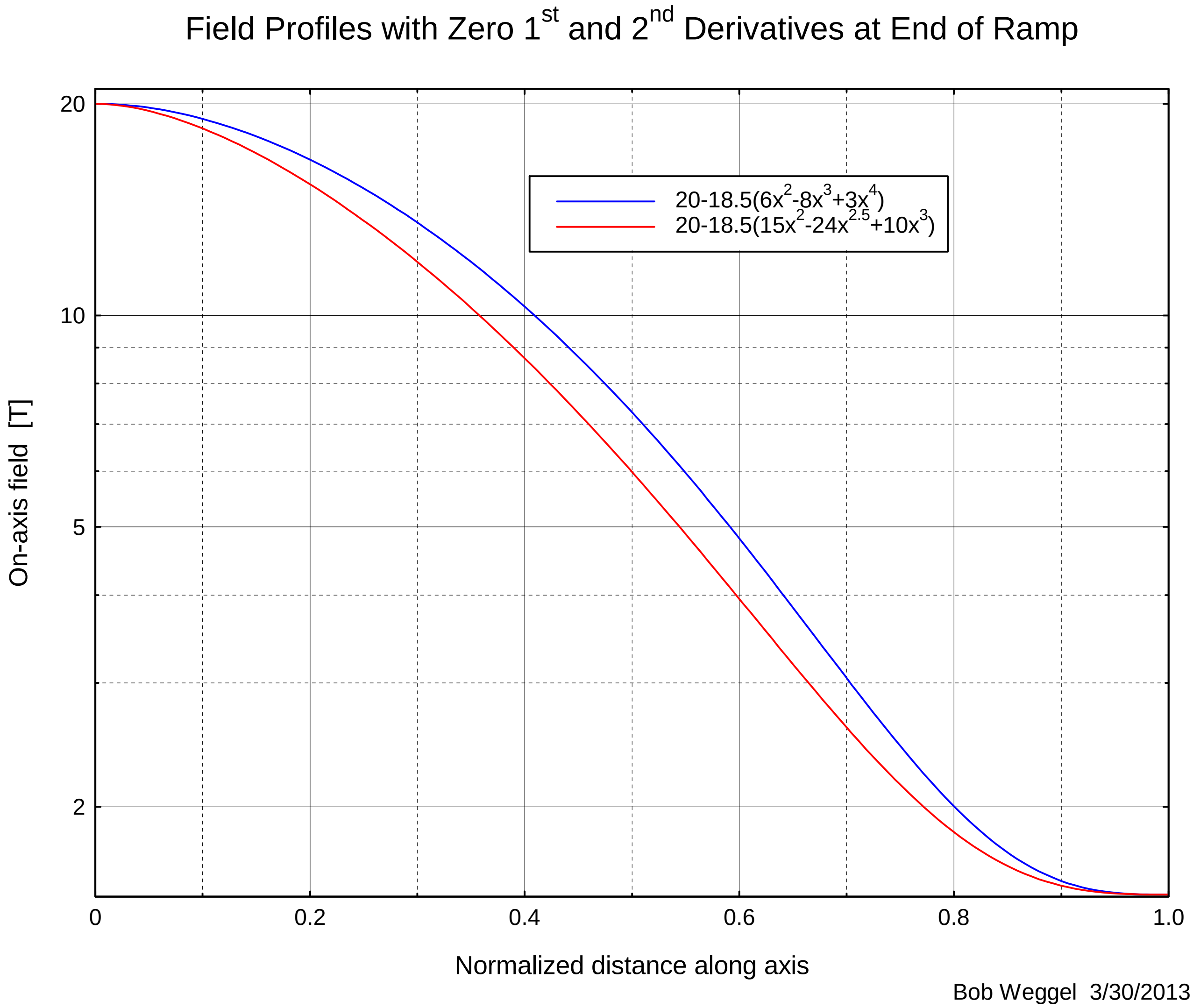

Fig. 1. Illustrative field profiles with zero 1st and 2nd derivatives at end of ramp.