Target-Magnet Field Profile that Ramps from 20 T to 1.5 T at 7 m

Bob Weggel
Magnet Optimization Research Engineering, LLC
(March 30, 2013)

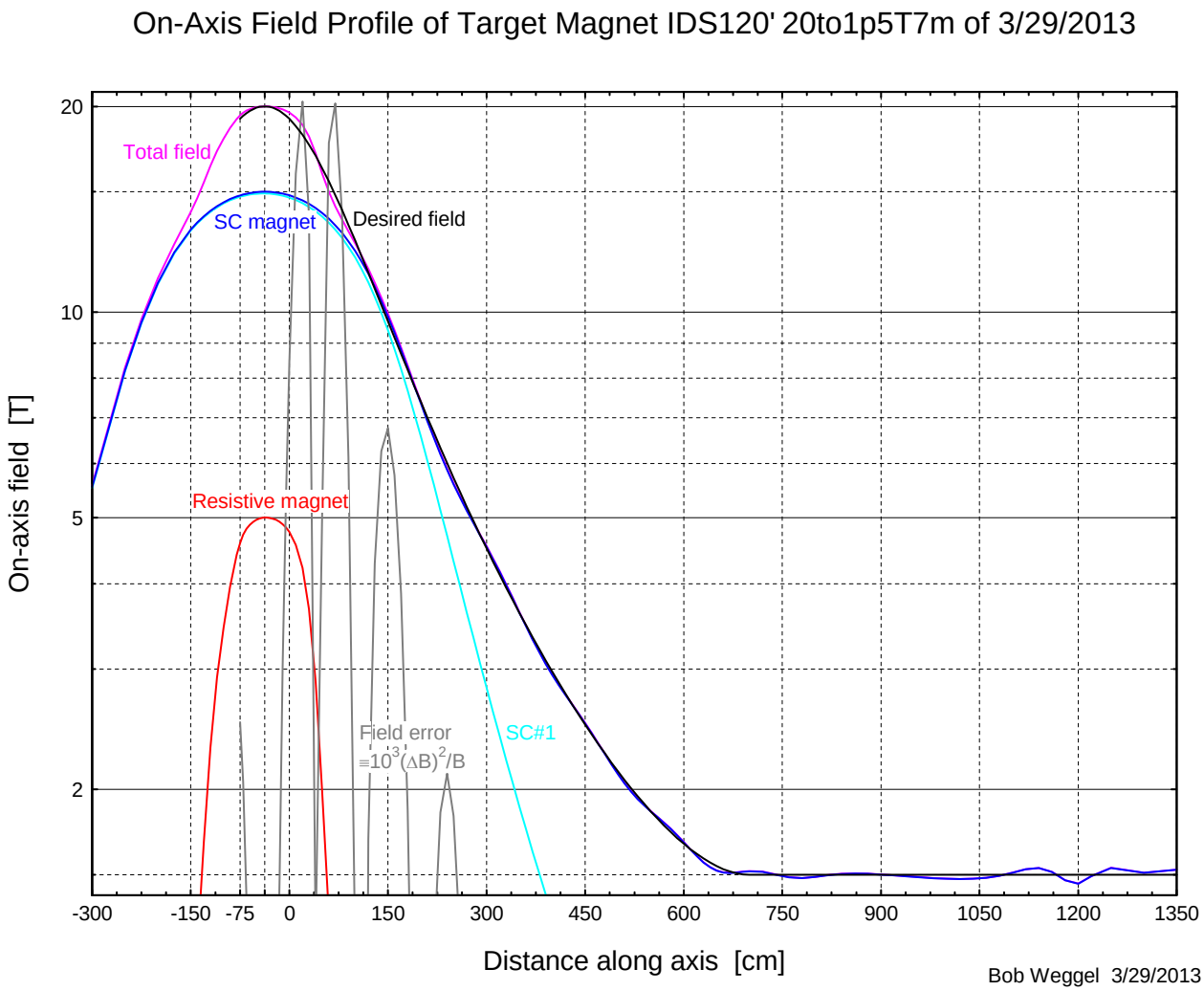

Fig.1. On-axis field profile of Target Magnet "IDS120'20to1p5T7m.xlsx" of 3/29/2013: 5-T, 10.8-MW resistive magnet (red); superconducting coil #1 (turquoise); 15-T superconducting magnet (blue); total field (magneta); desired field (black); and field error (grey), defined as $1000(\Delta B)^2/B$.

Table I: Parameters of Target Magnet "IDS120'20to1p5T7m.xlsx" of 3/29/2013

Current density $j\lambda_{coil}$	kA/cm^2	2.078	1.665	1.513	1.393	1.294	1.911	3.453	4.137	4.359
Coil length	cm	129.2	165.4	165.4	165.4	165.4	397.55	51.63	50.87	41.44
Gap between coils	cm							121.52	88.73	89.01
Upstream end	cm	-86.7	-122.9	-122.9	-122.9	-122.9	-238.2	280.9	421.2	561.1
Downstream end	cm	42.4	42.4	42.4	42.4	42.4	159.4	332.5	472.1	602.6
Inner radius [cm]	0.50	18.28	23.75	29.78	36.09	42.66	120.0	120.0	120.0	120.0
Radial depth of conductor	cm	4.760	5.318	5.579	5.815	6.031	79.63	16.501	8.069	7.945
Outer radius [cm]	50.0	23.04	29.07	35.35	41.90	48.69	199.63	136.50	128.07	127.94
Maximum on-axis field	T	20.00	18.83	17.77	16.78	15.86	15.00	4.89	2.74	1.82
Current density $j\lambda_{coil}$	kA/cm^2	4.570	4.588	4.548	4.680	4.716	4.715	4.715	4.715	4.715
Coil length	cm	78.10	358.62	30.00	20.00	467.14	20.00	20.00	467.14	20.00
Gap between coils	cm	70.00	17.86	17.86	50.00	17.86	17.86	40.00	17.86	17.86
Upstream end	cm	672.6	768.5	1145.0	1225.0	1275.0	1760.0	1820.0	1875.0	2360.0
Downstream end	cm	750.7	1127.1	1175.0	1257.1	1742.1	1780.0	1857.1	2342.1	2380.0
Inner radius [cm]	0.50	90.0	90.0	90.0	60.0	60.0	60.0	60.0	60.0	60.0
Radial depth of conductor	cm	2.658	2.403	8.474	4.214	2.449	6.302	6.302	2.449	6.302
Outer radius [cm]	50.0	92.66	92.40	98.47	64.21	62.45	66.30	66.30	62.45	66.30
Maximum on-axis field	T	1.51	1.48	1.53	1.51	1.51	1.53	1.51	1.51	1.53

The "desired field" in Fig. 1 is the inverse-polynomial $B(u) = 180/[9+37u^2(4-u^6)]$, where $u \equiv x/L$, $x \equiv z+37.5$ cm, and $L \equiv 737.5$ cm. B(u) involves only even powers of u, and therefore is symmetric about u=0—i.e., z = −37.5 cm. The derivative, dB/du is $53280u(u^2-1)[(u^2+1)^2-u^2]/(37u^8-148u^2-9)^2$, which is zero at u = 0 and u = 1—i.e., x = 737.5 cm, or z = 700 cm.

A more general expression, likewise with zero slope at x = 0 and x = L, is $B(u) = nB_0/[n+bu^2(n+2-2u^n)]$, where $B_0 \equiv B(u=0)$ and $b \equiv [B_0/B(L)]-1$. Its first derivative is $n[2bu(2u^n-n-2)+2bnu^{n+1}]/[bu^2(2u^2-n+2)-n]^2$. The equation for the 2nd derivative is two lines long; that for the 3rd derivative takes five lines—rather inconvenient for analytic prediction of the paraxial field by a power-series expansion in Legendre polynomials.

A form more amenable to analytic differentiation is $B(u) = B_0-\Delta B(au^2+bu^p+cu^q)$, where the parameters q and p need not be integers but should both be greater than 2, if the field near u=0 is to be dominated by the quadratic term. The three parameters a, b & c enable the expression to achieve at the end of the ramp not only the desired field and slope (zero), but also **zero curvature**. Figure 3 plots two illustrative field profiles.

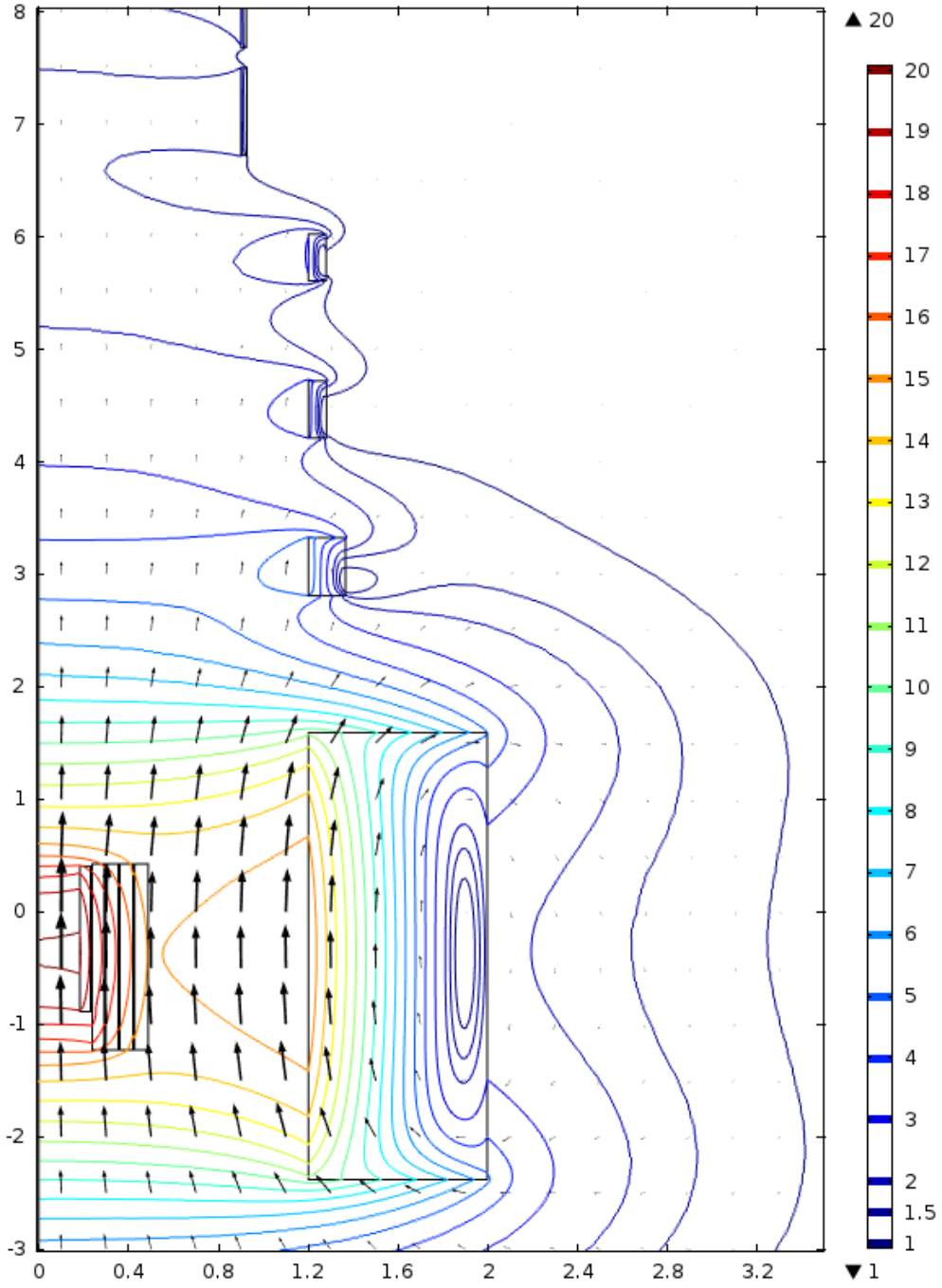

Fig. 2. Coil cross sections, and field magnitude & direction with B(r=0, z) = 20 T at z = −0.375 m, 1.5 T at z = 7 m.

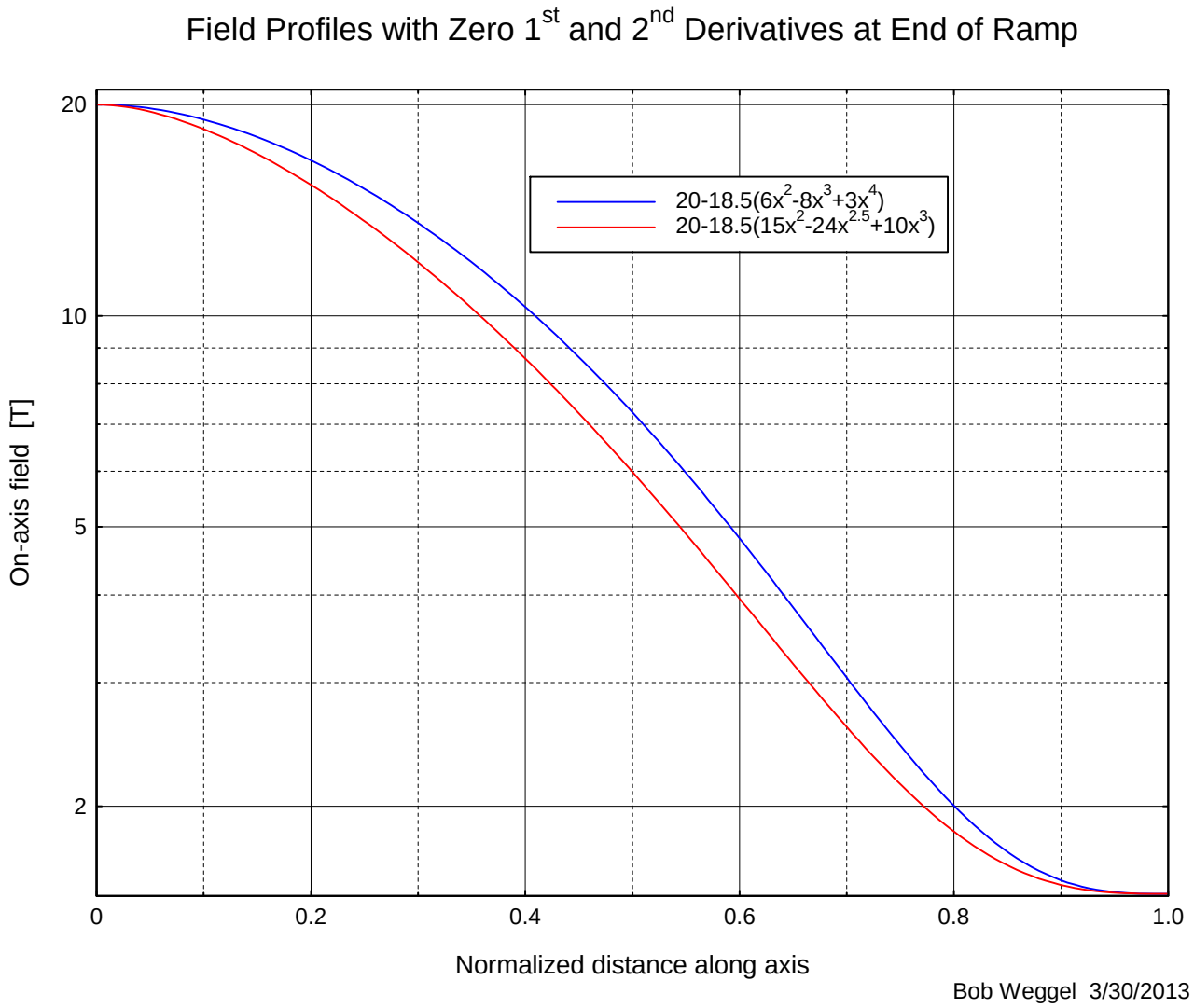

Fig. 3. Illustrative field profiles with zero 1st and 2nd derivatives at end of ramp.